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The development of analytical rubrics: An avenue to assess students' mathematical reasoning behavior

Rohati Rohati [a*], Universitas Pendidikan Indonesia, Department of Mathematics Education, Bandung, 40154, Indonesia https://orcid.org/0000-0002-4865-1326

Yaya S. Kusumah [b*], Universitas Pendidikan Indonesia, Department of Mathematics Education, Bandung, 40154, Indonesia https://orcid.org/0000-0001-5156-9115

Kusnandi Kusnandi [c*], Universitas Pendidikan Indonesia, Department of Mathematics Education, Bandung, 40154, Indonesia https://orcid.org/0000-0002-8900-4563

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Abstract

A rubric is a logical set of criteria that teachers can use to evaluate students to determine criteria for obtaining complex competencies. Therefore, this study was conducted to develop a rubric and assess its effectiveness in evaluating students' mathematical reasoning behavior when solving mathematical problems. This rubric was developed based on four stages: reflecting, listing, grouping, and applying using generic model research design in education. Furthermore, the meta-rubric and expert input results are applied to improve its quality. The results showed that the rubric developed was able to consistently provide more feedback and provide teachers with additional opportunities to optimally assess the mathematical reasoning behavior of the students. This study recommends that there be further studies on the ability of teachers to understand students' mathematical reasoning levels to help students progress from the lowest level of reasoning to the highest level. In the end, students have creative mathematical reasoning abilities.

Keywords: Analytical Rubric, Behavior, Development, Mathematical Reasoning, Meta-Reasoning

[*] ADDRESS OF CORRESPONDENCE: Rohati Rohati, Universitas Pendidikan Indonesia, Department of Mathematics Education, Bandung, 40154, Indonesia

Email address: rohati.fkip@unja.ac.id

1. Introduction

1.1 Conceptual Framework

Reasoning is a discursive activity which involves the justification of closely related claims (Kollosche, 2021). Students normally develop the ability to think logically and reason effectively when studying mathematics (Bronkhorst et al., 2020), and this further aid their ability to think correctly. Moreover, children utilize mathematics to justify arguments and generalize reasoning (Bragg & Herbert, 2018). Reasoning is also explained as the mental process which involves relating existing data to a conclusion while Brodie (2010) defined mathematical reasoning as the process of thinking about and using mathematical concepts. Most of these scholars concluded using relevant arguments that mathematical thinking is inextricably linked to students' capacity to reason effectively. This means this kind of reasoning is a critical skill required to be owned by mathematics students (Adnan et al., 2021).

Numerous experts have also emphasized the importance of reasoning abilities, particularly when it comes to understanding mathematics. For example, (Brown et al., 2020) showed its significance in constructing a mathematical concept and demonstrating its correctness. Reasoning is critical in mathematics because it is an active, flexible, and generative process required by mathematicians and users. It is also used by students and teachers to actively develop and comprehend mathematical ideas (Lam, 2012). Moreover, reasoning has the ability to improve students' understanding of mathematics and enhance the development of their creativity (Hansen, 2021). It is very important to develop students to participate in creative thinking activities (Nogerbek et al., 2022). The critical mathematical reasoning possessed by students requires teachers to analyze several parts of their abilities.

Loong et al. (2018) showed that one of the crucial functions of teachers is to assess the mathematical reasoning abilities of elementary school students. Therefore, this present study focuses on developing a rubric to be used by teachers in evaluating the mathematical reasoning ability of students in junior high schools. This is in line with a previous study by (Davidson et al., 2019) which was used to test classroom materials designed to assist elementary school teachers in planning and assessing their students' mathematical reasoning after which the capability of the teachers to use the materials was examined.

The primary reason for developing this analytical rubric was to provide teachers with a tool to evaluate students' mathematical reasoning. This is in line with Boer et al.'s (2021) definition of the rubric as a tool for feedback and assessment to improve the quality of learning from different perspectives. It is designed to facilitate the consistent assessment of each lesson while reducing the time required to design assignments and formulate appropriate evaluation criteria (Timmermana et al., 2011). A rubric is an invaluable tool to assist teachers to become more efficient and consistent in assessing the works of students while observing their overall behavior. Its usefulness for assessment is associated with its emphasis on the basic principles of equality and fairness (Stevens & Levi, 2005) and the ability to ensure the evaluation of the reasoning behavior of students is systematic and comprehensive, thereby, making it very important in the learning process.

1.2 Related Research

Rubrics play an essential role in authentic assessment regardless of level or discipline (Nkhoma et al., 2020). In the process of developing a rubric, there is a need to embrace ideas and seek to capture the specific dimensions of qualitatively relevant scientific inquiry. This can be accomplished by describing, explaining, controlling, and forecasting behavior and reviewing how these dimensions emerge during the learning process. This present study proposes that a rubric can promote authentic assessment strategies to capture more qualitative aspects of students' mathematical reasoning behavior when solving problems. The major benefit is its ability to establish a framework to demonstrate the behaviors such as the cognitive, affective, and meta-reasoning aspects which are fully described based on the reasoning style. This uniqueness is expected to serve as a benchmark for the efficacy of mathematics instruction in developing the creative mathematical reasoning of students.

The evaluation process is expected to provide the teachers with the platform to assess the overall learning process towards ensuring that the students attain the highest degree of reasoning which is creative mathematical reasoning which further determines their overall behavior in solving mathematical problems. This is in line with the opinion of Danielson & Marquez (2016) that standard test is unsuitable for the assessment of reasoning ability due to the fact that the skills linked to data analysis, formulation and testing of hypotheses, and pattern recognition are better tested using non-traditional methods. Moreover, the observations of students' behavior while solving problems serve as a guide to develop the rubric to assess their mathematical reasoning ability in finding solutions to mathematical problems.

Muir et al. (2008) showed that problem-solving behavior is inextricably linked to performance which has five major components of knowledge acquisition and use, control, trust, influence, and sociocultural setting. This means problem-solvers need to connect their knowledge to the current problem situation and the ability to do this affects their success. This explanation was used as the basis to determine the behavioral aspects of the rubric compiled in this study which include knowledge acquisition and utilization such as the cognitive and metacognitive aspects. Specific attention was placed in the metacognitive aspects due to its status as the meta-reasoning in the reasoning process while the affective aspect consists of confidence and self-confidence.

This indicates the three primary variables used to determine the mathematical reasoning behavior in this study are cognitive, meta-reasoning, and affective aspects. This is due to the possibility of observing the mathematical reasoning abilities of the students cognitively when they solve a problem based on mathematical reasoning indicators. The metacognitive aspect was also included because it has been confirmed to be a primary factor in the problem-solving process (Temur et al., 2019) and it is evident in the observation of more specificity in the mathematical reasoning processes of the students through the concept known as meta-reasoning. According to (Barnes, 2019), metacognitive talents in the context of mathematical reasoning are important to the tenacity of students in solving mathematical issues.

The affective aspect is another component observed to be playing important role in learning mathematics in addition to the cognitive and metacognitive aspects. Its main domains studied include self-confidence and belief. It has been previously reported that the students' justifications for decision while solving problems demonstrate their self-confidence and their beliefs was observed to be more influential than their mathematical understanding (Sumpter, 2013). Moreover, Self-confidence has been understood differently in the mathematics education literature where (Aulia et al., 2021; Moneva & Valle, 2020) explains that self-confidence is a belief in oneself and one's abilities that develops when there is someone who helps them and motivates them to believe in themselves. Foster (2016) also explained mathematical confidence as the student's view of their capacity to obtain satisfactory results as well as the confidence in their ability to deal with mathematical issues.

Standards-based competencies such as mathematical reasoning are notoriously difficult to quantify on a large scale and are frequently disregarded during the assessment process. This teaches pupils that their capacity for reasoning, problem-solving, collaborative work, and creative writing is secondary to facts and processes. This problem can be solved by incorporating rubrics into classroom assessment in order to allow the teachers to ensure the instructions developed meet all requirements and that they have the ability to assess the proficiency level of their students legitimately and reliably (Smit & Birri, 2014). This is possible because formative assessment enables teachers to collect evidence and provide feedback on students' progress during instruction.

Although empirical research on the use of rubrics in education has been conducted in various fields and for various goals, the process of creating analytical rubrics to improve the quality of rubrics, particularly in evaluating students' mathematical reasoning behavior, has received surprisingly little attention. A design-based research study in elementary classes, a rubric for evaluating mathematical reasoning, and other analytical rubrics have all been created in the past (Loong et al., 2018). Additionally, there are rubrics for evaluating problem-solving behavior (Muir et al., 2008) and the capability of reasoning and argumentation (Smit & Birri, 2014). The junior high school kids are the target audience for our unique rubric. The choice of rubric dimensions elements displayed in the indicators further demonstrates the novelty. The rubric focuses on cognitive and metacognitive aspects of reasoning (meta-reasoning) and affective.

Furthermore, reasoning behavior is developed for each reasoning category, including imitative, algorithmic, and creative mathematical reasoning. Students are expected to produce fundamental theories by using mathematical reasoning in solving math problems (Rohati et al., 2021). Students are expected to be able to produce fundamental theories by using mathematical reasoning to solve math problems. The produced rubric is evaluated by developing a meta-rubric, which is then evaluated by a panel of experts. Based on the changes to the meta-rubric, the rubric's quality was enhanced in several ways, making it a helpful assessment tool that teachers can use to gauge their students' mathematical reasoning behavior.

1.3 Purpose of the Study

This study attempts to answer the following research questions:

1. What are the steps to create a mathematical reasoning behavior rubric for junior high school students?

2. What is the quality of the mathematical reasoning behavior rubric designed?

2. Method

2.1 Research Model

The rubric developed has the same structure as the generic model proposed by Mckenney & Reeves (2013). The dimensions of rubric was explored and initial analysis was conducted based on literature review as indicated in Table 1, thereby, leading to the development and design of the analytical rubric to assess the mathematical reasoning behavior based on four key stages (Stevens & Levi, 2005) which include reflecting (stage 1), listing (stage 2), grouping and labeling (stage 3), and application (stage 4). The reflection process is described comprehensively in the construction and design section while the questions to be answered are listed in Table 3. Moreover, the rubric compiled through these stages was evaluated by analyzing the reviews from selected experts.

2.2 Participants

The participants in this study were three experts who were Doctors of Mathematics Education who provided input and assessment of the rubrics that the researchers had developed. In addition, three mathematics teachers who teach at different schools also provided input and suggestions on the rubric that was developed. These participants were selected based on their experience as practitioners, lecturers, and teachers in learning mathematics.

2.3 Data Collections

Research data was collected by first exploring the dimensions of the rubric and preliminary analysis based on the literature review, as shown in Table 1. An analytic rubric is a common lattice rubric regularly used by several teachers to evaluate the works of their students and has been indicated to be the best in providing detailed feedback. Its left column contains the criteria for student work, the top column contains the performance levels, left row describes the dimensions of the aspects of mathematical reasoning to be observed including the cognitive, meta-reasoning, and affective while the top row represents the level scale derived from the orientation of the continuum of mathematical reasoning behavior. The boxes contain specifications for students' mathematical reasoning behavior at each level. It is important to note that analytical rubrics are beneficial for both learning and assessment due to their ability to explicitly describe each criterion in a work (Brookhart, 2013).

Furthermore, after the researcher developed the rubric, input and suggestions were collected from experts to improve the quality of the rubric orally and in writing and assessment through meta rubrics.

Table 1. The Rubric's Dimensions.

Dimensions of Mathematical Reasoning Behavior	Indicator
Cognitive	Draw logical conclusions Present mathematical statements orally and in writing through pictures, diagrams, tables, and graphs. Estimate the answer and the solution process Use patterns and relationships to analyze mathematical situations, make analogies, and generalize Construct and test conjectures Provide counterexamples Compose valid arguments Draw up direct evidence
Meta-Reasoning	Metacognitive monitoring Metacognitive control (Written and oral)
Affective	Belief Self-confidence

2.4 Data Collection Process

Research data collection is done by asking permission and notifying the experts and mathematics teachers regarding the mathematical reasoning rubric developed by the researcher. Mathematics experts and teachers were asked to provide input and suggestions on the rubrics developed and evaluate the rubrics using meta rubrics to assess the overall quality of the rubrics. All notes and input, both orally and in writing, are well documented. It will take two months to complete all data collection processes.

2.5 Data Analysis

The meta rubrics were used independently and are typically unique with the checklists observed to be more convenient and efficient to use. Meta rubric is used to evaluate the quality of the rubric (Kim & Rosenheck, 2020). A back-and-forth switch occurred between the rubric and meta rubric criteria during the evaluation process and the meta rubric was observed to have assisted in refining and polishing the specific details in the rubric. Moreover, the "yes/no" element enables a cursory examination of key aspects of the rubric construction without delving into detail. The evaluation part of the rubric is related to the dimensions, description, scale, overall part of the rubric, fairness, and sensibility. The description of the meta rubric used to evaluate the entire rubric. In addition, peer evaluations (experts in rubric design) are also proposed to refine further the quality of the rubric (Gezie et al., 2012). Inputs and suggestions from mathematicians and teachers were analyzed. In addition, the assessments given by experts and teachers in the meta rubric evaluation are also calculated and tabulated by making frequency and percentage tables. In addition, the direct verbal responses from the experts were also summarized, as shown in Table 7 in the research results.

3. Results

The rubric was developed through four distinct stages with the first observed to involve reflecting on the task and context, the second focuses on outlining the learning objectives and expectations, the third entails grouping and labeling the objectives and criteria, and the fourth emphasizes the application of the objectives and criteria to a rubric grid format. The implementation of these stages in developing behavioral rubrics for mathematical reasoning and meta reasoning is further explained based on four key stages (Stevens & Levi, 2005), which include reflecting (stage 1), listing (stage 2), grouping and labeling (stage 3), and application (stage 4).

3.1 Stage 1: Reflecting

This stage addresses critical questions selected based on the opinion of the researcher and observed to be required during the early stages of developing rubrics (Stevens & Levi, 2005). The student's performance was measured in terms of mathematical reasoning ability which is the focus of this research and the most appropriate recommendation from this stage is to create a new rubric to serve as the foundation to assess students' mathematical reasoning orientation.

3.2 Stage 2: Listing

At this stage, we concentrate on the specific details of the rubric and the specific learning objectives that we hope to see in the final product. What are the expected outcomes of students completed mathematical reasoning behaviors? We then add a description of the highest performance level for each listed learning outcome to our list. Some people prepare for Stage 3 by jotting down the items for their list on sticky notes. Imitative, algorithmic, and creative are all concept words that convey expected performance levels. The expected level of mathematical reasoning behavior is determined by two distinct types of mathematical reasoning: imitative (memorized and algorithmic) and creative.

3.3 Stage 3: Grouping and Labeling

The first step is to define the four entire sections of the rubric, and this involved establishing the parameters to explain the students' mathematical reasoning behavior. In its most basic form, the rubric consists of a description of the measured aspects, indicators for each measured aspect, a specific level of achievement scale (mathematical reasoning behavior orientation category), and task dimensions (details of skills/knowledge for each indicator and behavioral orientation). Moreover, a description of the relationship between students' abilities and the material being taught is also included as indicated in the basic grid format presented in Table 2.

Table 2. Basic rubric grid format

	Scale level 1	Scale level 2	Scale level 3
Dimension 1			
Dimension 2			
Dimension 3			

The rubric is divided into three scales based on the reasoning of students which are imitative, algorithmic, and creative as well as three broad categories of cognitive, meta-reasoning, and affective. Moreover, the analytic nature of the rubric makes it entirely appropriate to use the format without a score as shown in Scale level in terms of categories are indicated in Table 3. It is also important to note that each aspect is presented in a separate table to allow the reader to concentrate on the important details.

Table 3. Initial Rubric of Mathematical Reasoning Behavior

Aspect	Indicator	Category		
		Imitative	Algorithmic	Creative
Cognitive		Performance Descriptor	Performance Descriptor	Performance Descriptor
Meta reasoning		Performance Descriptor	Performance Descriptor	Performance Descriptor
Affective		Performance Descriptor	Performance Descriptor	Performance Descriptor

3.4 Stage 4: Application

This stage was used to complete all the performance indicators based on relevant aspects and categories of mathematical reasoning behavior. The primary criterion is that the students that fall into the imitative reasoning category used the available information and this implies they possibly reserve rules for on-premises data, those oriented towards algorithmic behavior leverage the information provided and select from a variety of rules to reach the final target, while those with a creative orientation have the ability to develop their rules to reach the final target based on the provided information (Kusnandi & Rohati, 2020). It is important to note that each reasoning indicator affects the intended final target and the mathematical reasoning behavior rubric produced is presented in Tables 4, 5, and 6.

Table 4. Rubric of Students' Mathematical Reasoning Behavior (Cognitive Aspect)

Aspect	Indicator	Category of Students' Mathematical Reasoning Behavior: Imitative	Algorithmic	Creative
Cognitive	Draw logical conclusions	Only capable of utilizing the information provided but not yet capable of selecting a set of rules to reach a logical conclusion.	Can deduce logical relationships between number sequences and object configuration sequences based on the pattern and object configuration sequences.	Capable of utilizing the information provided and establishing their own rules to arrive at logical conclusions.
	Present mathematical statements orally and in writing through pictures, diagrams, tables, and graphs	Can only use known data but is unable to select a set of rules to present mathematical statements in the form of pictures, diagrams, tables, and graphs.	Can present mathematical statements orally and write on number pattern material using pictures, diagrams, tables, and graphs.	Capable of establishing their own rules to present mathematical statements in pictures, diagrams, tables, and graphs based on the information provided.
	Estimate the answer and the solution process	Capable of utilizing available data but unable to select rules to predict the answer and the desired solution process.	Make predictions and solve problems involving number patterns and object configurations.	Capable of creating custom rules to arrive at accurate answer predictions and processing solutions based on data provided.
	Use patterns and relationships to analyze mathematical situations, make analogies, and generalize	Can comprehend established patterns and relationships but unable to select a set of rules to analyze mathematical situations, draw analogies, and generalize.	Analyze mathematical situations, draw analogies, and generalize to number pattern material using patterns and relationships (e.g., triangular number patterns).	Can develop own set of rules to analyze mathematical situations, draw analogies, and generalize based on data provided.
	Construct and test conjectures	Capable of utilizing known data but unable to choose a set of rules to construct and test conjectures.	Generate and test hypotheses by arranging numbers into a specified amount using the rules provided for number patterns problems.	Capable of utilizing available data and establishing their own rules to construct and test conjectures.

Provide counterexamples	Capable of utilizing known information but unable to select rules that serve as a counterexample.	Can provide counterexamples to the given problem's incorrect number pattern.	Can set their own rules to provide examples of accurate disclaimers based on the information provided.
Compose valid arguments	Only capable of utilizing the information provided, but not yet capable of selecting a set of rules to prepare valid arguments.	Can develop valid arguments on the material of number patterns and object configuration by explaining the strategy selected and implemented as well as the reasons it worked or not.	Capable of utilizing the information provided to develop their own set of rules to construct valid arguments.
Draw up direct evidence	Only capable of utilizing the information provided but not yet capable of selecting a set of rules to prepare direct evidence.	Capable of utilizing provided information and selecting several sets of rules but unable to prepare direct evidence.	Capable of utilizing the information provided to develop their own set of rules to prepare direct evidence.

Table 5. Rubric of Students' Mathematical Reasoning Behavior (Meta-Reasoning Aspect)

Aspect	Indicator	Category of Students' Mathematical Reasoning Behavior		
		Imitative	Algorithmic	Creative
Meta-Reasoning	Metacognitive monitoring	Have a single rule or strategy to solve non-routine problems and adhere to the strategy even when all indicators show it is incorrect.	Have a single rule or strategy to solve non-routine problems but willing to try other rules or strategies to obtain the correct result.	Capable of developing novel rules or strategies to resolve non-routine problems and obtain the desired results.
	Metacognitive Control (in writing)	Metacognitive control was not observed in writing during the compilation or assembly of the intermediate targets or rules used from the information provided in the questions.	Metacognitive control was observed in writing during the compilation or assembly of the intermediate targets or the rules used but not up to the final target.	Metacognitive control was demonstrated in writing during the process of compiling or assembling intermediate targets or the rules used and obtaining the final target.
		The written responses are not neat and are littered with scribbles, thereby, indicating incorrect steps to the solutions.	The written response is neat but some doodles indicate incorrect completion of steps.	The written response is neat with few scribbles on incorrect completion steps.
		Capable of communicating	Student communicate	Capable of communicating thoughts

	Metacognitive Control (in oral)	thoughts and the results of the reasoning verbally only on a portion of the information provided.	thoughts and outcome of reasoning verbally using several sets of rules selected but was unable to reach the correct conclusion on the final target.	and the outcome of reasoning orally, coherently completely, and systematically, and also provides correct conclusion regarding the final objective.
		Have a single rule or strategy to solve non-routine problems and adhere to the strategy even when all indicators show it is incorrect.	Have a single rule or strategy to solve non-routine problems but willing to try additional rules or strategies to obtain the correct result.	Capable of developing novel rules or strategies to resolve non-routine problems and obtain the desired results.

Table 6. Rubric of Students' Mathematical Reasoning Behavior (Affective Aspect)

Aspect	Indicator	Category of Students' Mathematical Reasoning Behavior		
			Algorithmic	Creative
Affective	Belief	Not sure about the argument presented or selected rule to solve the mathematical reasoning problems.	Convinced by some of the arguments presented or rules selected but unable to reach the correct conclusion regarding the final objective.	Convinced by all the arguments presented or rules selected and reached the correct conclusion regarding the final target.
	Self-Confidence	Have self-confidence as indicated by the provision of answers quickly even when they are not sure the rules selected are correct or incorrect.	Frequently demonstrate confidence when solving mathematical reasoning problems but cannot reach the correct conclusions.	Demonstrate self-confidence and make sound judgments.

3.5Rubric Quality based on Meta-Rubric Evaluation

Table 6 shows the meta-rubric as described in the methodology section with the "yes/no" element observed to have enabled a quick assessment of key aspects of the rubric's construction without delving into the specifics. The three experts and three teachers were involved in the evaluation process including this meta-rubric and the results are expressed as the percentages of 'yes' or 'no' provided by the respondents. Moreover, the three experts also provided qualitative input on the overall content. The average score for the 26 evaluation criteria based on the rubric in the table was 94,88 percent for 'yes' and 5.12 percent for 'no.' These results indicate that the rubric developed is of good quality. In addition, the quality of the rubric is also analyzed qualitatively based on comments and input from experts.

3.6 Revision Following Expert Commentary

The experts evaluated the rubrics against meta-rubrics and provided feedback and suggestions. The inputs provided were used to improve the rubric's appearance and content with significant changes observed after the process as indicated in Table 7 while the summary of input provided for each prevision process is presented in Table 7.

Table 7. Expert Input on the Rubric Design

Revision	Input from Expert
First	The categorization of mathematical reasoning behavior as beginning, developing, consolidating, and extending was changed to imitative, algorithmic, and creative.
Second	The mathematical reasoning behavior is described in terms of cognitive, metacognitive, and affective dimensions. These characteristics were later used to determine each student's type of reasoning behavior including imitative, algorithmic, and creative reasoning.
Third	The indicators of mathematical reasoning were used to show the cognitive aspect of students' mathematical reasoning abilities. The affective aspect was represented by the students' self-confidence and ability to answer mathematical reasoning questions in writing and orally.
Fourth	The imitative, algorithmic, and creative mathematical reasoning need to be investigated further to determine the possibility of grading or classifying them based on levels.
Fifth	The mathematical reasoning indicators to be used in this study and organized into a rubric need to be selected based on the characteristics of junior high school students in line with expert opinions.
Sixth	There should be a clear connection between mathematical and meta-rational reasoning.
Eighth	The classification of the students' mathematical reasoning should be based on the following criteria. Imitative: the ability to select from multiple sets of rules to reach the desired outcome. Algorithmic: capable of utilizing available data and selecting from multiple sets of rules to reach the desired outcome. Creative: capable of creating personal rules to complete tasks based on the information provided.

4. Discussion

The rubric's cognitive aspects describe the behavioral orientation based on mathematical reasoning indicators with the expected level established based on the categories proposed by (Lithner, 2008) which include the imitative (memorized and algorithmic) and creative reasoning. This is in line with the two categories used by Lithner (2008) to explain the types of reasoning students frequently use to complete mathematical tasks which include imitative and creative mathematical reasoning. Moreover, Brookhart (2013) divided performance into different categories of imitation, ordinary or routine, creative, and very creative in an analytical rubric used to assess the creativity of students. However, the discussions with teachers showed that the only three possible orientations for mathematical reasoning behavior are imitative, algorithmic, and creative, and that their students did not appear to have the creative level.

The imitative reasoning has two components which include rote or memorized and algorithmic reasoning with the students observed to have the ability to recall or memorize answers in memory reasoning and recall the procedures to solve the problem in algorithmic reasoning (Lithner, 2017; Sumpter, 2016). Meanwhile, a creative level is when the reasoning is novel, flexible, plausible, and mathematically based (Lithner, 2008; Norqvist et al., 2019). The process of solving mathematical problems, especially non-routine ones, is expected to be creative and the students are expected to be taught how to solve problems creatively.

The rubric's meta-reasoning component is based on the metacognitive theory of problem-solving and since the rubric is designed to evaluate mathematical reasoning behavior, the concept of metacognitive processes is expanded to include meta reasoning. It is also important to note that the metacognitive conceptualization was extended to reasoning and this led to the development of meta-reasoning frameworks. This indicates reasoning is a precursor to meta-reasoning (Morsanyi et al., 2019). Moreover, the reasoning process normally starts with the identification of components and objectives such as the problems presented in the context of learning mathematics followed by the generation of an initial response naturally and finally through the application of analytical processing techniques.

The rubric developed also has affective components which reflect the overall behavior of the students while reasoning mathematically. It is important to point out that a student is cognitively involved in the learning process in terms of attitudes, behavior, and physical abilities. The feelings, emotions, and attitudes all fall under the affective domain (Hoque, 2016) and are critical to the process of learning mathematics. This led (Hannula et al., 2016) to introduce a new era of research on student affective learning in mathematics with the focus on the attitudes, beliefs, and motivation of the students. Moreover, Barnes (2019) focuses on the emotions of students engaged in mathematical reasoning activities. This rubric used self-confidence and belief as the central dimensions of the affective aspects of mathematical reasoning behavior.

This rubric developed shows differences from the rubrics that other researchers have developed. This signifies the rubric is focused on the cognitive, metacognitive (self-control), and affective aspects after which a reasoning behavior is developed for each reasoning category including the imitative, algorithmic, and creative mathematical reasoning with the students expected to produce fundamental theories using mathematic reasoning in solving mathematical problems. The rubric developed was assessed by creating a meta-rubric which was evaluated by a panel of experts. Several improvements were made on the quality of the rubric based on the revisions made to the meta-rubric and this makes it an adaptable and customizable assessment tool. It is important to note that a meta rubric was used to evaluate the rubric described in the findings section of this study (Stevens & Levi, 2005) based on the inputs from experts.

The two research questions were also answered by examining the expert's evaluation of the meta rubric. It was discovered that it is possible to determine the orientation level of the students' reasoning behavior based on the quality of the rubric. The findings also showed that the tool has high quality. Moreover, the rubric was easily evaluated by three experts and three teachers using the meta rubric after which feedback was provided for improvement. It is important to note that the inputs of the teachers were invaluable despite their small number considering the fact that they have experience in using rubrics as a tool for classroom assessment. Furthermore, the rubric developed was used to plan lessons including a sequence of formative assessments and was observed to have communicated what is expected from the students and what should be evaluated by the teacher. The demonstration from the teachers indicates the ability of the rubric to aid the evaluation of students' mathematical reasoning abilities and this was observed to be in line with the findings of previous studies that showed the efficacy of rubrics as an assessment tool (Boer et al., 2021; Nadolski et al., 2021; Timmermana et al., 2011). This implies the rubric developed is useful to provide the guidelines to assess students' mathematical reasoning behavior both for self-assessment and improvement. It also shows that the application of the rubric appropriately and optimally can be used to categorize or classify the mathematical reasoning behaviors of the students at each level in order to improve their capabilities up to the highest level.

5. Conclusion

The rubric was generally created to evaluate the mathematical reasoning abilities of junior high school students and also to ascertain the level of orientation of their reasoning and meta-reasoning behaviors. It was developed through four stages of reflecting, listing, grouping, and applying after which the evaluation of the meta-rubrics and expert input was used to improve its quality. Moreover, the difficulty and more schedule time expected to be used in the development of the rubric were reduced significantly. The findings showed that the rubric developed was able to consistently provide more feedback and provide teachers with additional opportunities to optimally assess the mathematical reasoning behavior of the students. This was observed to have assisted the teachers in improving the ability of each student from the lowest to the highest level towards ensuring the students develop adequate and creative mathematical reasoning abilities.

6. Recommendations

This study aims to develop a rubric to be used in evaluating the mathematical reasoning abilities of students with a focus on the three critical aspects of cognitive, metacognitive, and affective. The rubric was designed using a number pattern material and object configuration to assist teachers in developing mathematical reasoning questions. It is important to note that its format is basic but it is possible to expand its scope to include a variety of materials and grade levels. The rubric can also be used by secondary school teachers to assess the reasoning abilities of their students, especially when working on problems that require mathematical reasoning. Moreover, it is recommended that the study examine the relationship between this rubric and students' responses to mathematical reasoning questions and also analyze the overall effectiveness of the rubric. This study recommends that there be further studies on the ability of teachers to understand students' mathematical reasoning levels to help students progress from the lowest level of reasoning to the highest level. In the end, students have creative mathematical reasoning abilities

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