



Introducing a software to calculate the grade and ore deposit probability related problems

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Abstract

Different cut-off grades are usually considered for an ore deposit through feasibility study step and according to the distribution model, corresponding tonnage of ore reserve for each assumed cut-off grade is determined. Obviously, each determined tonnage of ore reserve has its individual mean grade. Thus, the tonnage of ore reserve and the mean grade of each hypothetical cut-off grade may be demonstrated as grade-tonnage curve. The tonnage of ore reserve and the mean grade corresponding to each desirable cut-off grade can be determined using alternative approaches. A simple method well-suited to grades having normal distribution has been taking advantage of probability tables and charts prepared based on some mathematical relations. Based on the expanded concept of methodology involved, in this work it was attempted to provide software by means of which the tonnage and grade can be determined and the probability may be analyzed and calculated. Input dataset analysis of less time and more accuracy can be indicated as benefits of this approach while manual operation is certainly time-consuming and of less precision.

Keywords: Normal distribution, Mean grade, probability, Tonnage of ore reserve.

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1. Introduction

Since the determination of recoverable ore and metal content depends on the model of distribution, the first step in estimating the tonnage and mean grade of an ore reserve is to identify the distribution model of grade values [1]. In addition, the estimated tonnage of ore reserve is related to the cut-off grade and corresponding recoverable ore tonnage and metal content illustrated in Grade-Tonnage curves. The distribution model of random variables is normal distribution in many cases as a diversity of ore reserves grade distributions could be referred to [2]. In these cases utilizing the table of cumulative probability of normal distribution [3], and the table of tonnage and mean grade determination in terms of cut-off grade [4], the ore reserve is estimated. In this work, based upon the methodology involved, software is provided to execute the procedure and give out the tonnage of ore reserve and mean grade.

2. Theoretical principles

2.1. Normal distribution

One of the most important continuous probability distributions in probability theory is normal distribution. Denomination root and also importance of this distribution is the coincidence of many obtained values during normal and physical fluctuations around a constant value with obtained values from this distribution. The main cause is the role of normal distribution in central limit theorem. In simple words, it is shown in central limit theorem that under some conditions, sum of obtained values from different variables each of which has limited mean and dispersion, will incline to normal distribution if variable number increases. This law which is also true under normal conditions and assumptions has caused that the result of different fluctuations from many unknown variables exhibit a normal distribution form in nature. For example, while so many variables effect on error measurement of a quantity (e.g., sight error, measurement device error, grades of an ore deposit, etc.), yet by several measurements, result of these errors has always been normal distribution that has been scattered around a constant value [2]. This distribution has been called Gauss function (Gaussian) for the sake of its usage in his own works [2].

1.1.1. Standardization of normal distribution

If normal distribution curve of variable x is like Fig.1 and the probability of the incident between x_1 and x_2 which is equal to the square under the curve in interval of $x_1 < x < x_2$ is desirable; this integral $\int_{x_1}^{x_2} f(x) dx$ should be calculated according to the fact that $f(x)$ is obtained from $f(x) = \frac{1}{(\sigma\sqrt{2\pi})} e^{-1/2((x-\mu)/\sigma)^2}$. Calculation of this integral is not possible in ordinary ways and must be calculated using approximate methods. On the other hand, providing a table of different μ and σ values is not possible. Thus by standardization of normal curve, it is possible to coincide all the societies which has normal distribution and provide a table for this standardized curve. Indeed, if the frequency distribution curve of different normal distributions is drawn, as illustrated in Fig.2, each one has its own particular character that make their comparison difficult. Thus, if the new variable of $x-\mu$ is drawn instead of main variable of x , as illustrated in Fig.3, mean value of all distributions will be coincided. But the curves ranges are still different. To solve this problem, variable $x-\mu$ can be divided by standard deviation to obtain new variable of $Z = (x-\mu)/\sigma$. Advantage of this variable alteration is that variations in the altered curve is appeared in terms of standard deviation, therefore, different curves can be compared easily because their ranges, as seen in Fig.4 are the same.

Standard normal curve with the mean value of zero and standard deviation of one is seen in Fig.5. Ordinary random variable which has the mean value of zero and standard deviation of one is called standard normal random variable. Indeed, the purpose of a variable standardization is to alter the variable such that the mean equals to zero and also the scale replacement is such that standard deviation (or variance) equals to one.

The area under standard normal curve for different z values as presented in the table of cumulative probabilities of normal distribution [3] has been calculated. Therefore, in each case of normal distribution, first z is calculated by $Z = (x - \mu)/\sigma$ and, subsequently, referring to the table of cumulative probabilities of normal distribution [3], the restricted area under the curve is calculated in terms of x. Since the values in the table of cumulative probabilities of normal distribution [3] have been calculated for positive z values, in the case that calculated z is negative, the following relation is applied [1]: $F(-z) = 1 - F(z)$

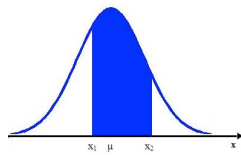


Figure 1. calculation of the square under the normal curve

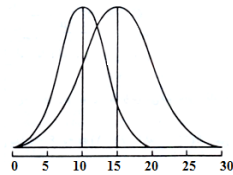


Figure 2. two different normal distribution

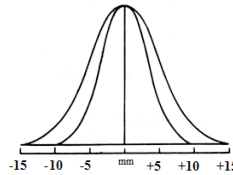


Figure 3. two distributions with common mean of zero

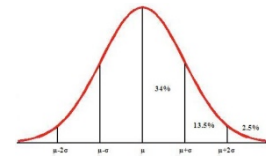


Figure 4. general form of a standard normal distribution

2. The tonnage of ore reserve and metal content of a certain cut-off grade

Within feasibility study step of proving a deposit, various values of cut-off grades are assumed and according to the distribution model of grades, the tonnage of ore reserve corresponding to each cut-off grade is determined [1]. Obviously, each determined ore tonnage has its individual mean grade. Therefore, considering the cut-off grade as a variable, the equivalent tonnage of ore reserve and mean grade may be demonstrated as grade-tonnage curve (see Fig. 5).

The tonnage of ore reserve and mean grade corresponding to each value of cut-off grade can be determined using alternative approaches. A simple method well-suited for the case grades have normal distribution is as follows:

With the grade distribution function of total reserve (universe) that has the mean grade (\bar{x}) available, the ratio of that portion of the ore reserve with grade values greater than the cut-off grade X_c equals:

$$(1) \quad t((x_c)) = \int_{x_c}^{\infty} f(x) dx$$

And the corresponding tonnage of latter portion equals:

$$(2) \quad T((x_c)) = t((x_c)) \times T_0$$

The metal content in grade range of x and $x+dx$ equals $T_0 f(x) dx$. Therefore, the metal content in ore reserve with grade values greater than the cut-off grade x_c equals:

$$(3) \quad q((x_c)) = T_0 \int_{x_c}^{\infty} x f(x) dx$$

As the total metal content is:

$$(4) \quad T_0 \bar{x} = T_0 \int_0^{\infty} x f(x) dx$$

Thus, the ratio of metal content of this part of reserve is:

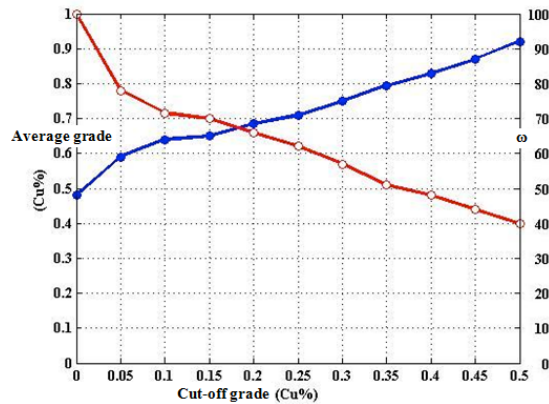
$$(5) \quad Q((x_c)) = (\int_{x_c}^{\infty} x f(x) dx) / (\int_0^{\infty} x f(x) dx)$$

Eventually, the mean grade (\bar{x}) of the (part/portion) with grade values greater than cut-off grade may be determined using following relation:

$$(6) \quad \bar{x}((x_c)) = (T_0 \int_{x_c}^{\infty} x f(x) dx) / (T_0 \int_{x_c}^{\infty} f(x) dx) \\ = (\int_{x_c}^{\infty} x f(x) dx) / (\int_{x_c}^{\infty} f(x) dx)$$

Thus, given the function $f(x)$, the curve of tonnage can be simply delineated in terms of various cut-off grades.

In the case that grade values are normal or lognormal distributed, remarkable relations can be derived.



Cut-off grade greater than mean grade		Altered cut-off grade	Cut-off grade below mean grade	
Ratio of ore tonnage	ω		ω	Ratio of ore tonnage
50	0.798	0	0.798	50
46.02	0.863	0.1	0.735	53.98
42.07	0.929	0.2	0.675	57.93
38.21	0.998	0.3	0.617	61.79
34.45	1.069	0.4	0.562	65.54
30.85	1.142	0.5	0.509	69.15
27.43	1.217	0.6	0.459	72.57
24.2	1.295	0.7	0.411	75.8
21.19	1.375	0.8	0.367	78.81
18.41	1.457	0.9	0.326	81.59
15.87	1.542	1	0.287	84.13
13.57	1.631	1.1	0.251	86.43
11.51	1.724	1.2	0.219	88.49
9.68	1.721	1.3	0.189	90.32
8.08	1.923	1.4	0.162	91.92
6.68	2.033	1.5	0.138	93.32
5.48	2.147	1.6	0.117	94.52
4.46	2.27	1.7	0.098	95.54
3.59	2.403	1.8	0.082	96.41
2.87	2.546	1.9	0.067	97.13
2.28	2.701	2	0.055	97.72
1.79	2.87	2.1	0.045	98.21
1.39	3.055	2.2	0.036	98.61

Figure 5. Grade-Tonnage curve of Songun porphyry copper deposit [5].

2.1. Normal distribution case

If $F(z)$ is a value derived from the table of cumulative probabilities of normal distribution [3], considering the relation (3) we have:

$$(7) \quad F(z) = \int_{-\infty}^z \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt$$

That is called the Gauss' Integral. We have also:

$$(8) \quad t((x_c)) = F((x_c - \mu)/\sigma)$$

And the mean grade of this part of ore reserve equals:

$$(9) \quad \bar{x}((x_c)) = \mu + \sigma \frac{e^{-1/2((x_c - \mu)/\sigma)^2}}{\sqrt{2\pi} F((x_c - \mu)/\sigma)}$$

Or

$$(10) \quad \bar{x}((x_c)) = \mu + \sigma \omega$$

In which ω is defined as:

$$(11) \quad \omega = \frac{e^{-1/2((x_c - \mu)/\sigma)^2}}{\sqrt{2\pi} F((x_c - \mu)/\sigma)}$$

The value of the factor ω as a function of altered cut-off grade is presented in Table 1 (Michael, 1987) [4]. This table also offers the ratio of the tonnage of the ore reserve having grade values greater than the cut-off grade [6].

For instance, take the total tonnage of ore reserve (without cut-off grade considered) that has been evaluated to be 20 million tonnes into account. Based on data from boreholes, the grade values have normal distribution with 1.4(%) and (0.36%²) for mean value and variance respectively. Now, if the cut-off grade is assumed to be 0.5, the probability of grades greater or less than 0.5, the tonnage of ore reserve and the mean grade may be calculated:

$$z = (x_c - \mu)/\sigma = (0.5 - 1.4)/\sqrt{0.36} = -1.5$$

Corresponding to the altered cut-off grade, the ratio of tonnage can be taken from the table [3]. Thus we have:

$$F(-1.5) = 1 - F(1.5) = 6.7$$

This conveys that a random grade value has a probability of 6.7 to be below and a probability of 93.3 to be greater than the cut-off grade. Thus, the tonnage of ore reserve equals:

$$20 \times 0.9332 = 18.664 \approx 18.66$$

Table 1 [4] can also be profitable. Utilizing this table, the ratio of tonnage corresponding to the altered cut-off grade can be obtained that is the same value as derived from Table 1 [4] So as just involved we have:

$$20 \times 0.9332 = 18.664 \approx 18.66$$

To determine the mean grade of the ore reserve corresponding to the cut-off grade 0.5, Table 1 [4] can be utilized again. The value $\omega = 0/138$ that matches the altered cut-off grade $z=1.5$ is taken from Table 1 [4]. The mean grade corresponding to this cut-off grade may be calculated utilizing following relation:

$$\bar{x}_{(x_c)} = \mu + \sigma\omega = 1.4 + 0.138 \times \sqrt{.036} = 1.483$$

3. Probability, mean grade and tonnage of ore reserve calculation using MATLAB programming language

There are two approaches to determining the probability of grade values greater or below the cut-off grade as involved. In the following sections the software model of calculating desired probability, mean grade and tonnage of ore reserve is revealed.

3.1. description of calculating the probability of normal distribution via MATLAB programming language

A subroutine named `Get_data_table.m` with the grades and cut-off grade as input data was coded. This subroutine exhibits the altered cut-off grade z , mean value, variance and standard deviation as output data. The table contents are entered the software through initial subroutine and subsequently the output of the first subroutine is used as input for the second subroutine `Ehtemal.m`. The recent subroutine takes the tenth and hundredth part of the altered cut-off from the table of cumulative probabilities of normal distribution values [3] respectively and then records the corresponding value as the probability of the grade values below the cut-off grade.

In the case that the altered cut-off value has thousandth part, the subroutine is coded to treat interpolation and calculate the corresponding value.

3.2. description of calculating the probability, tonnage of ore reserve and mean grade of normal distribution via MATLAB programming language

A subroutine named `Get_table2.m` was coded with the cut-off grade and altered cut-off grade values (derived from the second subroutine) as input. The subroutine introduces the Table 1 [4] to the software and then works out the prior parameters as well as the Table 1 [4] as output and, moreover, input for the fourth subroutine named `Final.m`. The fourth subroutine includes these data and controls the cut-off grade z , in the case that z is positive, investigates through the altered cut-off grade column of the Table 1 [4] and then takes the corresponding tonnage ratio and w from the left side of the Table 1 [4]. In the case that z is negative, the subroutine turns it to positive and then treats as in the prior condition but takes the corresponding tonnage ratio and ω from the right side of Table 1 [4]. If the altered cut-off grades is not exactly one of the prepared z values (more often), the subroutine treats interpolation to calculate corresponding tonnage ratio and ω .

3.3. Main program

A more comprehensive program named `TT` was coded. The program is comprised of all prior subroutines involved, i.e., recognizing the grade and cut-off grade values as the input data and working out the following as the output data: the mean value (m), variance (v), standard deviation (e), altered cut-off grade (z), the probability of grade values below the cut-off grade,

the probability of grade values greater than the cut-off grade, the ratio of tonnage (ω), the tonnage of ore reserve and the mean grade (x).

The program was coded to announce the total processing time as a value in terms of second.

4. Conclusion

Based on the expanded concept of methodology involved, in this work it was attempted to provide a software by means of which the tonnage and grade can be determined and the probability may be analyzed and calculated. Input analysis of less time and more accuracy can be indicated as benefits of this approach while manual operation is certainly time-consuming and of less precision. In the instance copper sulfide involved, the derived are $m=1.4$, $v=0.36$, $e=0.9$, $z=-1.5$, probability=6.7, $\omega=0.138$, $t=18.66$, $x=1.483$, and the processing time of 0.18s. Considering the reasonably high accuracy and processing velocity, the illustrated methodology could be suggested as a manner of approaching the solutions of tonnage and grade issues.

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