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Initial value problems: a didactic tool in evolution

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Abstract

Various science and engineering problems, generally time-dependent, are modeled with ordinary differential equations with conditions at the same value of the independent variable, called initial value problems. The use of technological resources during the learning process allows students to execute routine procedures quickly and simply, enabling them to have more time to discuss concepts. The use of these resources helps to generate a learning environment where, through the use of certain mathematical skills, students can achieve meaningful and comprehensive learning. This work aims to show an update of a self-designed tool to approximate the solution of initial value problems that allows comparisons between some one-step methods and multi-step methods and the analysis of the numerical solutions obtained when varying certain parameters. The update of the tool arises to satisfy different requirements discovered after the use of the previous tool.

Keywords: Digital; educational resource; initial value problems, SciLab.

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1. INTRODUCTION

Today's engineers, with the help of increasingly adequate mathematical models, use numerical approximation methods to solve problems whose mathematical representation is given by differential equations [1]. Hence the important for engineering students to learn these numerical methods to be able to solve the different situations that may arise, either in their academic training or in their jobs as professionals used as a method of assessment as discussed by Chen et al. [2] and Babović et al. [3] in their researches.

However, learning the numerical methods that allow for solving differential equations is not easy because, in general, their teaching is usually characterized by emphasizing mechanical procedures and the memorization of concepts, definitions, and techniques. Thus, Artigue [4] argues that various studies demonstrate that university teaching tends to focus on algorithmic and algebraic practice and essentially evaluate the skills acquired in this domain. This situation leads to a partial learning of numerical methods and many times difficulties arise in the understanding of fundamental concepts.

The rote learning of procedures is not enough to achieve the comprehensive training of future engineers. The new profile of students requires the development of skills where not only problem-solving knowledge is acquired, but also the skills to use them in daily and specific contexts of the profession [5].

One of the most important challenges when teaching is that of designing meaningful didactical situations for students which also allow them to take an active and leading role in their learning process. In this sense, digital resources become powerful partners, as they let students represent, discover, experiment, simulate, and validate built models, using numerical algorithms to make calculations.

The Engineering & Education Group, from Facultad Regional San Nicolás, Universidad Tecnológica Nacional, deals, among other tasks, with the design of didactical resources for teaching mathematics in engineering careers. Since 2008, many digital educational resources have been designed to solve many mathematical issues by applying numerical methods: nonlinear equations, linear equation systems, definite integrals, and ordinary and partial differential equations.

The first didactic tool was designed to help in the teaching of numerical methods for solving Initial Value Problems in 2011, implementing one-step methods [6]. In 2021 seemed to be interesting to add multi-step methods in this tool, to compare the two kinds of methods, therefore it was modified [7]. Nevertheless, some requirements were detected while using the new tool, for example, the value of the absolute error in the points of the discretization, or the implementation of Taylor´s methods. Hence, a new update of the tool was undertaken.

1.1. Purpose of study

The objective of this paper is to show the improvements made on the last version of the tool designed to approximate the solution of an Initial Value Problem, where comparisons can be made between one-step and multi-step methods.

2. METHODS AND MATERIALS

This app also makes it possible to analyze the numerical solutions obtained when varying certain parameters. The methods now available are Euler, Taylor of the second order, Runge-Kutta of the second order, Runge-Kutta of the fourth order, Adams-Bashforth, and Adams-Bashforth-Moulton.

Also, some classroom activities involving the use of the tool will be presented, together with the rubrics that were designed to analyze the mathematical abilities that students will display when solving those tasks.

3. RESULTS

3.1. Numerical methods for initial value problems

Many engineering problems are governed by ordinary differential equations (ODEs). Depending on the imposed additional conditions, two types of problems can be distinguished. If all the conditions are specified in the same value of the independent variable, the problem is an initial value problem (IVP), on the other hand, if the specification of the conditions occurs in different values of the independent variable, the problem is a boundary problem.

To approximate the solution of an IVP, it must be well posed, meaning that it admits a unique solution and that small variations in the equation or the conditions do not greatly affect the solution. Numerical methods developed to solve first-order IVP start from a discretization of the domain of interest, as a set of equispaced points. At these points, a recursive formula is applied, particularly for each method, which uses one or more of the previously calculated values. This allows us to classify them in one-step methods, or multi-step methods.

In general, the issue is tackled by first studying Euler's method to approximate the solution of a first-order IVP. This is due to its simplicity, both in its formula and in the error analysis. Then, based on the requirement of better approximations, higher-order Taylor methods are developed. In these cases, higher-order Taylor polynomials are required. This fact causes a major drawback in these methods: the computation of higher order derivatives, which can be quite complicated, and increases the number of calculations. For this reason, Runge – Kutta methods (RK methods) are presented, as they were developed in such a way that the order of the final global error is the same as the one of the corresponding Taylor´s method, but avoiding the evaluation of the derivatives. This fact makes it possible to implement RK methods in any programming language. The formula of RK methods includes the evaluation of the function at some points within the points of the mesh, in each iteration.

Multistep methods, although they do not improve the order of precision of the RK methods of the same order, optimize the number of operations needed to determine the numerical solution by considering values already obtained in the previous iterations [8-10].

3.2. Mathematical skills

Skills can be defined as a system of actions that allows carrying out a specific activity based on habits and acquired knowledge [11]. These are formed and developed through the exercise of mental actions and continuous training, becoming modes of action that provide solutions to theoretical and practical tasks. This is an evident situation in the process of teaching and learning mathematics because each content in this field, by nature, requires a way of acting with specific characteristics [12].

3.3. Revised Bloom's Taxonomy

One of the best-known classifications of cognitive abilities is that given by Bloom's revised taxonomy [13]. This taxonomy makes it possible to understand how students learn and lays the foundations at each level of learning to ensure meaningful learning and the acquisition of skills that allow the use of constructed knowledge. Bloom's revised taxonomy distinguishes six levels that students must overcome for a true learning process to take place. The first three levels correspond to the lower-order skills and the remaining, to the higher-order ones.

Table I shows these levels, a brief description, and some indicative verbs of the cognitive work that students should perform at each level.

TABLE I
COGNITIVE LEVELS IDENTIFIED BY THE REVISED BLOOM'S TAXONOMY

Level	Description	Indicator verbs
Remember	Students try to remember the acquired knowledge about facts, terminology, schemes, processes, or theories.	Quote, define, search, enumerate, write, memorize, say, enumerate, indicate, mention.
Understand	Students apprehend the content and try to generalize and relate it to each other.	Determine, conclude, estimate, associate, compare, generalize, distinguish, relate, explain, interpret.
Apply	Students use the acquired knowledge in an activity or practice.	Calculate, complete, execute, use, show, modify, choose, employ, operate, solve, tabulate, perform.
Analyze	Students break down the given problem into different parts and analyze the relationships between them.	Study, debate, criticize, deduce, differentiate, integrate, classify, associate.
Evaluate	Students make value judgments based on criteria through verification and criticism.	Estimate, check, formulate, choose, justify, argue, value, discuss.
Create	Students rearrange elements into a new structure through planning or production.	Generate, design, elaborate, devise, formulate, build, invent, modify, develop, produce.

3.4. Digital educational resources

Digital educational resources (DER) refer to digital resources such as applications (apps), software, programs, or websites that engage students in learning activities and support students' learning goals [14]. These types of resources not only help the learning of conceptual content but also the development of procedural skills [15,16].

3.5. Self-designed digital educational resources

To determine the main characteristics of a DER, it is necessary to define in advance which are the learnings that students are expected to achieve while using this tool, and the difficulties they would face [17]. Therefore, the DER shown in Fig. 1 was designed, seeking to approximate solutions of IVPs like the one presented in (1),

$$y'(t)=f(t,y) \qquad a\leq t\leq b \qquad y(a)=\alpha \quad (1)$$

by applying any of these methods: Euler, Taylor of second order, Runge-Kutta of second order, Runge-Kutta of fourth order, Adams-Bashforth, and Adams-Bashforth-Moulton.

To obtain an approximate solution for a particular IVP, users must first indicate the coefficients of the differential equation, the interval where the solution will be obtained, and the initial condition. If Taylor´s method is going to be used, the first derivative of the function f(t, y) must be written. It is possible to enter the law of the exact solution of the loaded PVI (if it is known), to be able to make comparisons and determine errors.

At the bottom of the DER, up to three options can be selected to perform different approaches at the same time, where the method and the number of points where the solution is calculated are chosen by the user.

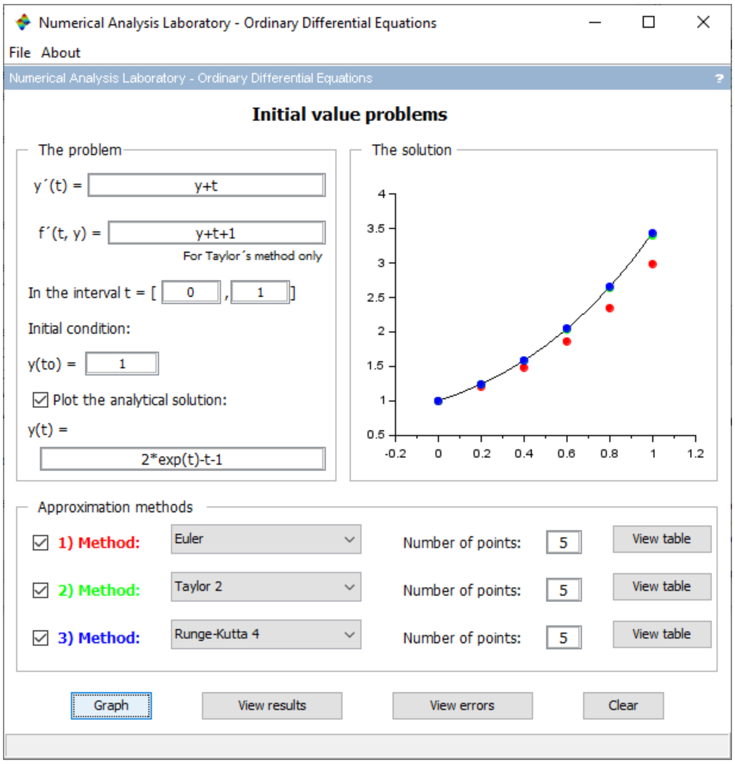
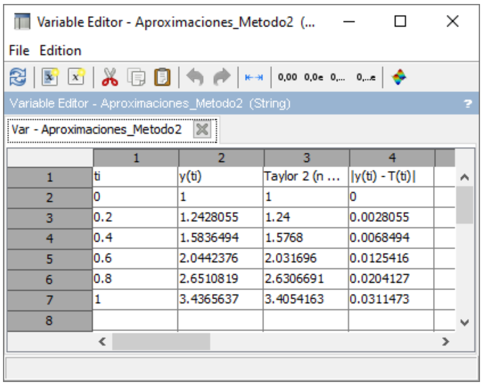


Fig 1. Self-designed digital educational resource

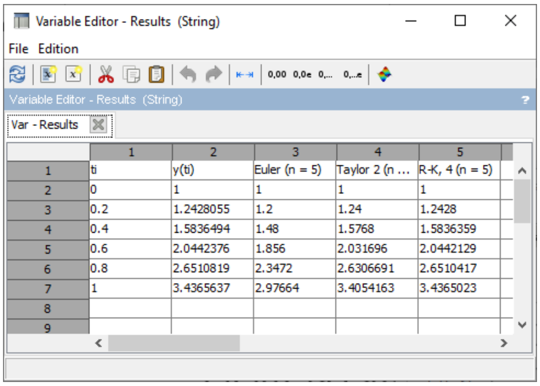
With this resource the numerical solution obtained can be analyzed from two points of view: the graphical and the tabular one. It is possible to obtain the graphic representation of the discrete solution in a system of coordinate axes using different colors to distinguish the solutions corresponding to the selected methods and also, the tabular representation can be shown to compare the different approximations calculated and analyze the behavior of the numerical solution concerning the exact solution, in case it was loaded. To obtain the graphical representation of the calculated approximations, it is necessary to press the "Graph" button. In this way, in "The solution" sector, the points corresponding to the different approximations are shown, using different colors, as indicated in the options at the bottom.



	1	2	3	4
1	ti	y(ti)	Taylor 2 (n ...	|y(ti) - T(ti)|
2	0	1	1	0
3	0.2	1.2428055	1.24	0.0028055
4	0.4	1.5836494	1.5768	0.0068494
5	0.6	2.0442376	2.031696	0.0125416
6	0.8	2.6510819	2.6306691	0.0204127
7	1	3.4365637	3.4054163	0.0311473
8				

Fig 2. Output is obtained by pressing the View Table button

The numerical solution represented in tabular form for each method selected is obtained by pressing the "View Table" buttons at the right of the option. In this case, as can be seen in Fig 2, the first column indicates the points where the solution is being calculated, the second shows the value of the exact solution, and the next pair of columns gives the calculations obtained by applying the method chosen and the absolute error in each one of the points. If the law of the analytical solution of the problem had not been loaded, the second and fourth columns are not shown.



	1	2	3	4	5
1	ti	y(ti)	Euler (n = 5)	Taylor 2 (n ...	R-K, 4 (n = 5)
2	0	1	1	1	1
3	0.2	1.2428055	1.2	1.24	1.2428
4	0.4	1.5836494	1.48	1.5768	1.5836359
5	0.6	2.0442376	1.856	2.031696	2.0442129
6	0.8	2.6510819	2.3472	2.6306691	2.6510417
7	1	3.4365637	2.97664	3.4054163	3.4365023
8					
9					

Fig 3. Output obtained by pressing the button View results

To compare the results obtained between the methods selected it is necessary to press the "View results" button. In this way, the output shown in Fig 3 is obtained. Before displaying the values, the number of points where the solution is going to be shown must be selected (as each method could have been applied with a different number of points).

3.6. Use of the digital educational resource in the classroom

The use of the presented DER while teaching methods for solving IVPs will allow to: obtain an approximation to the solution of the IVP loaded with a certain degree of precision; promote that the learning of numerical methods is not limited to the mere repetition of formulas and mechanical steps, but to the analysis and interpretation of results; compare the numerical solutions obtained and analyze how the different parameters involved in the numerical solution influence; address situations where the student, through the execution of examples selected by the teacher, discovers and understands concepts; strengthen and develop certain mathematical skills by solving different proposed problems [18].

3.7. Proposed activities

Some examples that can be worked on using the designed resource in class are presented here.

First example

Using the Runge-Kutta method of order two, approximate the solution of the following IVP,

$$y'(t)=y(t)+t \qquad 0\leq t\leq 2 \qquad y(0)=1 \quad (2)$$

taking as step size:

a) h = 0.5

b) h =0.25

c) h = 0.125

The analytical solution to the proposed problem is given by:

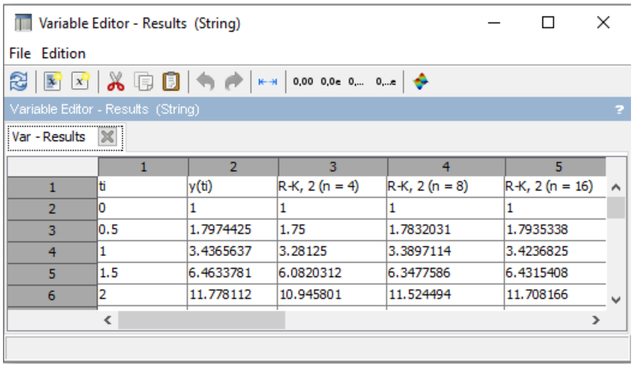
$$y(t)=2\,.e^{t}-t-1 \quad (3)$$

Which is the behavior of the numerical solution obtained as the number of points considered in the discretization is increased?

So as students understand the concept of convergence of a numerical method, the example proposes to solve the problem (2) by taking smaller and smaller step sizes.

When looking at the tabular output, students will be able to conclude that as the step size is smaller, the numerical solution becomes closer to the analytic one. It should be noted that this happens if the numerical method applied is stable for the considered problem and step.

Fig 4 shows the values of the three approximations obtained with the Runge-Kutta method of order two, with different step sizes, in the shared points of the different discretizations. The first column shows the domain points where the solution and the approximations are calculated, the second column presents the values of the analytical solution, and the third, fourth, and fifth columns display the numerical solution obtained using the specified step sizes.



Variable Editor - Results (String)

File Edition

Variable Editor - Results (String)

Var - Results

	1	2	3	4	5
1	ti	y(ti)	R-K, 2 (n = 4)	R-K, 2 (n = 8)	R-K, 2 (n = 16)
2	0	1	1	1	1
3	0.5	1.7974425	1.75	1.7832031	1.7935338
4	1	3.4365637	3.28125	3.3897114	3.4236825
5	1.5	6.4633781	6.0820312	6.3477586	6.4315408
6	2	11.778112	10.945801	11.524494	11.708166

Fig 4. The tabular output of the first example

This example is also interesting to analyze the relationship between the error and the method´s order. Table II shows the global final errors for the different step sizes considered. As it can be seen, when approximating y(2), the error decreases nearly by a factor of 0.25 when the step size is halved. This happens because a second-order method was used.

TABLE II
RELATIONSHIP BETWEEN STEP SIZE AND FINAL GLOBAL ERROR

Step size	**Number of steps**	**y (2) approximation**	**Final global error**
0.500	4	10.945801	0.8323114
0.250	8	11.524494	0.2536178
0.125	16	11.708166	0.0699464

Step size Number of stepy(2) approximation Final global error
0.500 4 10.945801 0.8323114
0.250 8 11.524494 0.2536178
0.125 16 11.708166 0.0699464

Second example

Consider the IVP:

y´(t)+0,5 .t^2=5 0≤t≤5 y(0)=0 (4)

a) Obtain an approximation of the solution using the Euler, Runge – Kutta of order two, and Adams – Bashforth methods with h=1.

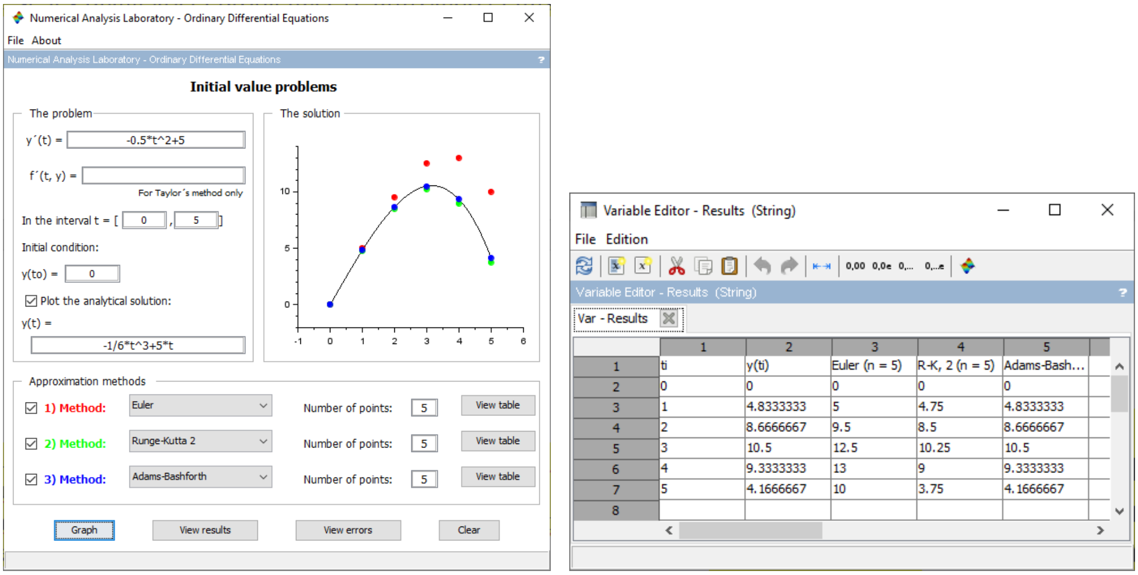
b) Considering that the exact solution of the proposed problem is given by:

y(t)=-1/6.t^3+5.t (5)

analyze the different numerical solutions obtained, and conclude.

To help students understand the influence of the precision order of a method on the numerical solution, they will solve problem (4), using the same step size.

By visualizing the graphical representations of the obtained solutions, as Fig 5. shows, students can predict that it is possible to obtain better approximations when using higher-order methods. This conclusion is enforced, as can be seen in Fig 5. b, with the analysis of the tabular register.



(a) (b)

Fig 5. Graphical and tabular outputs of the second example.

Caligaris, M., G., Rodríguez, G., B. & Laugero, L., F. (2023). Initial value problems: a didactic tool in evolution. Global Journal of Computer Sciences: Theory and Research, 13 (2), 109-120.

Third example

The function

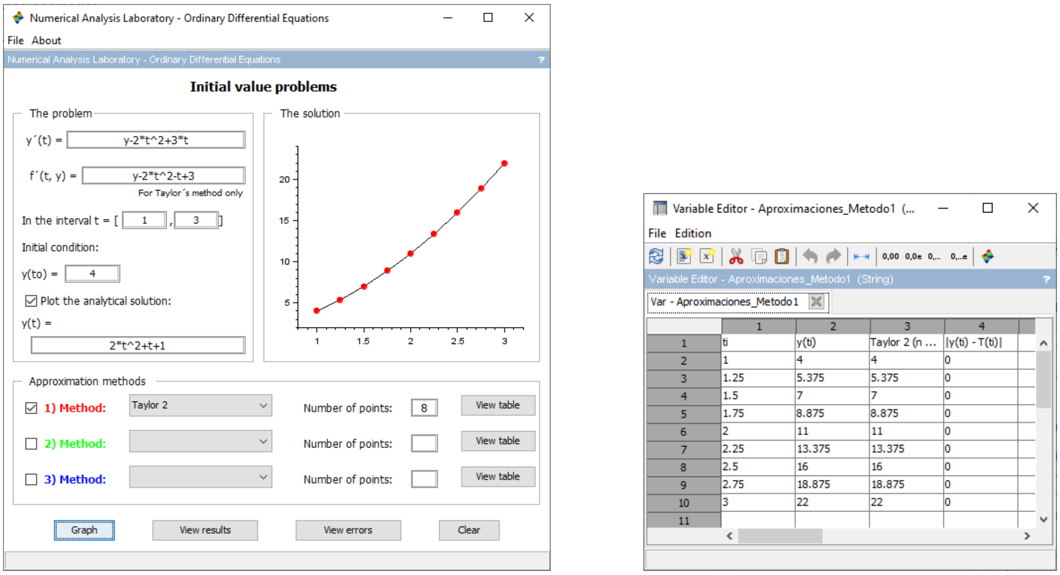
$$y'(t)=2.t^2+t+1 \quad (6)$$

is the exact solution to the IVP problem:

$$y'(t)=y(t)-2.t^2+3.t \qquad 1 \le t \le 3 \qquad y(1)=4 \quad (7)$$

Apply the second-order Taylor's method to approximate the solution, taking a step size h = 0.25. What can be said about the obtained numerical solution and the exact one?

This exercise aims to discuss with students if it is possible to obtain exact results with a numerical method.



(a) (b)

Fig 6. Graphical and tabular outputs of the third example.

When analyzing the results shown in Fig 6. students would say that the approximations obtained seem to be on the analytical solution´s graph. This appreciation is verified by analyzing the tabular output (Fig 6. b).

This example allows the introduction of the following property: the second-order Taylor´s method gives exact results when the solution of the initial value problem is a polynomial of degree two or less. It is easy to demonstrate, as it is a consequence of the expression of the truncation error of the method because it has the second derivative of the function as a factor, which is null.

Fourth example

Consider the IVP:

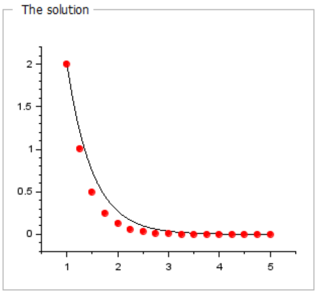
$$y'(t)=-2.y(t) \qquad 1 \le t \le 5 \qquad y(1)=2 \quad (8)$$

a) Use Euler's method to approximate the solution, taking 3, 4, 6, and 16 points respectively.

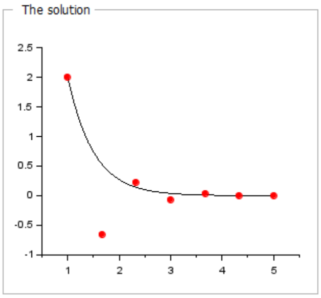
b) Knowing that the analytical solution of the proposed problem is given by:

y(t)=2 .e^(-2t+2) (9)

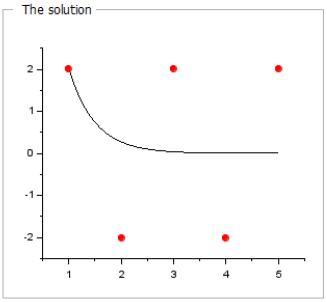
Analyze the numerical solutions obtained in each case. Are they adequate? Justify the answer theoretically.



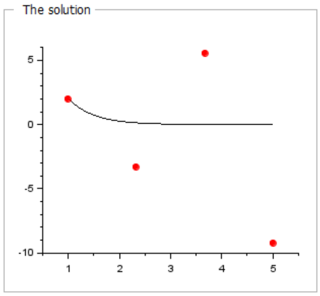
n=16



n=6



n=4



n=3

Fig 7. Graphical output of the fourth example.

Figure 7 shows the graphical output of the problem solution. Students will be able to determine that the numerical solution obtained, when 16 points are taken, provides a very good approximation to the exact solution.

When 6 points are considered in the discretization, the numerical solution oscillates around the exact solution. However, it gets closer to the exact one as the value of the independent variable increases.

When 4 points are taken, meaning that the step size h = 1, the numerical solution also oscillates around the exact solution but, unlike the previous case, it does not get closer to the exact solution while the independent variable increases.

Finally, when n = 3 (consequently, the value of the step size h is greater than 1) the numerical solution as the independent variable increases, grows exponentially.

Students should conclude that Euler's method is conditionally stable. For this particular case, the method is stable when $h \leq 1$. However, for step sizes close to this critical value, the numerical solution presents oscillations that make this solution unacceptable for describing a real physical model.

3.8. Evaluation of mathematical skills during the resolution of the proposed activities

To analyze the mathematical skills that students will deploy when solving the proposed activities, different rubrics were designed. The following Tables III, IV, V, and VI illustrate, for each level of Bloom's taxonomy, the mathematical skills that students must display when solving the second example.

TABLE III
MATHEMATICAL SKILLS OF THE REMEMBER LEVEL

Skill	**Not yet developed**	**Moderately developed**	**Developed**
Indicate the precision order of each method used to solve the IVP.	Does not correctly define the precision order of each method.	Defines the precision order of some methods.	Correctly defines the precision order of each method.
Evoke the concept of final global error.	Does not remember the concept.	Remembers the concept with some errors.	Remembers the concept.

TABLE IV
MATHEMATICAL SKILLS OF THE UNDERSTAND LEVEL

Skill	**Not yet developed**	**Moderately developed**	**Developed**
Recognize the information provided by the IVP proposed to execute the DER.	Does not identify the data that must be written in each field.	Identifies some of the data that must be written in each field.	Identifies the data that must be written in each field.
Interpret the information given by each of the outputs provided by the DER.	Does not relate the behavior of the final global error to the order of precision of the used method.	Partially relates the behavior of the final global error to the order of precision of the used method.	Relates the behavior of the final global error to the order of precision of the used method.

TABLE V
MATHEMATICAL SKILLS OF THE APPLY LEVEL

Skill	**Not yet developed**	**Moderately developed**	**Developed**
Calculate the number of points that constitute the discrete domain according to the step size used.	Does not correctly determine the number of points.	Determines the number of points with some errors.	Correctly determines the number of points.

TABLE VI
MATHEMATICAL SKILLS OF THE ANALYZE LEVEL

Skill	Not yet developed	Moderately developed	Developed
Analyze the relationship between the precision of the numerical solution, the used step size, and the order of the selected methods.	Does not explain the increase in the precision of the numerical solution depending on the order of the used method and the number of points.	Explain with some errors the increase in the precision of the numerical solution depending on the order of the method used and the number of points.	Properly explains the increase in the precision of the numerical solution depending on the order of the method used and the number of points.

4. CONCLUSION

The use of digital educational resources leads to substantial changes in the way that students appropriate different mathematical concepts as they are positioned in the center of the learning process and promote a positive attitude towards new knowledge. In this context, the challenge for teachers is to design activities that allow students, through interaction and experimentation, to learn the concepts to be taught and to develop different mathematical skills.

The authors of this paper consider that the digital educational resource presented in this paper, together with the proposed activities, is an example of what can be done if the mentioned objectives are to be achieved.

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