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Students' ability to solve function problems by using *handep* cooperative learning model

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Abstract

The cultural dimension can be a solution to identify the difficulties by learning about relations and functions. This study aims to describe the students' reasoning function concept, the solution variations on open-ended problems related to domain–codomains and function formulas, and to examine the effect of the *handep* cooperative learning model on the ability to solve function problems. This research used a quasi-experiment method involving 68 students. Data were collected through an essay test and analytical rubric. The effect of the model was tested on the problem-solving ability of functions by using an analysis of variance. Consequently, most students have good reasoning of the function definitions and the domain–codomain terms perfectly, very varied solutions in solving open-ended problems for domain–codomain and function formulas. The model has a significant effect on mastery of function concepts. The steps of the *handep* cooperative learning model supported the students' reasoning.

Keywords: Cooperative Learning, Domain-Codomain, Function Formula, *Handep,* Problem-Solving, Reasoning.

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1. Introduction

The term 'Function' has a strategic position in mathematics that needs to be well understood to study advanced mathematics, for example, calculus and trigonometry. Conceptual understanding and problem-solving skills are validated while learning functions in schools. Indonesian junior high school students learn the concept of function using the discovery learning model, which enables students gain mastery in mathematical reasoning and problem-solving skills. Students gain understanding on concepts such as definition, recognition, interpretation and problem-solving formed when they study about functions (Panaoura et al., 2017).

However, there are some challenges in understanding the concepts of relations and functions, for example, students still misinterpret symbols in functional equations (Sajka, 2003). Dubinsky and Wilson (2013) stated that students have difficulty in distinguishing functions from non-functions, understanding one-to-one properties, function representation, and notation. Students also have difficulty in problem-solving relations and functions. Jupri et al. (2014) and Jupri and Drijvers (2016) found that students still have difficulties in identifying problem situations and forming algebraic expressions, symbolizing, making mathematical models, and applying arithmetic operations.

Doorman et al. (2012) and Nitsch et al. (2015) developed a textbook to understand functions better using the Algebra Arrows tool, but not all students use this tool for learning functions. This is in contrast to a book written by Jin and Wong (2015) in Chinese to facilitate students' learning capabilities and also to make the basic concepts of algebra easy to understand. Parra and Trinick (2018) opined that indigenous languages play an important role in learning mathematics, especially in New Zealand. Other studies found that students' cultural background influences their mathematical thinking (Aikenhead, 2017; Louie, 2018; Yee & Bostic, 2014) and their inclination to learning design on mathematical problem-solving (Clivaz & Miyakawa, 2020; Perrenet & Taconis, 2009). The results indicate that the technology of learning design which is integrated into the cultural aspect has an effect on the learning process which functions as the basic concept of algebra.

Although students learn through collaborative strategies which are believed to have supported the problem-solving process, this method does not facilitate all students to learn well. This is because teachers still have difficulty implementing cooperative learning models, such as reports (Buchs et al., 2017) that more than 40% of teachers occasionally use cooperative learning. These challenges arise because the implementation of cooperative learning requires trained social skills (Gillies & Boyle, 2010), a collaboration process that in turn influenced by the cultural dimensions of students (Louie, 2018) and cultural transition (Corriveau, 2017).

The people in Indonesia have a cooperation culture base on tribes. Namely "*sambatan*" for the Java tribe, "*marumba*" for the Bali tribe, "*handep*" for the Dayak tribe. The cooperation mechanism is used to formulate the syntaxes of the cooperative learning model. H*andep* cooperative learning model has been formulated based on *the handep* cooperation mechanism. This study analyzed the effectiveness of the cooperative learning model that was developed based on the cooperation culture of the Dayak tribe in Indonesia. How the students' reasoning and ability to solve functional problems that are formed in students minds? How does it affect the function's problem-solving ability compared to the discovery learning model?

1.1. Conceptual framework

The research results showed that the cultural dimension is a phenomenon in mathematics learning, specifically in understanding concepts and solving function problems. This indicates that learning design can integrate cultural aspects into lesson plans. The social life of the Dayak tribe in Indonesia incorporates a mutual cooperation culture, namely *handep*. *Handep* is a mutual cooperation between two or more people, which is carried out as one person provides assistance to others (*andep*) and that person receives help from another person (*andep*) who has been helped. The essence of *handep* cooperation mechanism is as follows: *first*, the people reflected what their needs or problems are in order to get the job done, *second* they reveal what problems need other people's help, *third* people who want to help others happily and *fourth* people who have agreed to help each other. This collaboration mechanism inspired the design of learning model, the *handep* cooperative learning model.

The *handep* cooperative learning model was developed based on the Dayak culture of mutual cooperation which was used to facilitate the students to learn the rational exponents concept in high school (Demitra & Sarjoko, 2018). The learning model framework was built through the integration of *handep* cooperation mechanism and metacognitive questioning strategies to facilitate students' reflective thinking. Metacognition in mathematics learning has an important role in improving students' reasoning (Lestari & Jailani, 2018). Tachie (2019) recommended the use of metacognition skill to facilitate problem-solving. Mevarech and Fridkin (2006) had found that the use of metacognition question strategy can facilitate students reasoning to solve the problem of algebraic equation. Furthermore, the metacognition questioning strategy is used in the *handep* cooperative learning model to guide students to make reasoning on the involvement function.

The *handep* cooperative learning model is described as follows (Demitra & Sarjoko, 2018). The teacher forms a team of three or four members and then works in teams across the syntax of the learning model. The syntaxes of the model are as follows: *first*, student to understand and think the way of mathematical problem-solving, individually mediating throughout metacognition questioning strategy reflected and defined the difficulty. *Second*, all students present each other's difficulties in the team. *Third*, team agreed to solve the individual difficulty together. *Fourth,* all members help another member of the team who has difficulty in solving the problem, and then a member returns to help other members in the team. When students use reflective thinking to solve the math problems, their thinking is facilitated by using metacognitive questions strategy on the worksheets.

1.2. Related research

The research related to algebraic thinking ability has been carried out for years. According to Tajudin and Chinnappan (2016), the domain of reasoning of algebra including equations, formulas and functions is a part of higher-order thinking skills. And also, Panaoura et al. (2017) throughout the confirmatory research, found out four dimensions comprising the conceptual understanding of function definition, recognition, interpretation, and problem-solving. The other finding of research such as the ability to translate functional relationship thinking is studied in (Blanton et al., 2017; Chimoni & Pitta-Pantazi, 2017; Pitta-Pantazi et al., 2020; Wilkie & Ayalon, 2018). The translational skills consist of five competencies mastered during the functions studied, namely the ability to translate from graft to algebraic equations, graft to the numerical tables, graft to situational descriptions, numerical tables to algebraic equations, and situational descriptions to algebraic equations (Jupri & Drijvers, 2016; Nitsch et al., 2015).

Students can construct their reasoning on function definition related to the function property, in which the function properties are the elements that make up the definition of a function. Based on his research related to definition of function, De Bock et al. (2017) suggested that function properties are important to learn to embed the images of function. The strategy for exploring students' reasoning on function concept use example and non-example of function. Dubinsky and Wilson (2013) focused on students' learning of the concept of function using the approach of Action-Process-Object-Schema theory on students' ability to recognize the concept of function. The research finding is that high school students make correct choices of examples in function and non-function successfully.

Research related to the effectiveness of learning on the function concept mastery and problem solving was carried out by Kusumaningsih, et al. (2018), that there is an interaction between multiple representations using a realistic approach to improve the algebraic thinking ability. Lestari and Jailani (2018) also design the learning using a metacognitive strategy integrated into a cooperative learning context significantly can enhance mathematical reasoning. Students reasoning in mathematics which is composed of making a conjecture, providing an argument, and observing patterns can be improved using a metacognitive strategy through collaborative learning.

The research related to learning to enhance mathematical problem-solving skills and metacognitive skills has been carried out by Tachie (2019), who found that the development of mathematical problem-solving skills can teach using metacognitive strategies, such as task analyses, planning, monitoring, checking and reflection. Pennequin et al. (2010) found that metacognitive skill training enabled the lower achiever to make progress in mathematics problem-solving. Kramarski and Mevarech (2003) explained that metacognitive training uses a metacognitive questioning strategy in setting cooperative learning significantly effective to facilitate the students' graph interpretation with various explanations based on students' reasoning. Two dimensions of mathematical explanations of graph interpretation have been investigated such as fluency and flexibility. Mevarech and Fridkin (2006) showed that the use of metacognitive question strategy can significantly facilitate students during solving the function maximum and minimum problems.

Clivaz and Miyakawa (2020) explained that different countries have different styles of classroom management for mathematics learning and problem-solving lessons. The cultural factor influenced the mathematics learning system. Demitra and Sarjoko (2018) investigated the effect of *handep* cooperative learning model on the problem-solving skill of rational exponent. The problem-solving skills of students who learn using a cooperative *handep* model are better than those of problem-based learning.

1.3. Purpose of study

This study focuses on the effectiveness of the *handep* cooperative learning model on students' reasoning and skill of function problem solving which has been carefully taught to junior high school students. The study aimed to find an explanation as follow of, *first*, the students' reasoning to distinguish functional from non-functional concepts. *Second*, to describe the variation of students' solutions to solve co-domain open-ended problems. *Third*, the variation of students' solutions to solve open-ended function formula problems. And *fourth*, the effect of *handep* cooperative learning model on the function problem-solving skill.

2. Method

2.1. Research model

This research used quasi-experiment method using the pretest–posttest non-equivalent control group design. Subsequently, students in the experimental group learn to use the *handep* cooperative learning model, while those in control group learn to use discovery learning. Discovery learning model has been used to mathematics learning in Indonesian junior high school, so this model can be set as a learning model for the control group.

2.2. Participants

Participants involving in experiment are teacher and student of a junior high school. The populations are 205 students. Samples were collected through cluster random sampling, where power was 0.80 and effect size f = 0.35, at a risk of α = 0.05. A sub-sample of 34 students was obtained for each experimental and control group, thereby forming a total number of 68 samples.

2.3. Data collection tools

Data were collected through a function problem essay test and analytic rubric of mathematics problem-solving. The essay test of function problem-solving has a Cronbach Alpha reliability coefficient of r_{xx} = 0.81. The function essay tests including indicators and problems are presented in Table 1. Analytic rubric of mathematics problem-solving has been constructed by Charles et al. (1994), which is presented in Table 2.

Table 1. The essay test of function problems

No.	Indicators	The function problems
1.	The ability to distinguish between functional and non-functional concepts	Which one of the ordered pairs is a function or non-function? State the reason! The ordered pairs | Function/non-function [a] | Reason {(3,4), (5,5), (5,6)} | | {(a. 6), (b,6), (c,7)} | | {(Amin, 37), (Budi, 37), (Risma, 36), (Vera, 38)} | | [a] Select one of the two alternatives
2.	The abilities to solve the problem of the domain and codomain	A function *f* (*x*) = 2*x* − 4. Determining domain and codomain of that function, if the domain element of integer. Write your answer as you can do!
3.	The abilities to solve the problem of the function formula	Sukri cooked the meatballs, that children so like it. Through the 3 kg of meats, he gets 100 meatballs. The relation of 'the number of meats and the number of meatballs', state as a function of *f* (3) = 100. Reach the true function formula in some equations form, which fulfils that function, as you can!

Table 2. The analytic rubric for assess mathematical problem-solving skill

Aspects	Level of skills	Scores
Understand the problem	Complete understanding of problem.	2
	Part of the problem misunderstood or misinterpreted.	1
	Complete misunderstanding of the problem.	0
Planning a solution	The plan could have led to a correct solution if implemented correctly.	2
	Partially correct plan base on part of the problem being interpreted correctly.	1
	No attempt, or totally inappropriate plan.	0
Getting answers	Correct answer.	2
	Copying error; computational error; the partial answer for a problem with multiple answer.	1
	No answer or wrong answer based on an appropriate plan.	0

2.4. Data collection process

Data were collected by examining the students using the essay test of function problem-solving. Data of students' problems-solving skills were collected throughout pretest and posttest, that is given on students of the experiment group and control group in the classroom. And then, two raters assess students' solution to each problem by using an analytic rubric, to get scores of function problem-solving skills. The scores given by two raters were validated using the inter-rater reliability coefficient of $r_{xx'}$ = 0.90 (Cohen, 2017; Jones et al., 2014).

2.5. Data analyses

The data were analysed through the following steps: *in the first step*, the students' answer sheets for three questions of the essay test were analysed one by one qualitatively. The students' solutions to questions were analysed by grouping the solutions of each question to get the patterns of reasoning against the definition of function and skill of function problem-solving. *In the second step*, the effect of the *handep* cooperative learning model on the function problem-solving skills by using the analyses of variance (Anova).

3. Results

The use of the *handep* cooperative learning model in the learning functions has influenced students' reasoning and ability to solve the open-ended function problems. There are three main findings that describe how student reason and solve problems on the concept of function. The *first* is the reasoning ability to distinguish between function and non-function. The *second finding is* the ability to solve open-ended domain-codomain problem, and the *third* is the ability to solve the problem of open-ended function formulas, while the *fourth* is the effect of learning on the function of reasoning and problem solving.

3.1. The reasoning ability to distinguish function and non-function

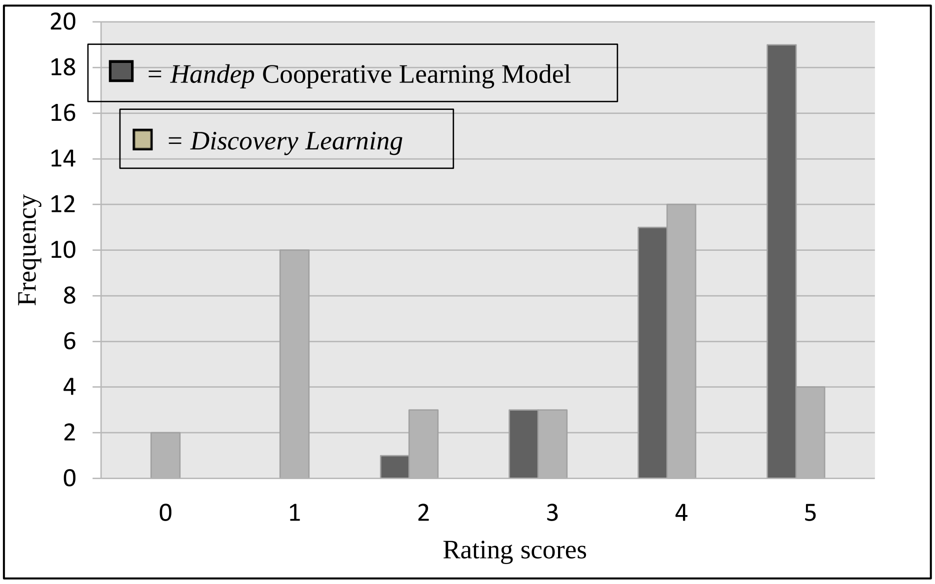
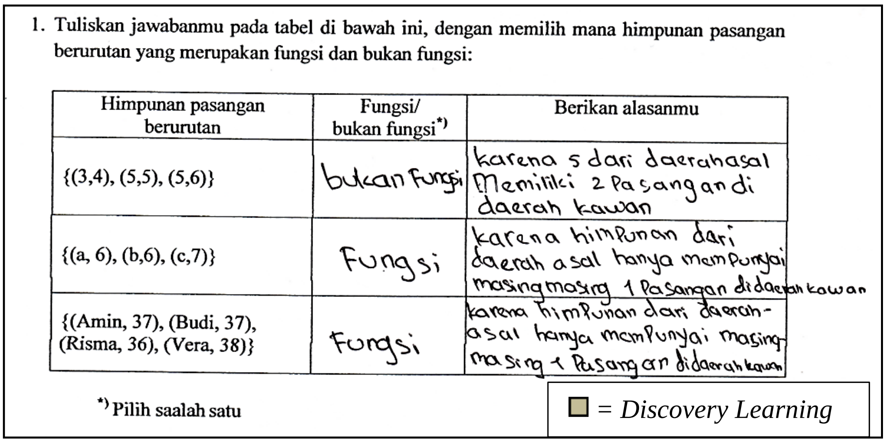


Figure 1. The distribution of scores to distinguish function and non-function

The *handep* cooperative learning model is able to establish students' reasoning to distinguish which set of ordered pairs is a function or non-function. The number of students who are able to distinguish a function and non-function of the experimental group was 88% with a control of 47%, as presented in Figure 1. The students' reasoning produced the right choice of set of ordered pairs for function and non-function, through the reasons given.



1. Tuliskan jawabanmu pada tabel di bawah ini, dengan memilih mana himpunan pasangan berurutan yang merupakan fungsi dan bukan fungsi:

Himpunan pasangan berurutan	Fungsi/ bukan fungsi*)	Berikan alasanmu
{(3,4), (5,5), (5,6)}	bukan Fungsi	karena 5 dari daerah asal Memiliki 2 Pasangan di daerah kawan
{(a, 6), (b,6), (c,7)}	Fungsi	karena himpunan dari daerah asal hanya mempunyai masing masing 1 Pasangan di daerah kawan
{(Amin, 37), (Budi, 37), (Risma, 36), (Vera, 38)}	Fungsi	karena himpunan dari daerah asal hanya mempunyai masing masing 1 Pasangan di daerah kawan

*) Pilih saalah satu

= *Discovery Learning*

Figure 2. The students' reasoning for distinguishing function and non-function

Figure 2 presents a snippet of students' answers who can distinguish sequential forms according to function and non-function. According to students' argument that the ordered paired set of {(3, 4), (5, 5), (5, 6)} is non-function, because the number element '5' of a domain has two pairs in the codomain. Meanwhile, the two ordered pairs are functions of {(a, 6), (b, 6), (c, 7)} and {(Amen, 37), (Budi, 37), (Risma, 36), (Vera, 38)} because each codomain element has its pair. The findings of students' reasoning on the problem of distinguishing functions and non-functions from a set of ordered pairs are represented in Figure 2**.** When students' reasoning sentences were analysed, the formulation is based on the definition of relations and functions. In the textbooks, function *f* is a process (rule) from a set *A* to *B*, in which each element of set *A* matches one exactly to set *B*. Set A is the field of definition or domain of function *f.*

According to Table 3, there are four levels of students' reasoning based on their sentences. The table also shows the formation of students' understanding of the function definitions in the variants of the four perfection levels. The students' reasoning reflected their understanding of function definition. Hence, the sentences were analysed to find their reasoning patterns. R^1 presents the definition of the most perfect reasoning level. The data showed that students gave their definition based on their understanding of the function requirements, before it was synthesized with the concepts of domain and codomain. The three indicators of function that have been mastered by students are presented at this level, namely (1) function requirements, (2) function definitions, and (3) domains and codomains.

Table 3. The students' reasoning deducted from all their sentences

Variation of reasoning		Formation of reasoning	Reasoning ability level
Non-function	Function		
Because one domain element corresponds to two codomains. Because element 5 has been paired with elements 5 and 6 co-domains. Because element 5 in the domain has two element pairs to the codomain.	Because each domain element has only one correspondent to one element in the codomain. Because all domain elements have pairs and only a pair in the codomain. Because the set of one domain codomain and only one pair in the codomain.	Reasons were established in order to apply students' understanding of the definitions and terms of functions to the concepts of domains and codomains perfectly.	R^1 Very good
Because there is one element that has more than one ordered pair element.	Because each member of the set {a, b, c} has only one ordered pair {6,7}.	Reasons were established in order to apply students' understanding of the definitions and terms of functions, but they were unable to relate them to the concepts of domain and codomain perfectly.	R^2 Good
Because 5 has two friends in the codomain.	• Because a, b, c, have a friend. • Because Amin, Budi, Risma, and Vera have friends in the codomain.	The reasons formed during the application of students' understanding of the definition of a function with incomplete sentences show a	R^3 Poor

		reflection of the definition of a function that is less than perfect.	
Because the element set 5 has more than one friend (5, 6).	Because the set of 5 only has one friend.	The stated reason is applied using a function requirement, that is, every element of set A corresponds to one of the elements of set B, imperfectly.	R^4 Very Poor

R^2 shows a good level where students apply the definition and requirements of function, but are not able to use the terms domain and codomain in their reasoning. The two dimensions in this concept are (1) function requirements and (2) function definition. The poor level recorded in R^3 is a reflection of the students' inability to define only one function concept completely. Furthermore, the very poor skill level of R^4 entails functional requirement which states that each element of set A corresponds to an element of set B. At this level, students focus their attention on the domain element that corresponds with codomains. However, they are not able to use the term domain and codomain in their reasoning. Students receive only one dimension of the concept of function, namely the function definition at level R^3 and the function requirements at level R^4. R^3 and R^4 indicate that students think the construction of the concept of the function set is not complete, but this level reduces the number of students.

3.2. Student solution of the domain-codomain open-ended problem

In the experimental group, students were rated from 5 to 8. Based on the frequency, 15 students scored 8 with a percentage of 46.88%, while 2 students scored 5 with a percentage of 5.88. This means that the *handep* cooperative learning model is able to facilitate student's learning process to achieve the best skills in solving domain-codomain problems. Mastery learning is higher than students in the control group. Students create perfect solutions when solving domain–codomain problems. The frequency distribution of students' answer assessment scores for solving domain and codomain problems is as shown in Figure 3.

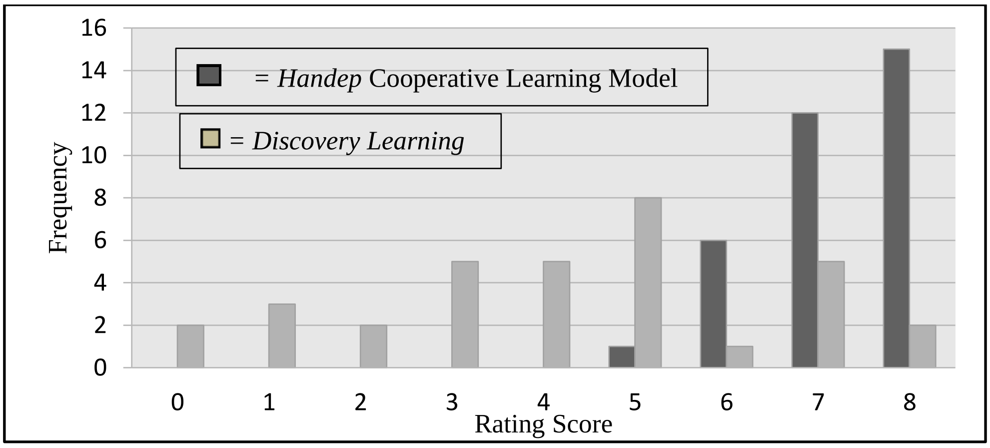


Figure 3. The scores distribution of the domain-codomain.

Furthermore, the perfect solution has been described in the following important findings. First, students generate fourteen variations of codomain–domain open-ended problem solutions, by substituting each domain element into the function $f(x) = 2x - 4$, and then find the codomain value. Second, this solution has been produced by students who study with the *handep* cooperative learning model. The findings of the variation of students' solutions to solve the domain-codomain open-ended problem are shown in Figure 4.

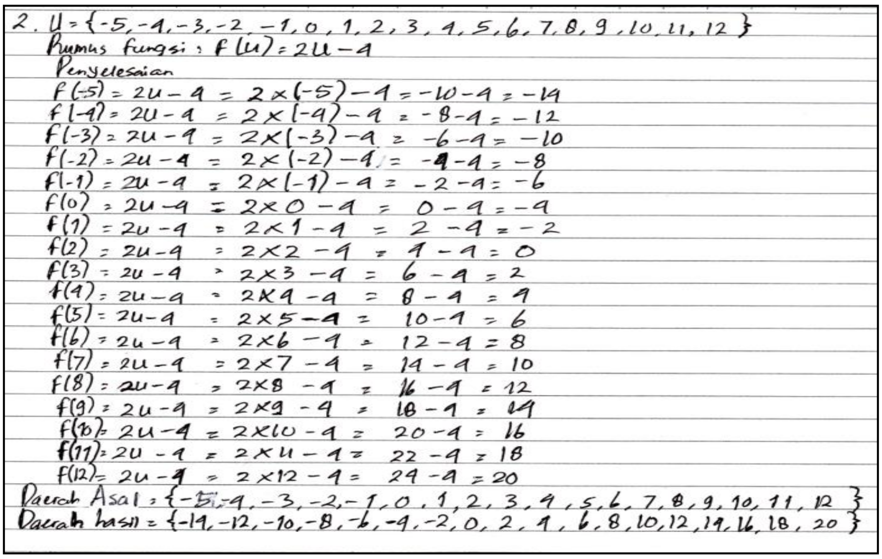


Figure 4. The variation solutions of domain-codomain open-ended problem

At the early stage, students determine the domain of {−5, −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. Afterward, they substitute the domain values into $f(x) = 2x - 4$ *to* get the codomain of { −14, −12, −10, −8, −6, −4, −2, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20}. Third, students' solutions show that they have understood the symbolic representation of equation $f(x) = 2x - 4$.

3.3. The open-ended problem-solving ability of function formula

The finding is the frequency distribution of the experimental group students' scores collection tendencies at rating values of 7 and 8. This means that almost all of students who have studied through the *handep* cooperative learning model are able to solve open-ended problems related to function formulas perfectly. Meanwhile, the scores in the control group spread from 0 to 8, which indicates that some of students' abilities in solving problems related to function formulas are still lower than the experimental group. The frequency distribution of scores is present in Figure 5, and students answer present in Figure 6.

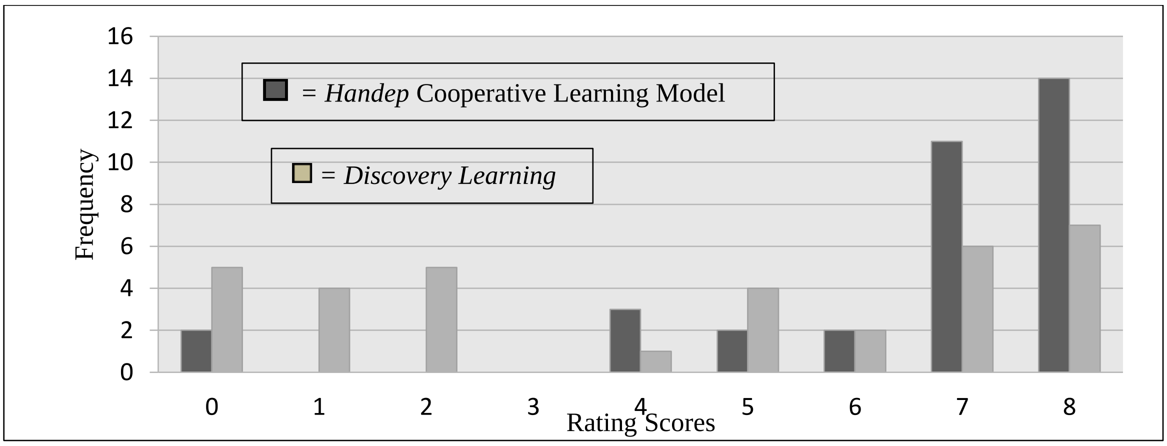


Figure 5. The scores distribution of solve the function formula open-ended problem

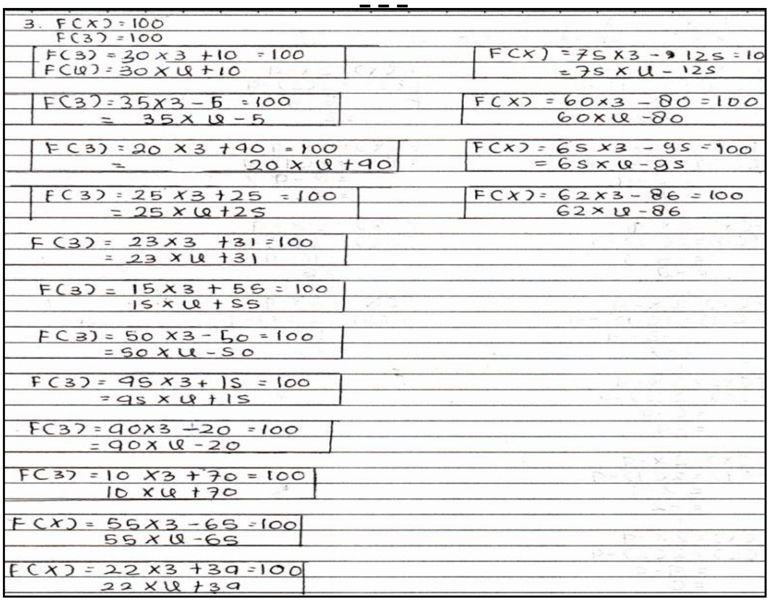


Figure 6. The variation solutions of the function formula open-ended problem

According to Figure 6, the variation of students who were taught by using the *handep* cooperative learning model was greater, compared to those taught by direct learning. The function formulas that have been formed in various coefficients of variable *x* and constant *b*, produced by students are based on the general equation $f(x) = ax + b$ which satisfies the equation of function $f(3) = 100$ *expressed* in the form of coherent logical thinking. It is also seen that students know the questions that was asked. These results indicate that students have rules in constructing various equations that meet the function formula $f(3) = 100$ and also understood algebraic expressions as function formulas.

3.4. The learning effect on the reasoning and problem-solving of function

The effect of the *handep* cooperative learning model on the problem-solving ability of functions was analysed by using Statistical Package for the Social Science Version 17.00. Figure 7 presents the average pretest value of the experimental group of 7.15 with a standard deviation of 2.03, while the posttest score has an average of 18.06 with a standard deviation of 2.63. Meanwhile, in the control group, the average pretest score was 6.50 with a standard deviation of 1.93, and the posttest average was 11.88 with a standard deviation of 3.17. These values indicate the effectiveness of all differences in problem-solving functions in students who are taught by the *handep* cooperative learning model and discovery learning.

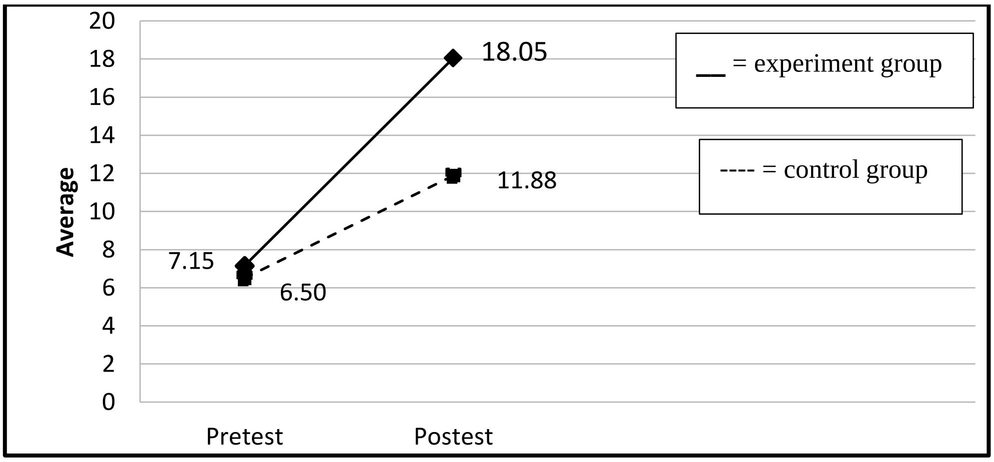


Figure 7. The average score of the ability to solve of function problems

Table 4. The homogeinity test of variance

	Levene statistic	df1	df2	Sig.
Pretest	1.18	1	66	0.28
Posttest	3.22	1	66	0.08

The preliminary test of the fulfilment of the homogeneity assumption is presented in Table 4, where the Levene Statistic is the ability to solve functional problems in both the pretest (1.18, $p > 0.05$) and posttest (3.22, $p > 0.05$), hence, the variance is homogeneous. The assumption of homogeneity being met means that ANOVA needs to be used to examine the effect of cooperative hands-on learning and discovery learning on the ability to solve functional problems.

Table 5 presents the value of $F = 3.09$ on the pretest line with a significant level of 0.08 ($p > 0.05$), which means that the initial ability is the same in solving function problems both in the experiment and control. The results of the initial ability test through the pretest score showed that students in both groups had the same initial ability to function. The ability of students to complete the functions learned through the *handep* cooperative learning model was significantly different from discovery learning ($F = 68.81$; $p < 0.05$). This means that the model has facilitated a good understanding of students' function concepts; however, it was able to achieve function reasoning, solve domain–codomain problems, and function formulas.

Table 5. The ANOVA of the model effect on function problem solving

Dependent variable: posttest	Tests of between-subjects effects					
Source	Type II sum of squares	df	Mean square	F	Sig.	Partial Eta squared
Corrected model	674.96[a]	2	337.48	39.39	0.00	0.55
Intercept	649.60	1	649.60	75.81	0.00	0.54
The model of learning	589.61	1	589.61	68.81	0.00	0.51
Pretest	26.43	1	26.43	3.09	0.08	0.05
Error	556.98	65	8.57			
Total	16,472.00	68				
Corrected total	1,231.94	67				

[a] $R^2 = 0.55$ (adjusted $R^2 = 0.54$).

4. Discussion

Handep's cooperative learning model is able to improve students' ability to solve functional problems significantly. The ability to solve function problems is related to a conceptual understanding of the properties of functions and formulas. Furthermore, the problem-solving ability of students who were taught by using the *handep* cooperative learning model was higher than those using discovery learning model. This is relevant to (Louie, 2018) finding that local community culture influences students' mathematics thinking and (Aikenhead, 2017) that is able to reduce the problems inherent in conventional school mathematics.

The research finding of students' abilities can improved by using *handep* cooperative learning model. *The first*, students are able to distinguish which set of ordered pairs are functions or non. Students can define the meaning of function, throughout their reasoning of function and nonfunction. The reasoning pattern reflected perfect reasoning level, which students gave reason base on definition. This finding are relevant to the findings of (Dubinsky & Wilson, 2013) students are able distinguish which set of ordered pairs are function or non-function. The ability to distinguish the case of a function or non-function is a characteristic of higher-order thinking skills (Tajudin & Chinnappan, 2016).

Second, students can make solution in some variation to solve the open-ended problem to find out domain and codomain of function $f(x) = 2x - 4$, because most of students has an understanding of symbolic representation of function. According to Panaoura et al. (2017) symbolic representation is the first factor in a meaningful understanding of function definition which refers to (Kusumaningsih et al., 2018) as a kind of multiple representation of algebraic thinking. An important conclusion has been drawn that students have a good understanding of the domain concepts, codomain, and function equation. Students then skillfully substitute each member of the domain into the equation and use arithmetic calculations to find the final solution or codomain. The ability to substitute the value of x in an equation into $f(x)$ is functional thinking. Functional thinking abilities and computational processes described by (Sajka, 2003; Wilkie & Ayalon, 2018) has been developed through the *handep* cooperative learning model.

Third, students can make some variation of solution to solve the open-ended problem of function formula. It means most students have good understanding of algebraic expressions. These results indicate that students have rules in constructing various equations that meet the function formula $f(3) = 100$ and also understood algebraic expressions as function formulas (Sajka, 2003). Furthermore, students use external representations to express simple functions in different forms (De Bock et al., 2017) and create equations using functional thinking (Wilkie & Ayalon, 2018) from these equations.

The research findings mentioned above can be explained from the aspect of the advantages of this model. The *first* advantage is that the *handep* cooperation stages facilitates students to work independently and support them in reflecting on their difficulties. In this way, students will better understand their sense of difficulty when they learn function concepts and solve function problems. Students make two complementary mathematical reasonings in two ways, one focusing on the final answers, and the other on written explanations provided by students to justify their mathematical ideas. The study of Demirel et al. (2015) found that reflective thinking affects mathematical thinking to solve related problems.

Meanwhile, the reflective thinking mediated through metacognitive questioning strategies integrated with *handep* cooperation learning to solve functional problems are able to support students' reasoning processes. Lestari and Jailani (2018) and García et al. (2016) showed that the reflective thinking process, which involves mediation through metacognitive thinking question, is able to improve the reasoning process in understanding mathematical problems. Kramarski and Mevarech (2003) and Smith and Mancy (2018) report similar finding, metacognitive training strategy integrated into cooperative learning improves mathematical reasoning.

The *second* advantage is the hand-held collaboration stage include individual problems are presented to other members of the group in turns, rotation agreements help members in the group, and supports be given in a collaborative manner to solve functional problems. The cognitive burden of individual students is lightened and the knowledge sharing during the process of solving individual problems together in groups is carried out. When students have difficulty in understanding the material and solving problems, the other members explain to their friends to enable the group members who have difficulty to complete their work. The process in the *handep* cooperative learning model facilitates collaboration, sharing of knowledge, and promotes peer tutors. Agani (2021) and Edwards and Jones (1999) stated the collaboration classroom have influence on students' math performance. Kutnick et al. (2017) also found that student work groups are able to increase teacher-student interaction (teacher-peer-tutorial) and also improve students' skills.

5. Conclusion

The model has facilitated student's learning process to achieve the best skills for domain-codomain problem-solving. Based on the result, most of students are able to create perfect variation solutions of open-ended problems in domain-codomains. Students understand the symbolic representation of the functions, their functional thinking is well developed, and they understand the concept of domain-codomain and function equation. The model also facilitates students to solve the open-ended problem from function formulas, helps them to make various large equations that fulfil the function $f(x)$, and understand the concept of the variable coefficients. This shows that students have a good understanding of algebraic expressions, and mastery of functional thinking.

6. Recommendation

Generally, the model of learning developed based on the cooperation culture influences students' thinking about the concept of function. The implication of this finding is that teacher needs to be skilled in metacognition questioning strategies that are integrated into the *handep* cooperative learning syntax. Metacognitive questions need to be trained on students to facilitate their reflective thinking about the concept of function and solve open-ended functional problems. Further research to complement teaching materials, it is necessary to develop student worksheets containing metacognitive questions.

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