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High school students' attitudes towards geometry: An exploratory factor analysis

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Abstract

While there are several scales that measure students' mathematical attitudes, few are focused on measuring students' attitudes towards geometry. Some of the scales present in literature measure different dimensions of students' attitudes towards geometry, such as their enjoyment of the subject, motivation to learn it, and perception of the usefulness of learning geometry. In the present study, we present an instrument that can be used to measure students' attitudes towards geometry by focusing on four factors: (1) students' enjoyment, (2) perception of the usefulness of learning geometry, (3) negative (or positive) factors underlying the learning of geometry, and (4) students' motivation to learn geometry. The instrument has been tested on a sample of 242 Italian high school students. An exploratory factor analysis has shown that the instrument is valid and the reliability of the instrument is high.

Keywords: geometry, attitude, PCA, EFA, high school.

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1. Introduction

Geometry is an important branch of mathematics (Abdullah & Zakaria, 2011) which is present in our everyday lives (Graumann, 2011) and has had a profound impact on the development of humanity (Vargas & Araya, 2013). Due to its importance on the development of modern society, mathematics teachers have the responsibility to present geometry to students correctly, precisely, and holistically, starting from primary schools (Bayrak et al., 2014; Sinclair & Bruce, 2015; Marchiş, 2008; Rolet, 2003; Boo & Leong, 2016) or even kindergartens (Casey et al., 2008; Zaranis & Synodi, 2017; Huleihil & Huleihil, 2011). The teaching of geometry is important because students learn how to justify their answers (Bayrak et al., 2014), as well as how to prove theorems (Herbst et al., 2010; Harel, 1999). Moreover, the teaching of geometry develops students' abilities of visualizing, critical thinking, intuition, perspective, problem-solving, guessing, and deductive thinking (Jones, 2002; Jones et al., 2012; Juperi, 2018).

According to the findings of several studies (Mogari, 1994; Turk & Akyuz; McAndrew et al., 2017; Ünlü & Ertekin, 2017), students' attitudes towards geometry significantly impact their achievements and learning. It has been stated that students will attain better results in geometry when they have a positive attitude towards the subject (Abdullah & Zakaria, 2011; Nichols, 1996; Liu et al., 2018). To measure students' attitudes towards geometry, specific instruments which have been validated and used in other studies have been proposed (cf. Abdullah & Zakaria, 2011; Mogari, 1994; Utley, 2004, 2007; Tsao, 2017; Salifu et al., 2021). In the literature, however, we have not been able to identify instruments measuring Italian high school students' attitudes towards geometry.

1.1. Factors underlying geometry attitudes

Learning and teaching geometry is considered to be one of the most difficult tasks in mathematics (Adolphus, 2011). A number of studies have investigated the factors underlying students' and teachers' difficulties in teaching and learning geometry. Students' attitudes towards geometry and, in particular, their willingness and readiness to learn it, have been identified as one of the factors than negatively impact students' attainments in geometry (Adolphus, 2011). Mathematics attitudes are defined as positive or negative responses towards mathematics that are relatively stable, similar to dispositions (Hemmings et al., 2011). Thus, attitudes towards mathematics are defined as students' self-reported enjoyment of the subject, their interest, and the levels of mathematical anxiety (Pilli & Aksu, 2013). According to the findings of several studies (Ma & Xu, 2004; Hemmings et al., 2011; Hagan et al., 2020; Asante, 2012), positive attitudes towards mathematics increase students' achievements and learning. At the same time, negative attitudes towards mathematics, and specifically geometry, hinder mathematical learning and engagement (Garden, 1997). Several factors are believed to be underlying students' attitudes towards geometry and mathematics in general, such as confidence, anxiety, value, enjoyment, motivation, and usefulness (Lim & Chapman, 2013)

The research of Mata et al. (2012) and Atanasova-Pacemska et al. (2015) shows that positive attitudes towards geometry reflect a positive emotional disposition, while negative attitudes are related to a negative emotional disposition. In addition, these studies found that positive attitudes towards mathematics in general reflect a positive self-confidence, enjoyment, value, and emotional disposition. Moreover, students' attitudes towards geometry are related to their beliefs of the usefulness of this subject and their intention of using it in their future careers, as well as their willingness to solve geometry problems and their feeling about the difficulty of geometry (Sunzuma et al., 2013; Tsao, 2017; Dede, 2012). Furthermore, there is a correlation between the perception of the usefulness of learning geometry, the enjoyment in learning it, and the confidence related to learning geometry (Tsao, 2017).

Enjoyment towards learning mathematics, and, specifically, geometry, is defined as the extent to which students like working on math tasks and/or how boring they think mathematics (or geometry) is (Stipek, 2002; Dede, 2012). The usefulness refers to the extent a task fits one's future plans, such as the intention to take future mathematics classes in college (Wigdield & Eccels, 2002; Hwang et al., 2009), but also the perception that geometry is useful in today's world (Atweh, 2007; Dede, 2012). Mathematical anxiety is defined as fear of mathematics, which is, moreover, related to a feeling of frustration when someone faces a mathematical problem or a situation in which manipulating numbers and figures is needed (Ashcraft & Faust, 1994). Generally speaking, anxiety towards mathematics is defined as a combination of several negative attitudes towards mathematics, such as the fear of success or the lack of self-confidence and overall pressure (Bessant, 1995). Similarly, geometry anxiety can be defined as a feeling of repulsion towards this particular subject (Sağlam et al., 2011).

1.2. Measuring geometry attitudes

In the last two decades, a number of studies have been aimed at constructing and validating instruments that measure students' attitudes towards geometry. One of the reasons that encouraged such kind of research is that positive attitudes towards the subject may lead to better results (Abdullah & Zakaria, 2011).

In the research of Abdullah and Zakaria (2011), which involved 161 students, the authors used the Attitude towards geometry survey (ATGS), which consisted of 24 items, divided into three constructs: (1) enjoyment of learning geometry topics (e.g., "I enjoyed being in geometry class", "Geometry is a fun topic"), (2) valuation of geometry (e.g., "Geometry can be applied in the situations outside the classroom", "I need geometry in my daily life"), and (3) motivation towards geometry topics (e.g., "I am looking forward to learning geometry as much as possible in the future", "I always want to learn geometry more and more"). Both positive and negative statements were used in the survey and all the questions applied a 5-point Likert scale. The internal consistencies, measured with the Cronbach's alpha coefficients, were .85, .77 and .69 for the three constructs measured; the overall Cronbach's alpha for the ATGS was .90. The authors found an average of M=83.35 (SD=16.46), which suggests that students have a positive attitude towards geometry since the score is higher than 50. The factor analysis suggested that the ATGS can be used to measure students' attitudes toward geometry in the Malaysian context.

Also Dede (2012) used the Attitude toward Geometry Scale (ATGS). After a factor analysis, 17 items were retained, which measured three constructs: (1) students' enjoyment (11 items; e.g. "Geometry subjects are boring for me", "I like geometry subjects", "I would not like to learn geometry subjects"); (2) usefulness of learning geometry (4 items; e.g. "I can apply geometry subjects in daily life"); and (3) anxiety towards geometry (2 items; e.g. "I'm afraid of geometry subjects"). The reliability of the sub-scales were .93, .61 and .57 respectively.

In another research in which the participants were 74 students, Mogari (1994) explored students' attitudes towards Euclidean geometry. The study evaluated the following constructs: (1) enjoyment, (2) motivation, (3) importance, and (4) fear of geometry. The reliability of the instrument was good (α=.77). In another research, Mogari (2004) developed an attitudinal scale for measuring attitudes towards Euclidean geometry. The instrument measured four constructs: (1) enjoyment, (2) value, (3) motivation, and (4) belief and had a high reliability (α=.85).

Avcu & Avcu (2015) translated the Utley Geometry Attitude Scale (UGAS) to Turkish. Their sample consisted of 750 undergraduate students who were randomly divided into two subsamples to perform a factor analysis: the first subsample (n=371) was used to perform the exploratory factor analysis (EFA), while the second (n=379) to perform the confirmatory factor analysis (CFA). The initial 32-items UGAS was reduced to 25-items instrument through EFA and four factors have been constructed: (1) confidence ("Geometry problems often scare me", "I am confident I can get good grades in geometry"), (2) enjoyment ("Geometry problems are boring"; "Geometry is fun"), (3) future use ("I believe that I will need geometry for my future", "I do not expect to use geometry when I get out of school"), and (4) every day use ("I often see geometry in everyday things"). The CFA confirmed that the proposed model has a very good fit.

A recent study by Salifu et al. (2021) investigated pre-service teachers' attitudes in terms of (1) usefulness, (2) enjoyment, and (3) confidence in teaching geometry. The authors adapted the aforementioned UGAS scale by adding some open-ended questions.

From the studies mentioned above we may conclude that instruments measuring students' geometry attitudes measure several constructs, among which the most common are: (1) students' enjoyment towards geometry; (2) negative feelings and attitudes related to studying geometry; (3) students' motivation to learn geometry; and (4) students' perceptions of the usefulness of learning geometry.

1.3. Aims of the research

The aim of the present study is to contribute to the literature on instruments used to assess students' attitudes towards geometry by designing a questionnaire for high school students and evaluating its validity and reliability.

2. Method and Materials

2.1. Methodology

A non-experimental quantitative research methodology was used.

2.2. Participants

The sample was composed of 242 randomly chosen Italian high school students from North-Eastern Italy. 144 (59.5%) were female, and 98 (40.5%) were male. The average age was 15.7 (*SD*=1.43; *min*=14; *max*=20) years old.

2.3. Data collection tools

Based on the reviewed literature, we designed a 29-item questionnaire, divided into four factors: (1) personal beliefs about geometry and enjoyment, (2) perception of the usefulness of geometry, (3) cognitive factors and motivation in learning geometry, and (4) aversion towards geometry. Students reported their agreement with the statements on a 5-point Likert scare, where 1 represents "Strongly disagree" and 5 represents "Strongly agree".

The face validity of the instrument (cf. Taherdoost, 2016) was checked with a group of four experts in the field of mathematics education and psychology. Among them, two were full professors of mathematics education, one was a researcher in the field of mathematics education with at least one publication in the field of mathematics education, and one was a psychologist with at least one publication in the field of mathematics education. The experts were asked to assess the validity of the questionnaire. The attitudinal scale was considered appropriate for the given purpose. All the statements of the attitudinal scale were reported to be relevant for the respective component variables.

The content validity of the questionnaire was measured for each of the 29 items regarding students' attitudes towards geometry. The draft was reviewed by the abovementioned group of experts. They were instructed to assess the relevancy of the items with a 4-items Likert scale (1=not relevant, 4=very relevant). We dichotomized the scale into "agreed" (ratings 3 and 4) and "not agreed" (ratings 1 and 2). The level of content validity of an item was determined as the ratio between the number of "agreed" and the number of experts. We accepted an item to be content-valid if its item content validity was at least .90 (Polit et al., 2007). All the questions were included in the final instrument.

Participants completed the questionnaire in November 2021. The instrument was administered online, due to the COVID-19 restrictions. After collecting students' or their parents' (in case of minors) signed informed consents, which included the purpose of the research and a statement of anonymity, the participating students received a link on their school e-mail addresses. The order in which the items appeared on each questionnaire changed randomly to avoid response order bias (Kitchenham & Pfleeger, 2002; Malhotra, 2006).

2.4. Data analysis

In the present study, both the Principal Component Analysis (PCA) and the Exploratory Factor Analysis (EPA) was used. The PCA was performed to identify the number of dimensions in the scale. Orthogonal rotations were used (Kaiser, 1969) which are easier to interpret (Tabachnick et al., 2007). Several iterations of the PCA were applied to retrieve a clear factor structure (cf. Avcu & Avcu, 2015). The following criteria were applied in order to decide which items should be retained:

(a) item loadings have to exceed .40 on at least one factor;

(b) in items with factor loadings >.30 on more than one factor, a minimum gap of .10 is needed;

(c) a factor needs to be identified with at least three significant loadings (Zwick & Velicer, 1986).

A reliability analysis was performed, as well an item analysis to identify and eliminate possible problematic items from the final version of the instrument. The item analysis was conducted on the basis of the calculation of the item-total correlations. Values greater than .40 are considered acceptable (cf. Gilem & Gilem, 2003). Finally, the reliability of the overall instrument and its factors were examined by computing both the Cronbach's alpha coefficient and the McDonald's omega (Ravinder & Saraswathi, 2020).

All the data were analyzed using the *Jamovi* statistical software.

3. Results

3.1. Principal Component Analysis

Initially, the factorability of the 29 items was examined. The Kaiser-Meyer-Olkin measure of sampling adequacy was .889, above the commonly recommended value of .600 (Dziuban & Shirkey, 1974). Moreover, all factors had a *KMO* measure of sampling adequacy greater than .700. The Bartlett's test of sphericity was significant ($\chi 2(406)=2087$; $p<.001$). Given these indicators, the factor analysis was deemed to be suitable with all the 29 items.

A principal component analysis was used to identify and compute composite scores for the factors underlying the 29-items geometry attitude questionnaire. With the parallel analysis (Hayton et al, 2004; Çokluk & Koçak, 2016), two components were identified, i.e. personal attitudes and enjoyment (Component 1) and usefulness of geometry (Component 2). The two factors explained 39.9% and 8.2% of the variance respectively, for a total of 48.1% of the variance. Solutions for two factors were each examined using the varimax and oblimin rotations of the factor loading matrix. Little difference was found in the two rotations for the component loadings, thus the varimax rotation was considered (Table 1). The two factors explained 25.2% and 22.9% of the variance respectively, for a total of 48.1% of the variance.

Table 1. Loadings with the parallel analysis and the varimax rotation

Item code	Item	Component 1	2	Uniqueness
GEO1	I like geometry.	0.590	0.503	0.399
GEO2	I like solving geometrical problems.	0.653	0.448	0.373
GEO3	Geometry is fun.	0.603	0.573	0.308
GEO4	I like geometry lessons.	0.669	0.347	0.433
GEO5*	Geometry lessons are boring.	0.579	0.487	0.428
GEO6*	During geometry lessons I feel uncomfortable.	0.700		0.446
GEO7	The funniest part of mathematics is geometry.	0.615		0.578
GEO8	I would like geometry to get more attention in school.	0.546	0.339	0.587
GEO9*	During geometry classes I am stressed.	0.671		0.528
GEO10*	Geometry topics scare me the most of all mathematics topics.	0.739		0.448
GEO11	I use the geometry I learn in school in my everyday life.		0.650	0.506
GEO12	Geometry can be used in everyday life.		0.720	0.435
GEO13	I feel geometry is very useful in my everyday life.		0.698	0.434
GEO14*	I would prefer geometry not to be taught in schools anymore.	0.663	0.334	0.448
GEO15	I would like to learn as much geometry as possible.		0.650	0.494
GEO16	I would like to increase my geometry knowledge.		0.573	0.670
GEO17	I want to learn geometry as much as possible.	0.498	0.500	0.502
GEO18	Geometry teaches me to think logically.		0.684	0.532
GEO19	I believe I'm good at geometry.	0.577	0.366	0.533
GEO20*	In the future I will not use geometry anymore.		0.479	0.725
GEO21*	Geometry is a very difficult subject.	0.676		0.543
GEO22*	Doing geometry is a waste of time.	0.516	0.503	0.480
GEO23*	Geometry is not useful to society.		0.602	0.565
GEO24*	Only the best students can understand geometry.	0.413		0.802
GEO25*	Geometry is the worst part of mathematics.	0.751		0.424
GEO26*	I would prefer not to learn and use geometry in college.	0.454		0.710
GEO27	Knowing geometry would help me increase my mathematical knowledge.		0.510	0.680
GEO28	Geometry helps me to develop correct thinking.		0.618	0.608

GEO29	Geometry is useful in everyday life.	0.694	0.455

*Reversed score.

Considering the Kaiser's rule, i.e. the eigenvalues-greater-than-one rule (Cliff, 1988), six components were identified, i.e.: (1) students' enjoyment of geometry and personal beliefs about geometry (Component 1); (2) usefulness of geometry (Component 2); (3) motivation for learning geometry (Component 3); (4) self-efficacy and anxiety towards geometry (Component 4); (5) willingness to learn geometry in the future (Component 5); and (6) the person most suitable to learn geometry (Component 6). The first component explains 15.2% of the variance, the second 13.0%, the third 12.6%, the fourth 11.3%, the fifth 8.8% and the sixth component explains 6.6% of the variance. Overall, the six components explain 67.5% of the variance.

According to Zwick and Velicer (1986), the Kaiser's criterion is not suitable to extract factors, since it overestimates the number of factors and therefore the parallel analysis is recommended in addition to the eigenvalues criterion (see also Morton & Altschul, 2019; Patil et al., 2008).

Before deciding the final number of factors, we checked the Rotated Component Matrix. We retained the items that had a loading exceeding .40 on at least one factor (a). For the items with factor loadings that exceed .30 (b) more than one factor, a minimum gap of .10 between the loadings is required and at least 3 significant loadings are required for factor identification (c). Considering Table 3 and criterion (b), the difference between the loadings for items GEO1, GEO3, GEO5, GEO8, GEO14, GEO19, GEO21 and GEO28 was insufficient and these items were thus deemed unsuitable.

After deleting the aforementioned items, the second PCA with varimax rotation was performed. The number of extracted factors with the eigenvalue rule reduced to four. The factors explained 17.4%, 16.7%, 15.3%, and 11.5% of the variance respectively; overall, 60.9% of the variance was explained.

Moreover, item 9 did not respect the (b) criterion and was thus removed. Therefore, a third PCA was performed. The number of factors remained four (see Table 2), each of which was considered since they respected the (c) criterion. The four components regarded: (1) the perception of the usefulness of geometry (Component 1); (2) the enjoyment of geometry and geometry anxiety (Component 2); (3) negative attitudes towards geometry (Component 3); and (4) the motivation towards learning geometry (Component 4). The eigenvalues of the components were 7.271, 1.898, 1.463, and 1.092; the explained variance was 18.0, 16.7, 15.1, and 11.9 respectively. The total explained variance was 61.7%.

Table 2. Loadings with the eigenvalues criterion and the third varimax rotation

Items	Component				Uniqueness
	1	2	3	4	
GEO2		0.634			0.319
GEO4		0.621			0.377
GEO6*		0.467	0.569		0.423
GEO7		0.707			0.395
GEO10*		0.809			0.326
GEO11	0.692				0.405
GEO12	0.766				0.289

GEO13	0.811				0.229
GEO15				0.725	0.303
GEO16				0.826	0.292
GEO18	0.612				0.526
GEO20*			0.647		0.491
GEO22*			0.654		0.336
GEO23*	0.401		0.641		0.367
GEO24*			0.650		0.546
GEO25*		0.795			0.282
GEO26*			0.551		0.613
GEO27				0.680	0.457
GEO29	0.761				0.300

*Reversed score.

The items loaded on the first factor were related to the perception of the usefulness of geometry and were therefore named "Usefulness". The items loaded on the second factor highlighted students' enjoyment of geometry and their geometry anxiety and were named "Enjoyment & Anxiety". The items loaded on the third factor were related to negative attitudes towards geometry and were named "Negative attitudes". The items loaded on the fourth factor were related to the students' motivation for learning geometry, hence they were named "Motivation". Items covered by each factor are presented in Table 3 so the final version of the questionnaire consist of 16 items.

Table 3. Factors from the PCA and the questions related to them

Factor	Items
Usefulness	[GEO11] I use the geometry I learn in school in my everyday life.
	[GEO12] Geometry can be used in everyday life.
	[GEO13] I feel geometry is very useful in my everyday life.
	[GEO29] Geometry is useful in everyday life.
Enjoyment & Anxiety	[GEO2] I like solving geometrical problems.
	[GEO4] I like geometry lessons.
	[GEO7] The funniest part of mathematics is geometry.
	[GEO10*] Geometry topics are those mathematical topics that scare me the most.
	[GEO25*] Geometry is the worst part of mathematics.
Negative attitudes	[GEO6*] I feel uncomfortable during geometry lessons.
	[GEO20*] In the future I will not use geometry anymore.
	[GEO22*] Doing geometry is a waste of time.

	[GEO23*] Geometry is not useful to society.
	[GEO24*] Only the best students can understand geometry.
	[GEO26*] I would prefer not to learn and use geometry in college.
Motivation	[GEO15] I would like to learn as much geometry as possible.
	[GEO16] I would like to increase my geometry knowledge.
	[GEO27] Knowing geometry would help me increase my mathematics knowledge.

3.2. Exploratory Factor Analysis

An Exploratory Factor Analysis (EFA) was used to identify and compute composite scores for the factors underlying the geometry attitude questionnaire. We used a parallel analysis (Hayton et al, 2004; Çokluk & Koçak, 2016) and the minimum residual extraction method. Four factors were identified, i.e. geometry enjoyment and anxiety (Factor 1), perception of the usefulness of geometry (Factor 2), negative attitudes towards geometry (Factor 3), and students' motivation for learning geometry (Factor 4). Varimax rotation was used to explore factor loadings (see Table 4).

Table 4. The EFA results

Item	Factor				Uniqueness
	1	2	3	4	
GEO11		0.677			0.412
GEO12		0.743			0.307
GEO13		0.682	-0.328		0.358
GEO29		0.676	-0.307		0.373
GEO2	0.594	0.377		0.350	0.350
GEO4	0.571		-0.323	0.320	0.433
GEO7	0.613				0.517
GEO10*	0.718				0.467
GEO25*	0.762				0.332
GEO6*	0.444		0.516		0.498
GEO20*			0.520		0.646
GEO22*			0.703		0.311
GEO23*		0.308	0.669		0.401
GEO24*			0.393		0.772
GEO26*			0.392		0.737
GEO15				0.714	0.355
GEO16				0.710	0.470

GEO27	0.515	0.643

The first factor explained 15.2% of the variance, the second 14.3% of the variance, the third 13.5%, and the fourth 10.5% of the variance respectively, for a total of 53.5% of the variance.

The EFA model fit was evaluated by the standards proposed by Hu & Bentler (1999) and Brown (2015). An excellent fit was found for the four-factor model for the data:

- the RMSEA was .000 (90% CI [.000; .0384]), lower than the recommended <.06 (Hu & Bentler, 1999);
- the TLI was 1.02, higher that than the recommended ≥.95 (Bentler, 1990; Hu & Bentler, 1999);
- the $\chi 2$ test is significant ($\chi 2(87)=78.5$; $p=.730$), and also the preferred $\chi 2/df$ ratio (Marsh et al., 1988) has shown an excellent fit ($\chi 2/df=78.5/87=.902<3$; Kline, 2011).

3.3. Reliability analysis

After the EFA, an item analysis was performed to identify and eliminate possible problematic items from the final 16-items instrument (cf. Avcu & Avcu, 2015). The item discrimination was checked through the corrected item-total correlation values. All the identified item-total correlations were >.30, thus the correlations are considered to be acceptable (cf. Zijmans et al., 2018). The corrected item-total correlations (*r*) are presented in Table 5.

Table 5. Item-total correlation coefficients

Item	*r*
GEO15	0.575
GEO16	0.382
GEO27	0.459
GEO11	0.608
GEO12	0.650
GEO13	0.658
GEO29	0.655
GEO2	0.712
GEO4	0.650
GEO7	0.529
GEO10*	0.398
GEO25*	0.556
GEO20*	0.469
GEO22*	0.690
GEO23*	0.582
GEO24*	0.387
GEO26*	0.456

*Reversed score.

As it seen in Table 5, the corrected item-total correlation values of each item ranged from a minimum of .387 and .690. This suggests that all items work well and there is no need to eliminate any of them from the scale (Gilem & Gilem, 2003).

The reliability of the final version of the questionnaire was assessed by means of internal consistency measures. The reliability of the overall instrument was measured with the Cronbach's alpha (α) and McDonald's omega (ω) coefficients. The overall reliability is excellent (α=.898; ω=.901). Moreover, the internal consistency of the "Enjoyment & Anxiety" (α=.872; ω=.874), "Usefulness" (α=.841; ω=.842), "Negative attitudes" (α=.788; ω=.798), and "Motivation" (α=.725; ω=.735) was checked resulting in a good to very good internal consistency. The best internal consistency was found in the "Enjoyment & Anxiety" item, while the lowest was identified in the "Motivation" item.

4. Discussion and Conclusions

Literature has shown that several factors influence students' attitudes towards mathematics and, specifically, geometry, such as students' confidence, mathematics anxiety, perceived value of mathematics, mathematics enjoyment, motivation, and usefulness (Lim & Chapman, 2013; Adolphus, 2011; Hemmings et al., 2011; Pilli & Aksu, 2013; Ma & Xu, 2004; Hemmings et al., 2011; Hagan et al., 2020; Asante, 2012; Garden, 1997; Sunzuma et al., 2013; Tsao, 2017; Dede, 2012). Despite the fact that geometry attitude scales measure all the above dimensions, most of them usually focus on selected items, such as enjoyment, usefulness, anxiety, and value (Abdullah & Zakaria, 2011; Dede, 2012; Mogari, 1994, 2004; Avcu & Avcu, 2015; Salifu et al., 2021). The instrument presented in our study considers several dimensions, i.e. students' geometry enjoyment and anxiety, perception of the usefulness of studying geometry, negative (or positive) attitudes towards learning geometry and their motivation for studying it.

The aim of the present study was to construct and validate an instrument that teachers and researchers could use in assessing students' attitudes towards geometry. Based on the analysis of the literature, a 29-items questionnaire was developed which aimed to assess four dimensions: enjoyment and anxiety, usefulness, negative attitudes, and motivation. Through the PCA, the initial instrument was cleared of some items that did not comply with the established criteria. A final version of 16 items was then analyzed through an EFA. The EFA has shown an excellent model fit. After the EFA, item analyses and reliability analyses were performed to determine the suitability of the items. The item-total correlations were satisfactory since all the correlations were higher than .30. Moreover, the reliability of the instrument was high, and the reliability of the four subscales was medium to high.

In conclusion, while this instrument proved to be valid and reliable for high school students, it might also be tested for use with middle school or college students. Teachers and researchers might gain a better insight into their students' attitudes towards geometry, thus making it possible to develop personalized geometry curricula.

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