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Cognitive-constructivist model and the acquisition of mathematics knowledge according to Gagné's taxonomy

Lea Kozel, University of Primorska, Faculty of Education, Cankarjeva 5, 6000 Koper, Slovenia

Mar Cotic, University of Primorska, Faculty of Education, Cankarjeva 5, 6000 Koper, Slovenia

Daniel Doz 1, University of Primorska, Faculty of Education, Cankarjeva 5, 6000 Koper, Slovenia

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Abstract

School reforms aim to achieve lasting knowledge for pupils. Some researchers have shown that lasting knowledge can be achieved through a cognitive-constructivist model of instruction, as pupils discover concepts themselves and are active in the classroom. In this paper, we aimed at investigating whether pupils who are exposed to a cognitive-constructivist model of instruction are more successful in solving mathematical tasks than those who are not exposed to a cognitive-constructivist model of instruction. For this purpose, we have designed an experiment involving 252 pupils of the 3rd grade of primary school: of these, 100 were included in the experimental group receiving constructivist mathematics instruction, while the remaining 152 pupils constituted the control group. The study showed that the experimental group performed better in solving mathematical problems which involve all three taxonomic levels according to Gagné.

Keywords: cognitive-constructivist model, acquisition, mathematics, Gagné taxonomy, experimental group.

* ADDRESS FOR CORRESPONDENCE: Daniel Doz, University of Primorska, Faculty of Education, Cankarjeva 5, 6000 Koper, Slovenia
E-mail address: daniel.doz@pef.upr.si

1. Introduction

Contemporary school reforms around the world aim to achieve comprehensible, meaningful, and lasting knowledge for pupils (Bone, 2022; Du, 2021; Reimers, 2021). Within this goal, constructivist theories are also being put forward (Chapman et al., 2005; Lofti et al., 2012; Scott, 2006; Thompson, 2020; cf. Vogel-Walcutt et al., 2011). Constructivist theories of knowledge are based on the assumption that individuals construct their knowledge through their own building of knowledge (Applefield et al., 2000; Felda, 2011; O'Connor, 2022; Plut - Pregelj, 2003), and therefore, constructivist advocates argue that it is the constructivist theories of knowledge which should become the main starting point for modernisation of the curriculum (Černilec, 2019; Henson, 2015; Rutar Ilc, 2002; Young, 2008).

Numerous research (Bransford et al., 2000; Ng et al, 2020; Strommen & Lincoln, 1992; Ucus, 2015; Ünal, 2017; Wingfield & Black, 2005) have shown that the permanency and usefulness of the knowledge gained in the so-called active manner are greater than if this knowledge is purely by default, as pupils who discovered certain mathematical concepts through active investigation applied this knowledge in new and atypical situations, while pupils who were only familiarised with the same mathematical concepts (rote learning) failed in new situations (Lithner, 2015; Rutar Ilc, 2002).

Cognitive psychologists (Piaget, Vygotsky, Bruner, Bransford, Marzano; according to Rutar Ilc, 2002) have stressed the importance of an activity in which the learner, with the teacher's thoughtful support in the process of exploration and discovery, analysis and integration, comes to or builds his/her own knowledge by himself/herself. An active approach, grounded in the learner's own discovery and building of knowledge through various activities and thought processes and procedures which these activities stimulate, is the one which enables the internalisation of concepts, principles, and laws, and thus the sustainability and transfer value of knowledge, into a more passive uptake of ready-made knowledge (Rutar Ilc, 2002; Terwel et al., 2009).

Too often in modern school practice, primarily due to time constraints, teachers look for a way of teaching which would lead to the goal quickly (Andrew, 2007; Lord, 1999). Sometimes the path can indeed be shorter, especially when passive methods of teaching are used, such as explanation, demonstration, discussion, etc. (Felda & Bon Klanjšček, 2017). However, we fail to realise that it is precisely this learner's passive acceptance of material that leads to learning and performing at lower taxonomic levels, which prevents the learner from developing the appropriate thought processes and skills which lead to useful knowledge (Tabriyi & Rideout, 2017).

The abovementioned issue of rote learning of mathematical procedures (Felda & Bon Klanjšček, 2017), combined with the lack of conceptual thinking in the curriculum of mathematics at the primary school level (Ahmad et al., 2011), might be partially solved by the adoption of the cognitive-constructivist model of teaching mathematics (D'Souza & Wood, 2003). However, despite the previously mentioned literature has shown the beneficial effects of the cognitive-constructivist model of acquiring mathematical knowledge, little is known about the effects of this model on various taxonomic levels. For instance, the cognitive-constructivist model might help students to understand better and interiorise the studied concepts (Thompson, 2020), especially new ones (Ahmad et al., 2011), however it is not clear whether it could also increase students' problem-solving skills (cf. Amin & Mariani, 2017). Therefore, it is important to answer the question of whether the cognitive-constructivist model affects the acquisition of mathematical knowledge according to Gagné's taxonomy (Gagné, 1985). Specifically, the aim of this paper is to explore whether adopting the cognitive-constructivist model of learning might positively affect pupils' learning of mathematics on all three levels of Gagné's taxonomy: (1) basic and conceptual knowledge, (2) procedural knowledge, and (3) problem-solving skills.

Hence, in this paper, we wish to test whether pupils who learn mathematics constructivistically are better at solving exercises of all taxonomic levels (according to Gagné, 1985) than pupils who do not receive the constructivist manner of instruction. The experiment involved 252 pupils from the 3rd grade of primary school, of whom 100 were included in the experimental group and the remaining 152 pupils formed the control group.

1.1. Gagné classification of knowledge

Mathematical knowledge can be classified in different ways, one of the most popular of which is Gagne's taxonomic scale (Ahmad et al., 2011; Tang et al., 2020). We describe pupils' achievement in terms of the level of skills attained using the taxonomic scale proposed by Gagné (1985) and shown in Table 1. Later on, we will describe each level separately.

Table 1. Gagné's classification of knowledge (Gagné, 1985).

Basic and conceptual skills - Basic skills and knowledge - Conceptual skills
Procedural skills - Routine procedural skills - Complex procedural skills
Problem skills - Problem-solving strategies - Applicative skills

1.2. Basic and conceptual knowledge

Basic skills and knowledge mainly include knowledge of concepts and facts and the retrieval of knowledge (Hiebert & Lefevre, 1986; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Siegler, 1998). Conceptual skills denote the understanding of concepts and facts. The basic elements of basic and conceptual skills are: knowledge of specifics (multiplication knowledge, knowledge of isolated information and factual information); knowledge of specific facts (knowledge of definitions, formulas, axioms, theorems, relations, basic properties; knowledge of terminology and basic symbols (parallelism, orthogonality, +, -, %, etc.; rectangle, function, equation, kilogram); knowledge of classifications and categories (recognition of different mathematical objects and their classification); recognition of concepts (e.g., triangles on a plane, solids, in nature, etc.); representation (e.g., two congruent right-angled triangles form a right-angled triangle); recognition of terminology and symbolism in a given situation; connections (similarities, differences, integration).

1.3. Procedural skills

Procedural skills comprise of knowledge and effective mastery of algorithms and procedures (Hiebert & Lefevre, 1986; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Siegler, 1998). It is divided into routine (procedural) knowledge and complex (procedural) knowledge. The basic elements of procedural knowledge are: performing routine procedures; using rules and forms; solving simple non-composite tasks with little data; knowing and effectively mastering algorithms and procedures (methods, procedures); application /not recalling/ of rules, laws, procedures; selection and execution of algorithms and procedures of a procedure, while having to justify or verify the choice and execute the procedure, as well as application of complex procedures (composite tasks with multiple data).

1.4. Problem skills

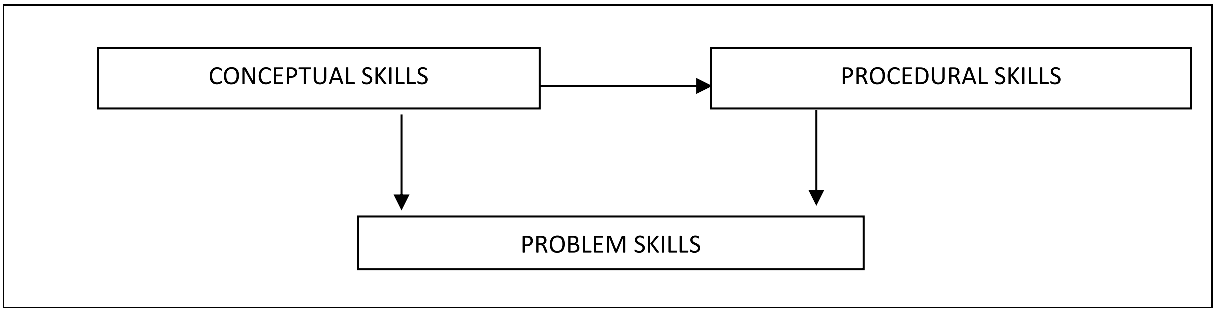
Problem skills are used for the application of knowledge to new situations, the use of combinations of multiple rules and concepts in dealing with a new situation, and the ability to apply conceptual and procedural skills (Ahmad et al., 2011; Gagné, 1983; Liljedahl et al., 2016). The core elements of problem skills: setting a problem (problem identification and formulation, asking meaningful questions); data verification (analyse if the problem has enough data to be solved, if the problem has too much data to be solved, if the data are contradictory. etc.); solution strategies (application of a set of the following processes: communication, operational, thinking and note-taking processes); application of skills or transfer of knowledge (to apply the mathematical concept learnt in another context); thinking skills (analysis, synthesis, induction, deduction, interpretation); metacognitive skills (to judge whether a mathematical concept has been correctly applied in a given context; to justify one's position, thus demonstrating that a cognitive conflict has been overcome).

1.5. Problem skills

Conceptual skills are, to some extent, a prerequisite for procedural skills (Byrnes & Wasik, 1991; Rittle-Johnson & Alibali, 1999; Rittle-Johnson & Siegler, 1998): we can perform a procedure even if we do not understand it, but we usually need to have at least partial understanding of the object we are operating on. Problem skills are partly general (general strategies, etc.), partly related to concrete content and require solid conceptual and procedural skills, even understanding of procedures.

It is undeniable that skills have an impact on each other: knowledge of procedures has some impact on understanding of concepts (although not as much as we sometimes imagine). What is more difficult is to give the right weight to each type of skill. The importance of different types of skills depends on external circumstances, the purpose of schooling, the subjective judgement of the teacher, etc. Knowledge is interrelated. Even in application, for example, we never use only procedural or problem skills, but rather intermix them (Rittle-Johnson et al., 2009; Silver, 2013; Star, 2005; Surif et al., 2015). Therefore, it is impossible to give more weight to one type of skills than to other, as they are so intertwined that they cannot be easily separated. The relationship between different types of skills is illustrated in Figure 1.

Figure 1. The relation between types of knowledge.



Modern learning and teaching strategies have brought about major changes in mathematics education and thus also in the assessment and evaluation of knowledge. The objectives of mathematics education, which were oriented towards the acquisition of concrete content and, above all, procedural skills, are nowadays increasingly complemented by conceptual and procedural skills, or skills oriented towards finding pathways and problem-solving strategies, which are also transferable to other subject areas and beyond the curriculum (Chapman & Aspin, 2013; Cotič, 2010).

In today's world, the importance of procedural skills has declined and the need for problem skills has increased significantly (Aydoğdu & Ayaz, 2008). There is an increasing emphasis on the importance of complex knowledge, which ranges from basic reading and numeracy skills to awareness of complex problems and ways of solving them (English, 2008). There is a change in the way we think about what knowledge is, from seeing knowledge as singular and unchanging to complex and dynamic. Theories and classifications of skills provide a starting point on types and aspects of knowledge (Žakelj, 2003). It is important that both classroom teachers and teachers of mathematics and didactics of mathematics are aware of the different aspects, types, and levels of skills, to be able to judge which to prioritise in different situations, and to know how to introduce, address, consolidate, and ultimately test and assess them in mathematics lessons (Cotič, 2010).

1.6. Cognitive-constructivist model of instruction

Cognitivism and constructivism are the fundamental theories of learning. It is the belief that human thinking and learning is similar to a computer - a robot, computer information processing. In education, cognitivism focuses on the transformation of knowledge from the real world to the learner through the teacher's transmission. Success is achieved if at the end of the lesson, the learner has the same mental construct of the subject after studying it with the teacher. The problem that arises is how to be sure that the learner's mental construct is indeed such. Cognitivism is entirely focused on internal mental processing systems or learning schemas in the context of how the brain receives, internalises and retrieves information. In constructivism, the learner constructs his or her meaning, an image based on new knowledge, which helps him or her construct new knowledge (Leonard, 2002).

Constructivism is the belief that pupils have a certain background knowledge and experience on which their assumptions (hypotheses) are based and which form the basis for creating the context, the basis for solving a concrete problem raised by the teacher. Constructivism is a learner-oriented pedagogical paradigm in which content is constructed from the learner in a team-based group in the form of collaborative learning and in the context of a constructivist learning environment open to constructions. Constructivism theory focuses on the learner's thinking, on his/her learning activities. In the active learning paradigm of constructivism, the teacher is no longer the primary mediator and the only channel for the delivery of knowledge. The entire knowledge first goes directly to the learner. The teacher is the catalyst, the advisor, and the programme manager of the implementation of projects for solving a concrete problem, rather than the barrier between the learner and the content. With constructivism comes learning inquiry, exploration, and the discovery of the autonomous, self-motivating learner, which is a critical moment important for the successful implementation of the learning process (Leonard, 2002).

Marentič Požarnik (2000) defines cognitivism as a psychological trend which emphasises the importance of people's internal mental processes, especially cognitive processes, in learning and achieving deeper understanding. Constructivists, she says, go one step further, believing that knowledge in a ready-made form which cannot be given to or received from someone else, but must be reconstructed by each person through his or her own mental activity. Thus, supporters of constructivism argue that knowledge is not received from outside but constructed by our own activity in the process of making sense of our experiences. It follows that knowledge is not something which exists objectively, independent of the one learning, but a subjective construct created by each learner in the process of reflecting on his or her experiences (Marentič Požarnik, 2000).

Požarnik conceives constructivism as a holistic view of learning that is not merely a cognitive activity of the human being, but necessarily integrates the emotional, motivational and social dimensions of the individual. In this way, a person's intrinsic motivation for a particular subject or problem is stimulated being built or constructed in the process of solving meaningful problems and promotes a different quality of learning than predominantly extrinsic motivation (Marentič Požarnik, 2004).

1.7. Goals of the study

If we take a closer look at mathematics teaching in the first three years of formal education in Slovenia, we find that teaching is mainly focused on the teacher's delivery of the material, and the pupils are mostly passive recipients of the content (Felda & Bon Klanjšček, 2017). This model of instruction is a sequence of three basic learning components: content - teacher - pupils. This type of teaching is based on the classical frontal form of work and the methods associated with it (explanation, discussion, etc.).

Strmčnik (2001) points out that it is important to realise that teaching is primarily an interaction between the participants in the classroom, while learning is a distinct subjective intra-activity of the learning subject himself/herself. Therefore, in terms of its purpose, teaching is subordinate to learning since there is no learning without teaching. The purpose of teaching is to help and encourage pupils to learn as independently and creatively as possible, since teaching itself precedes or parallels learning, and the quality of learning depends on it (Strmčnik, 2001).

So far, didactics has paid more attention to teaching than to learning. Komensky was already striving for a closer integration of teaching and learning, looking for ways to make the teacher teach less and the pupils learn more. Only modern didactics is seeking to align teaching appropriately in favour of learning (Strmčnik, 2001).

Despite the cognitive-constructivist model of learning might have some positive effects on students' learning of mathematics (Lumbantoruan & Natalia, 2021; Thompson, 2020), especially of new mathematical concepts (Ahmad et al., 2011), and it has been hypothesised that constructivism might enhance students' problem-solving abilities (cf. Amin & Mariani, 2017), literature on this topic is still scarce. In particular, to the best of our knowledge, no research has investigated the effect that the cognitive-constructivist model has on the acquiring of mathematical knowledge on all three levels of Gagné's taxonomic scale.

The research problem is therefore focused on the design and evaluation of an experimental cognitive-constructivist model of mathematics instruction based on the active involvement of the learner in the learning process, which we have built on the basis of foreign research (Jennings, 1994; Butler & Winne, 1995; in Šteh 2000), taking into account the Slovenian context. In school practice, we conducted an experiment with 3rd-grade pupils to determine whether the use of an experimental cognitive-constructivist model of instruction would have a statistically significant effect on pupils' mathematics achievement on all three taxonomic levels according to Gagné.

1.8. Study hypotheses

The study aims to answer the following general research hypothesis:

H: Pupils who receive an experimental cognitive-constructivist model of mathematics instruction will be more successful at solving mathematical problems at all three levels of skills according to Gagné's taxonomy.

From the general research hypothesis, we derive three specific research hypotheses:

H1: The experimental group of pupils will perform better than the control group of pupils in solving mathematical problems of the first taxonomic level according to Gagné (conceptual skills).

H2: The experimental group of pupils will be more successful than the control group of pupils in solving mathematical problems of the second taxonomic level according to Gagné (procedural skills).

H3: The experimental group of pupils will perform better than the control group of pupils in solving mathematical problems of the third taxonomic level according to Gagné (problem skills).

2. Method and Materials

2.1. Methodology

We used a causal experimental method of pedagogical research. The experiment was performed in the existing primary school classrooms. This means that no equating of the classrooms to random differences had been done prior to the experiment. The group of pupils who was subjected to the experimental factor was called the experimental group. The group of pupils that did not receive the experimental factor was called the control group. The experimental factor included the experimental cognitive-constructivist model of mathematics instruction, which we built on the basis of foreign research (Jennings, 1994; Butler & Winne, 1995; in Šteh, 2000), taking into account the Slovenian context. The experimental and control groups were formed by female classroom teachers, who had a similar level of education.

In the study, a pedagogical experiment (the researcher deliberately introduces the experimental factor into the research situation) was used as part of the empirical research approach, as it is appropriate to study novelties (the use of an experimental cognitive-constructivist model of mathematics education).

In order to obtain the most complete data, it was collected quantitatively by means of knowledge tests (two knowledge tests in mathematics).

We designed the experimental model with school classes as comparison groups. The comparison groups were existing third-grade classes at different primary schools.

In this model, there was no randomisation (matching of comparison groups by random selection) of factors related to pupils as individuals, as in the school field, the possibility for the use of models with randomisation is quite limited. However, randomised experimental models are better in terms of internal validity in comparison with non-randomised designs, thus we controlled the most relevant factors (mathematical knowledge) related to pupils as individuals at the beginning of the experiment, by using the analysis of covariance. Teachers were controlled by equating the level of education and years of teaching in the control group (CG) and the experimental group (EG).

The experimental group received the full experimental treatment, which included: (1) modern working methods (problem-based learning, constructivism, project work, etc.) and (2) pupil-centred classroom instruction.

2.2. Sample

The study was performed on a sample of 252 pupils of the 3rd grade in four randomly selected primary schools, namely a100 pupils constituted the experimental group and 152 pupils the control group.

In addition, 10 female classroom teachers with the average age of 36 years participated as well. The average age of the teachers who taught the control group pupils was 37 years, while the average age of the teachers who taught the experimental group pupils was 34.5 years. The age of the female teachers ranged from 31 to 38 years. In terms of educational level, the teachers of the control and experimental groups were completely equal, as all of them had a high level of education.

2.3. Mathematics lessons in 3rd grade of the control group (CG) and the experimental group (EG)

Pupils in CG and EG were taught mathematics according to the current curriculum (Učni načrt, 2011). In the control group, the teachers taught lessons using traditional methods, with mostly frontal instruction. The traditional teaching methods used were mainly explanation, demonstration, and discussion. In the experimental group, the teachers used more modern teaching methods (discussion, debate, experiment, experiential learning, project work, problem-based learning, etc.) and performed pupil-centred teaching in all mathematics subjects. Thus, the experimental group received one mathematics lesson each week, in which they actively participated with the help of prepared mathematical materials on a specific mathematical topic.

The mathematics teaching in the experimental group was dominated by problem-based learning, which is a way of learning in which the pupils, alone or in a group, with great or little help from the teacher, find their own way from a problematic situation to its solution. The teachers placed great emphasis on the path/method of solving a problem, as pupils learned to acquire new knowledge through their own mental activity and through their own cognitive structures and abilities.

As the pupils were performing project-based learning, the teachers encouraged, guided, and helped them to learn or to perform the activities which the pupils had taken on when planning the execution of the project. In this way, the pupils learned independently with the indirect support of the teacher. At the beginning of the project work, the pupils, with the help of the teachers, set appropriate mathematical objectives, where both the pupils and the teachers took into account that the pupils were the main actors of the activities, and the teachers were merely the initiators and advisors. The important thing was that the implementation and the content of the activities followed a set plan.

Teachers had a crucial task in the experimental group, as they had to make sure that they offered pupils varied and, above all, relevant mathematical problem situations, being extremely careful not to give the pupils any clues for solving the mathematical problems, but at the same time guiding the pupils towards the appropriate solution path.

2.4. Measuring instruments

In this study, we complemented the traditional methodology of empirical research with a qualitative methodology of pedagogical research. The study used a pedagogical experiment as part of the empirical research approach. For the purpose of the study, we designed two knowledge tests for the pupils.

The initial and final mathematical knowledge of the experimental and control groups was tested by means of an initial (Appendix A) and a final (Appendix B) knowledge test. The tasks in both tests were designed considering Gagné's taxonomy and the 3rd-grade mathematical content (Učni načrt, 2011). The characteristics of both knowledge tests were demonstrated on a pilot sample of 102 third-grade pupils in two randomly selected coastal schools. The initial knowledge test consisted of 16 tasks. The pupils in the study took the test in one day, for 2 school hours. The final knowledge test consisted of 19 tasks. This test was also taken in one day, for 2 school hours. The initial and final knowledge tests were comparable both in terms of content and taxonomic levels.

The initial and final tests of mathematical knowledge were initially tested in one primary school in order to guarantee the objectivity, validity, and reliability of the instrument. The objectivity of the instrument was checked by comparing the scores assigned by three independent evaluators: two teachers (T1, T2) and one researcher in the field of mathematics education (R). The Pearson correlation coefficient between T1 and T2 was $r=1.00$ ($p < .001$), between T1 and R was $r=.99$ ($p < .001$), and between T2 and R was $r=.99$ ($p < .001$).

The face validity was checked with the aid of two independent primary school mathematics teachers, who assessed the measure to which the instrument is supposed to measure students' mathematical knowledge. The initial draft of both the initial and final test of mathematical knowledge was checked by the two teachers and all the problems were deemed to be suitable for the aims of the present study. The content validity was assured since all questions regarded the topics present in the National Curriculum of mathematics (Učni načrt, 2011).

To assess the reliability of both the initial and final tests, the method of parallel forms was applied (Revelle & Condon, 2019). The same pupils in the pilot study were tested twice; on the second testing session, we adopted a parallel test (i.e., a test that was constructed in such a way that it measures the same constructs as the first one). To this end, two initial tests (1A and 1B) and two final tests (2A and 2B) were given to the pupils. The reliability was assessed by comparing pupils' achievements on both tests with Pearson's correlation coefficient. For the initial tests, the correlation was $r=.99$ ($p < .001$), while for the final test, it was $r=.82$ ($p < .001$).

2.5. Data collection procedure

The initial knowledge test (Appendix A) was administered to the control and experimental groups before the beginning of the experiment, and the final knowledge test (Appendix B) was administered after concluding the experiment, under the same conditions and with the same tester. The study was performed over a period of 8 months.

During the time of the study, the experimental group was being taught mathematics in the 3rd grade by using an experimental cognitive-constructivist model of teaching. In mathematics lessons, the experimental group teachers delivered the current teaching material by using the experimental cognitive-constructivist model of teaching, which we had built on the basis of foreign research (Jennings 1994, Butler & Winne 1995, in Šteh 2000), in consideration with the Slovenian context.

At the end of the study, we measured the differences in maths knowledge between the experimental and control groups with a final test, consisting of 19 tasks. In the final knowledge test, as in the baseline test, the tasks required pupils' knowledge of all taxonomic levels of Gagné's taxonomy to be adequately solved and included tasks that tested basic, conceptual, procedural, and problem-solving skills. The pupils of the experimental and control group took the test on the same day, for two school hours.

2.6. Study process

The study was performed in four phases over a 2-year period (end of 2019 - beginning of 2021). More detailed information on the process of the study is presented in Table 2.

Table 2. Study phases of the introduction of an experimental cognitive-constructivist model of teaching.

Phase	School year	Description
1st phase	2019/20	Training of researchers (study of literature, contacts abroad, purchase of didactic resources, workshops, job interviews, etc.). Formation of the experimental and control groups of teachers and preparation of the experimental group teachers for the experiment.
2nd phase	Beginning 2020/21	The first empirical recording - testing of baseline knowledge prior to the introduction of the experimental factor in the 3rd grade of primary school.
3rd phase	2020/21	Introduction of the experimental factor into the experimental group.
4th phase	End 2020/21	The second empirical recording was performed - testing the pupils' knowledge at the end of the experiment.

2.7. Data analysis

In the present research, both descriptive and inferential statistical tools were used. In particular, we computed the means, standard deviations, modes, and medians of all data, as well as the skewness and kurtosis of points of specific taxonomic levels. Both groups followed a normal distribution but did not have the same variance (which was checked using Levene's test of equality of variances). In order to determine possible differences between the achievements of the experimental and control group on the final knowledge test, Welch's t-test was used. All data were analysed using the SPSS (v. 26.0) statistical software. The results were interpreted in line with proving the hypotheses. In doing so, we took into account that the maximum tolerable risk of rejecting a hypothesis is 5% (the value chosen for the significance level is 0.05).

2.8. Ethical considerations

The experiment was conducted following the ethical considerations of pedagogical experimentation. Signed informed consent was collected from the parents of all the involved pupils. Parents and pupils were informed of the aims of the present research, the anonymity of the collected data, and the possibility of withdrawing from the study at any time without consequences. All researchers referred to The European Code of Conduct for Research Integrity (Allea, 2017).

3. Results

3.1. Analysis of mathematical knowledge differences at baseline

There were no statistically significant differences in mathematics knowledge between the experimental (M_{EG}=84.5) and control group (M_{CG}=76.5) at the initial knowledge test (see Table 3), as all the classroom teachers included in the study taught according to the concept of National Curriculum (Učni načrt, 2011) with traditional teaching methods and forms of work. In particular, no differences between the experimental and control group were detected among the three Gagné's taxonomic levels ($p > 0.05$).

Table 3. Display of the differences between the experimental and control group pupils' performance on the initial knowledge test (*t*-test).

	Levene's test		*t*-test	
	F	*p*	*t*	*p*
Total	3.950	0.060	1.725	0.087
I	1.797	0.183	1.816	0.072
II	0.073	0.869	0.075	0.947
III	0.089	0.766	0.484	0.629

3.2. Analysis of mathematical knowledge differences on all three levels of skills in the final state

Table 4 presents descriptive statistics for the dependent variables included in the analysis of variance. The analysis of differences in mathematical knowledge among the pupils from the experimental and control groups was performed using the analysis of variance and the *t*-test.

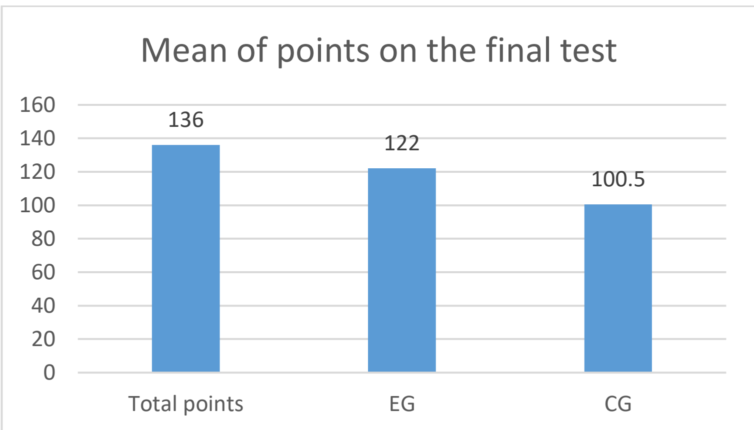
Table 4. Descriptive statistics parameters for the dependent variables of the final test.

Taxonomic level	*n*	*M*	SE_M	*Mdn*	*Mo*	*SD*	*VAR*	*Skew*	*Kurt*	*min*	*max*
I	240	38.950	0.718	41.000	45.000	7.875	62.031	-1.218	1.151	13.000	48.000
II	240	48.183	0.908	51.000	58.000	9.948	98.975	-1.164	0.985	15.000	59.000
III	240	22.300	0.565	24.000	28.000	6.196	38.397	-0.970	0.229	4.000	29.000

The table of parameters for each variable shows that most of the variables are not normally distributed, therefore, in addition to the arithmetic mean and standard deviation, we also calculated the degree of asymmetry, which is shown by the skewness coefficient. In our case, there is a predominant asymmetry to the left. From the table of parameters, we can also see the degree of flattening, indicated by the kurtosis coefficient. In our case, more than half of the variables are conically distributed and less than half are flattened.

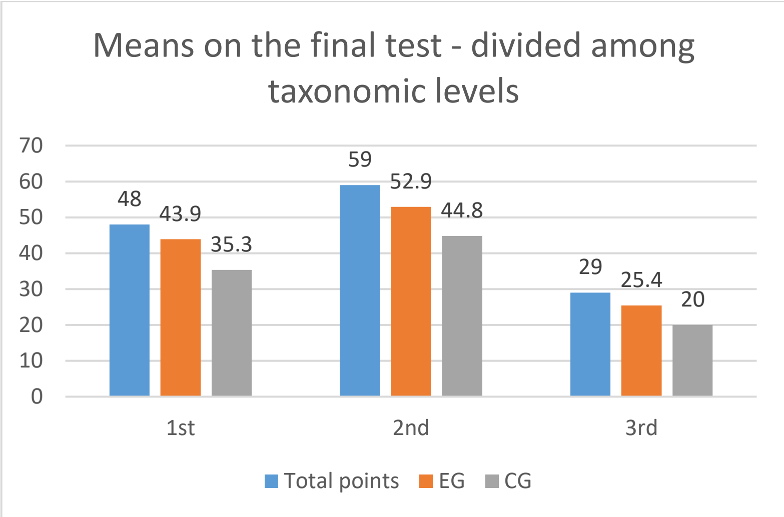
In the final knowledge test, it was possible to score 136 points. The graph (Figure 2) shows that when comparing the arithmetic means of the experimental and control groups, the differences in the number of points scored are very large, since the arithmetic mean of the points scored in the final knowledge test for the experimental group is 122 points, while for the control group, it is 100.5 points, which means that the difference in arithmetic means between the two groups is 21.5 points. With the use of the analysis of variance and the *t*-test (Table 5), we found statistically significant differences in the points scored on the final knowledge test among the pupils of the experimental and control group.

Figure 2. Comparison of the arithmetic means of the scored points obtained between the experimental group (EG) and control group (CG) in the final knowledge test.



The graph (Figure 3) shows that the arithmetic means of the scores on the final test of all taxonomic levels between the experimental and the control group differ on all three taxonomic levels, that is on the basic and conceptual, procedural and problem skills, in favour of the experimental group. Looking at the results in more detail, it can be observed that the experimental group performed on average 8.6 points better on the final knowledge test on the basic and conceptual skills tasks, 8.1 points better on the procedural skills tasks, and 5.4 points better on the problem skills tasks. Using the analysis of variance and the *t*-test revealed that all arithmetic means of the pupils' achievement on all taxonomic levels were statistically significant.

Figure 3. Comparison of the arithmetic means of the scored points at the end of the study between the experimental (EG) and control (CG) group on all taxonomic levels.



The *t*-test was used to determine the statistical significance of the differences between the performance of the experimental and control groups on the final knowledge test (Table 5). The experimental group achieved better results on the entire final knowledge test, on all the tasks of each taxonomic level. The table above reveals that all the differences measured between the experimental and control groups are statistically significant.

Table 5. Display of the differences between the experimental and control group pupils' performance on the final knowledge test (*t*-test).

	Levene's test		*t*-test	
	F	*p*	*t*	*p*
Total	9.471	0.003	6.018	0.000
I	14.851	0.000	6.967	0.000
II	5.794	0.018	4.774	0.000
III	5.472	0.021	5.175	0.000

4. Discussion

Despite the need of achieving lasting and meaningful knowledge for pupils (Bone, 2022; Du, 2021; Reimers, 2021), teachers often look for ways of teaching which would lead to the goals quickly (Andrew, 2007; Lord, 1999). To this end, many teachers prefer to explain the material and the lessons are generally teacher-centred (Felda & Bon Klanjšček, 2017). Rote learning and pupils' passive acceptance of the material might lead to worse performances and learning (Tabriyi & Rideout, 2017). To overcome this obstacle, many researchers have proposed adopting a cognitive-constructivist way of teaching (Chapman et al., 2005; Lofti et al., 2012; Scott, 2006; Thompson, 2020), where pupils construct their own knowledge. This model of teaching, especially mathematics, was seen as a starting point for a possible modernisation of the curriculum (Černilec, 2019). However, despite numerous research have shown the usefulness of the cognitive-constructivist model of learning (Bransford et al., 2000; Ng et al, 2020; Strommen & Lincoln, 1992; Ucus, 2015; Ünal, 2017; Wingfield & Black, 2005) and highlighted the importance of using it in mathematics education (D'Souza & Wood, 2003; Lithner, 2015; Rutar Ilc, 2002), no extensive research has explored its impact on pupils' mathematical knowledge according to Gagné's (1985) taxonomy. In particular, the cognitive-constructivist model helps pupils to learn more effectively new mathematics concepts (Thompson, 2020), however the question whether this model of teaching and learning is effective also on other taxonomic levels remains unanswered by previously mentioned researches. Therefore, the aim of the present study was to examine whether students included in groups that adopted the cognitive-constructivist model of teaching and learning have higher achievements than those students who are not included in such classes.

With the present study, we aimed at validating the general hypothesis that the cognitive-constructivist model of teaching and learning positively affect students' mathematical knowledge at all Gagné's taxonomic levels, i.e. (1) conceptual knowledge, (2) procedural knowledge, and (3) problem-solving. To validate this hypothesis, we performed a pedagogical experiment with an experimental and a control group. Both groups had similar achievements in the initial test of mathematical knowledge, while the achievements of the experimental group in the final test was significantly higher for the experimental group. The final knowledge test included tasks from all four mathematical areas, namely arithmetic and algebra, geometry and measurement, logic and language, and data processing. The test, which was given to 3rd-grade pupils at the end of the school year, was designed to test their knowledge of the mathematical content covered in the third grade. Both the control and the experimental group demonstrated a good level of mathematical knowledge, which is the basis for building on the mathematical content in the following years of schooling.

The pupils of the experimental group scored higher on the overall final knowledge test on all three taxonomic levels (I, II, III). The control group pupils performed best on tasks requiring procedural skills, i.e., knowledge of knowing and applying simple and complex procedures, which can be divided into routine procedural skills (performing routine procedures, applying rules) and complex procedural skills (applying and mastering procedures and algorithms, solving routine text tasks), as they scored highest on these tasks compared to the tasks on other taxonomic levels.

In conclusion, pupils of the experimental group performed best on tasks requiring basic and conceptual skills, which include knowledge of concepts and facts and retrieval of knowledge (cf. Thompson, 2020); knowledge of particulars: knowledge of isolated information, knowledge of specific facts, knowledge of definitions, formulas, theorems, knowledge of terminology, familiarity with basic symbols and terminology, and conceptual knowledge (cf. Thompson, 2020), which in turn consists of understanding of concepts and facts, namely recognition of concepts and recognition of terminology and symbolism.

Both the experimental and control group performed worst on the problem skills tasks, which include problem identification and formulation (formulating a problem from a text or a concrete situation), data checking (checking whether there is enough data, or whether there is missing data), selecting solution strategies (how to solve the problem), applying knowledge and thinking skills (how to solve the problem) and metacognition (did I choose the best way to solve the problem). In these, pupils from both groups scored the lowest of the given points compared to other taxonomy levels. These findings are not surprising, since literature has found that pupils have non-negligible difficulties in planning the solution of mathematics problems (Kusmaryono et al., 2018; Mulbar et al., 2017). Nevertheless, the obtained results disclose that the statistically significant differences in achievements between the experimental and control group in the final knowledge test occurred due to the introduction of the experimental cognitive-constructivist model of classroom instruction, thus shifting the focus of classroom instruction from the teacher to the pupil, and the use of modern methods of work (problem-based learning, constructivism, project work, cooperative learning, etc.). Hence, experimental group pupils performed better on problem-solving tasks, as hypothesised (cf. Amin & Mariani, 2017).

Although the teachers who taught in the experimental and control group taught mathematics according to the curriculum (Učni načrt, 2011), there were statistically significant differences in the experimental group as the concept of mathematics teaching in that group was different. The teachers of the experimental group were very attentive during the educational process in mathematics to design the lessons in such a way that the learner's own activity in the process of acquiring knowledge was at the forefront, by applying our cognitive-constructivist model of teaching (Bransford et al., 2000; Ng et al, 2020; Strommen & Lincoln, 1992; Ucus, 2015; Ünal, 2017; Wingfield & Black, 2005). In this way, the pupils built self-confidence in their own abilities, as the teachers used modern methods of work (interview, discussion, experiment, experiential learning, project work, problem-based learning, etc.; Černilec, 2019) to encourage them to be independent, to be creative and, above all, to be metacognitive, that is to say, to think about the process of solving problems, about the solutions they got and what they learned from the problems (Lithner, 2015; Rutar Ilc, 2002; cf. Amin & Mariani, 2017). In this way, the teachers of the experimental group took on the role of modeller and not only the transmitter of knowledge (Rutar Ilc, 2002; Terwel et al., 2009).

Based on the analysis of the results obtained, we confirmed our general hypothesis: the pupils who receive the experimental cognitive-constructivist model of mathematics instruction will be more successful in solving mathematical tasks on all three levels of skills according to Gagné's taxonomy. Accordingly, it can be concluded that the experimental cognitive-constructivist model of classroom instruction introduced in the experimental group increased memorisation and the knowledge of concepts and facts, as well as the retrieval of knowledge, i.e., knowledge of particulars: knowledge of isolated information, knowledge of specific facts, such as knowledge of definitions, formulas and theorems.

The present study has potential limitations. Pupils from the experimental group had better achievements than pupils from the control group, however we did not check whether these difference are lasting. Therefore, future studies might want to consider follow-ups as well. Moreover, some considerations about the teachers' way of teaching in both groups that were described in the discussion of this paper are based on the authors' in-class observations. Future studies might triangulate our findings with more rigorous qualitative research, such as interviews and focus groups, in order to have a wider picture about the effect of adopting a cognitive-constructivist model of teaching and learning. Furthermore, since in the present experiment randomisation was not performed, results should be interpreted with caution and additional studies are needed to possibly generalise our findings.

5. Conclusions

In today's world, there are many research and studies on how to improve teaching methods. Most of them lead to the same solution, namely a change from teaching to learning (cf. Bone, 2022; Du, 2021; Reimers, 2021). This means that the teacher is no longer at the centre of the educational process, as the learner is taking on an increasingly important role (Lithner, 2015; Rutar Ilc, 2002). Nevertheless, the teacher has a key role in this process, by preparing the material to be covered, guiding and indirectly leading the learners through the learning process, by identifying the achievement of pre-set learning objectives, and by 'correcting' the learner's misconceptions. All this can be done by using modern teaching methods in which the learners' own activity is the main guiding principle (Applefield et al., 2000; Felda, 2011; O'Connor, 2022; Plut - Pregelj, 2003). In learning to learn, the learners build on their previous learning experiences and life experiences in different circumstances (Official Journal of the European Union, 2006).

In the present research, the cognitive-constructivist model for learning mathematics, applied to Gagné's taxonomy levels, was examined. Findings suggest that students who were taught according this model had better achievements on all three Gagné's (1985) taxonomic levels. The study results are useful for planning the learning and teaching of mathematics and for the education of teachers and future teachers of classroom education. A change at the level of educating future teaching staff is imperative or almost necessary, as they often use the very teaching style they themselves received during their education, which is mainly the traditional, transmission mode (Felda & Bon Klanjšček, 2017). This certainty then leads to the fact that the teachers mainly require from their pupils to memorise definitions and concepts and to perform computational algorithms in an automated way (ibid.).

Our study has shown that we can expect a higher quality of knowledge from a pupil if he/she is actively involved in the acquisition of knowledge in the classroom; in conversations with the teacher, through which the teacher determines whether the pupil's findings are in line with what is expected, and engages with classmates to the extent that he/she listens to them and thus obtains different opinions and arguments, which can be compared with his/her own, builds upon or refutes them, and may adopt new ones. The key here is to embed each new piece of mathematical knowledge in mental connections, as this is how the learner increases his/her level of mathematical literacy.

If the cognitive-constructivist model of teaching is to be translated into school practice, it must also be introduced in the teaching of future teachers at the faculties of education. This means that prospective teachers themselves need to become familiar with this model of teaching, since, in addition to their knowledge and beliefs, the experiences they acquire during their schooling influence the way they teach.

Ethical Approval: Informed consent was obtained from all participants for the abovementioned experiment.

Conflict of Interest: Authors declare no conflict of interest.

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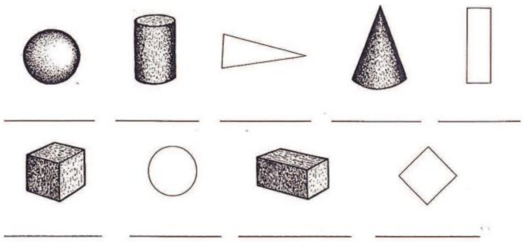
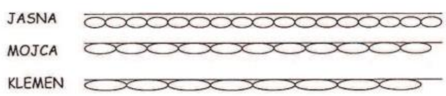
Appendix A

1. Write the names of the following shapes. [9 points]



2. Jasna, Mojca and Klemen measured the length of a rope with their shoes. [6 points]



   a. The rope is long:

      i. ___ Jasna's shoes

      ii. ___ Mojca's shoes

      iii. ___ Klemen's shoes

   b. Whose shoe is the shortest? _____________

   c. Whose shoe is the longest? ____________

   d. Whose measure is the biggest? ___________

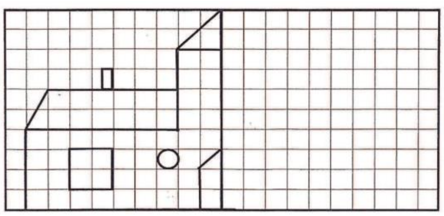
3. Write the following numbers or determine the units (U) and tens (T). [8 points]

   a. 4T 3U = _________
   b. 8T 0U = _________
   c. 10T = _________
   d. 9U = _________
   e. 28 = ____ T ____ U
   f. 60 = _____ T ____ U
   g. 7 = ____ T ____ U
   h. 93 = ____ T ____ U

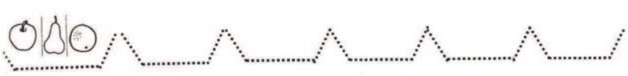
4. Write the symbol >, <, =. [4 points]

   a. 56 ___ 65
   b. 28 ___ 32
   c. 9 ___ 90
   d. 69 ___ 96

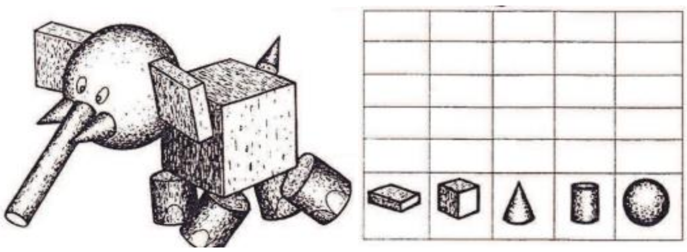
5. Complete the figure in such a way that it will be symmetrical. Colour the picture in a symmetrical was with at least 4 different colours. [3 points]



6. Arrange the following fruits in all the possible ways. [5 points]



7. Count, how many shapes are there in the picture of the elephant. Represent the data with columns [histograms] and answer the questions. [10 points]

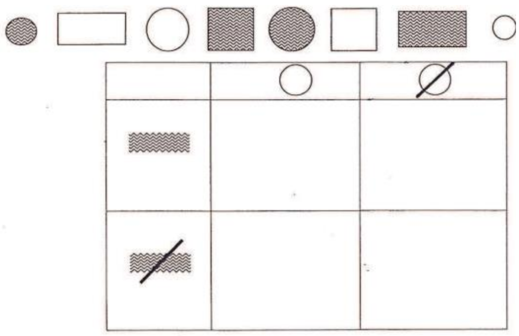


a. How many shapes are there? ______

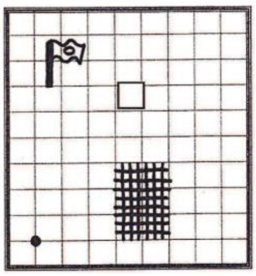
b. Which shape is the least often? _______

c. Which shape is less common than cylinders, but more than rectangular solids? _____

8. Arrange the following shapes in the table. [8 points]



9. Move on the grid as in the instructions. Start from the dot. What are you getting? _______ [2 points]



10. Measure the length of the following lines. On each line write its length and unit of measure. [3 points; 3 lines: the first is 4 cm long, the second is 6 cm long, the third is 2 cm long]

11. Order the following numbers. Start with the biggest. [1 point]

43, 34, 4, 30, 73

12. Order the following numbers. Start with the smallest. [1 point]

69, 96, 66, 99, 9

13. Complete the table. [6 points]

Precedent number	Number	Following number
	28	
		91
	60	

14. Continue the sequence. [4 points]

a. 56, 57, 58, __, __, __, __, __

b. 74, 73, 72, __, __, __, __, __

c. 44, 46, 48, __, __, __, __, __

d. 37, 35, 33, __, __, __, __, __

15. Compute. [32 points]

a. 51+8 = ____

b. 14+4 = ____

c. 11+40 = ____

d. 36+50 = ____

e. 22+34 = ____

f. 51+17 = ____

g. 28-16 = ____

h. 47-14 = ____

i. 92+7 = ____

j. 56+3 = ____

k. 47+20 = ____

l. 38+30 = ____

m. 17+21 = ____

n. 43+25 = ____

o. 39-23 = ____

p. 25-11 = ____

q. 29-2 = ____

r. 48-8 = ____

s. 63-40 = ____

t. 45-20 = ____

u. 51+24 = ____

v. 52+25 = ____

w. 56-23 = ____

x. 64-32 = ____

y. 77-5 = ____

z. 59-4 = ____

aa. 74-60 = ____

bb. 99-30 = ____

cc. 65+31 = ____

dd. 25+34 = ____

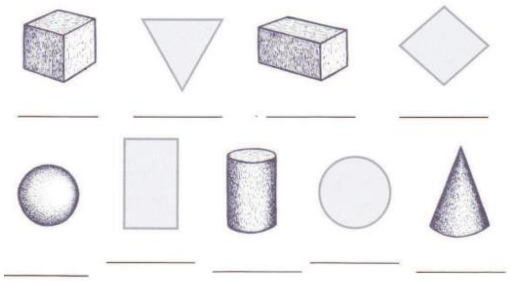
ee. 95-12 = ____

ff. 78-37 = ____

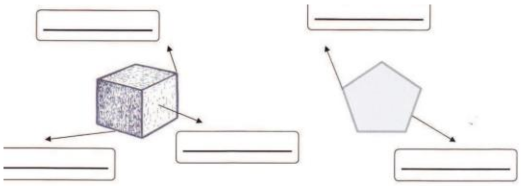
16. Martin had to read a book. The first day he read 24 pages, the second day he read 31 pages. [6 points]
    a. How many pages did he read in two days?
    b. The book has 99 pages. How many pages does Martin still need to read to complete the book?

Appendix B

1. Colour the solids in green and shapes in red. Underneath each geometric shape write its name. Which solid is missing? [12 points]

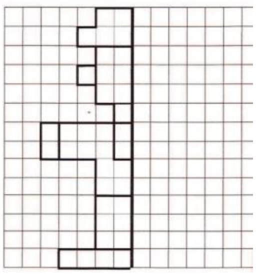


2. Complete. [5 points]



3. Which part of the shape is coloured? [3 points]

4. Complete the figure so that it is symmetric and colour it symmerically. [2 points]



5. Complete the table. [6 points]

Precedent number	Number	Following number
	59	
		341
	999	

6. Compute. [4 points]

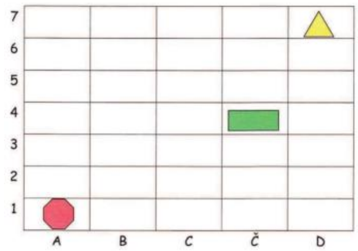
a. 56+23 = ___

b. 628+261 = _____

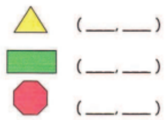
c. 83-50 = _____

d. 758-613 = _____

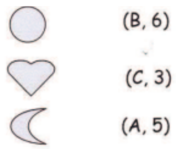
7. Some children are playing hide-'n-seek. Write, where in the following net are collocated the following shapes and draw in it the following [other] shapes. [6 points]



a. Where are the following shapes?



b. Draw the following shapes in the net.



8. Continue the number sequence. [3 points]

a. 29, 30, 31, 32, __, __, __, __, __, __

b. 100, 200, 300, __, __, __, __, __, __

c. 450, 440, 430, __, __, __, __, __, __

9. Order the following numbers. Start with the smallest. [2 points]

500, 100, 450, 1000, 54

Order the following numbers. Start from the biggest.

135, 300, 35, 315, 531

10. Help Ana to complete the table. Measure the lables and write its length in the table. Don't forget the units of measure. [11 points; red = 5 cm; blue = 4 cm; green = 7 cm; yellow = 9 cm; grey = 3 cm]

Lable	Length
Red	
Blue	
Green	
Yellow	

Grey	

a. The longest label is _______

b. The shortest lable is _____

c. By how much is the green lable shorter than the blue one?

d. Order the lables from the longest to the shortest.

11. Three girls and two boys play basketball. Write the pairs of the children in the following table. How many boy-girl pairs are there? [6 points]

	MAJA	URŠA	NEŽA
JANI	JANI, MAJA		
ROK			

12. Write the following numbers [5 points; H = hundreds, T = tens, U = units]

a. 5T 7 U = _____

b. 9T = ___

c. 3H 1T 4U = _____

d. 9H 9D = _____

e. 5H 6U = _____

13. Determine the number of hundreds (H), tens (T), and units (U). [5 points]

a. 290 = ___ H ___ T ___ U

b. 6 = ___ H ___ T ___ U

c. 73 = ___ H ___ T ___ U

d. 898 = ___ H ___ T ___ U

e. 403 = ___ H ___ T ___ U

14. Write the symbol >, <, =. [5 points]

a. 12 __ 21

b. 134 __ 340

c. 300 __ 300

d. 789 __ 700

e. 999 __ 1000

15. Put the following numbers in the following cicles. [9 points; 1st circle = multiple of 3; 2nd circle = not multiple of 3; 3rd circle = not multiple of 2; 4th circle = multiple of 2]

6, 21, 1, 10, 14, 15, 11, 12, 18

16. Compute and write the password. [14 points]

$4 \cdot 3 =$ ___	U	$42 : 6 =$ ___	L
$7 \cdot 8 =$ ___	T	$30 : 3 =$ ___	E
$5 \cdot 6 =$ ___	M	$56 : 7 =$ ___	P
$9 \cdot 9 =$ ___	L	$8 : 2 =$ ___	K
$6 \cdot 10 =$ ___	O	$27 : 3 =$ ___	B
$5 \cdot 7 =$ ___	A	$14 : 7 =$ ___	J
$8 \cdot 6 =$ ___	E	$45 : 9 =$ ___	O

4	30	35	7	12	9	60	8	5	81	48	56	2	10

17. In the 3.A class there are 19 pupils, in the 3.B class there are 26 pupils. [6 points]
    a. How many pupils are there in both classes?
    b. On Monday, 9 pupils of the 3.B went on a math competition. How many students were then in the 3.B class?