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Mathematical communication of students with low performance in mathematics

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Abstract

This study aims to help students with poor mathematical skills communicate through mathematics. Two twelfth-grade senior high school students consisting of a male student (S1) and a female student (S2), with low mathematical performance, were selected as respondents. The subjects were determined by using the data pairing technique developed by Milles. Data were collected through test and semi-structured interviews and then analyzed qualitatively. The result of this study revealed that there are mathematical communication differences between male and female students. Both subjects presented mathematical ideas by writing down important points from the test. They utilized mathematical notation correctly and represented ideas in graphics adequately. The male student communicated their understanding better by connecting various concepts. At some point, they could not describe thoughts using written words properly. This study is expected to provide benefits for teachers in designing learning that can improve the learning outcomes of low-ability students.

Keywords: Gender; mathematics ability; mathematical communication; student.

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1. Introduction

Mathematics is close to the human scene, and human action cannot be separated from it (Lutfianto & Hartono, 2013). The importance of this science makes mathematics universally studied and has characteristics such as diverse mindsets (Putri & Aisyah 2020). Mathematics students should process communication skills using various mathematical symbols (Qohar & Sumarmo, 2013). Thus, communication becomes essential for students to learn mathematics (Lanya et al., 2020).

Communication is crucial for mathematics and mathematics education (Morgan et al., 2014). A crucial component of learning mathematics is mathematical communication ((Rusyda et al., 2020; Sumaji et al., 2020). Communication is central for students to build teamwork and formulate concepts (Lestari et al., 2019; Mulawarman et al., 2021). Through communication, learners can organize, reflect, and clarify ideas, relationships, and mathematical processes and develop mathematical ideas that impact future careers (Vale & Barbosa, 2017). Students can also form new ideas and express them orally, in writing, or visually (Lanya et al., 2020).

The learners have different skills in mathematics communication because many elements can affect their ability to learn mathematics. One of them is gender. Gender differences impact differences in skill in mathematics ability and act as a way of acquiring knowledge of mathematics (Wang & Degol, 2017; Zhao & Perez-Felkner 2022; Xie & Liu 2023). Gender factor was caused by biological differences between male and female brains (Zhu, 2007). Male students are more interested in mathematics than female students (Halpern et al., 2007; Triana & Zubainur 2019).

The PISA, (2018) placed Indonesian students at 72 out of 78 countries regarding the soft skills of Indonesian students in understanding mathematics (PISA, 2018). This low mathematical ability shows that Indonesian students have relatively low mathematics communication. Therefore, Indonesian students need to improve their skills in mathematics communication to compete with other students from various countries (Kamaliyah et al, 2013). One of the exciting and essential mathematical concepts to learn is the concept of limit (Oktaviyanthi & Supriani, 2015). Therefore, in this study, researchers are interested in using the limit concept to figure out the mathematics communication skills of students with low performance in mathematics.

1.1. Conceptual or theoretical framework

Communication in mathematics was known as mathematical communication. Mathematical communication was defined as expressing mathematical ideas (Ontario Ministry of Education., 2005; Rohid et al., 2019; Yusra & Saragih, 2016). Generally, mathematical communication consists of written and oral mathematical communication. Written mathematical communication expresses ideas, understanding mathematics, and realizing it in writing (Ontario Ministry of Education., 2005). At the same time, oral mathematical communication is expressing ideas or mathematical understanding verbally by speaking, and art to sharing knowledge with others (Pantaleon, et al., 2018a; Pantaleon, et al., 2018b).

The importance of mathematical communication for students as a concept of mathematical reflection, a tool for organization, reinforcing learning, and personal development (Asikin & Junaedi, 2013). In this aspect, the teacher's chosen learning paradigm may impact how well mathematical communication skills are improved (Chasanah & Usodo, 2020). This shows that mathematics communication needs to be expanded in mathematics education (Kamid et al., 2020).

Students are challenged to convey their ideas orally and in writing when communicated. It might enhance their conceptual knowledge. Mathematical communication allows the student to share his ideas with others (Chung et al., 2016; Klang et al., 2021). Mathematical communication involves sharing support about mathematics (Kaya & Aydin, 2016). In addition, it was needed to know the extent of students' understanding of symbols and mathematical concepts (Ratnasari et al., 2018). Communication can help teachers understand students' ability to interpret the various forms of their mathematical understanding (Harianja, et al., 2020).

Research from (Firdiani et al., 2020) shows that only a few students can express, understand, and present mathematical ideas. (Lanya et al., 2020) found that female learners are more prominent than male learners in solving the mathematical problem in writing, which was obtained by studying both mathematical communications. (Pantaleon, et. al., 2018a) Points out that this research also focuses on the mathematical communication of men, which is more accurate than that of women. Responding to this situation requires further study of mathematical communication. Knowledge about students' mathematical communication can help the teacher design the learning process to optimally develop mathematical communication to achieve the learning goals (Sfard, 2001).

Middle school students from Vietnam reacted in mathematics communication and presentation to research conducted in which an uncomfortable learning environment caused students to lose motivation to engage in learning activities; as a learning environment is also an important factor in learning (Ramazan et al., 2023; Berlin & Cohen 2020). A necessity for boosting self-confidence and inspiring students to participate in learning activities and be flexible in using mathematics to solve real-life problems is the construction and structuring of learning circumstances for students to be involved in mathematical communication. The inventive orientation of education by the growth of students' mathematical competence is another benefit of mathematical communication. Therefore, it is concluded that it is necessary to renovate education and build steps to grow students' mathematical communication competence in teaching mathematics to be competitive and high-quality (Uyen et al., 2021).

1.2. Related Research

There are several previous studies related to this research. A study (Qohar & Sumarmo, 2013) suggests that students' mathematical communication abilities affect their mathematical skills. Independent learning can be used to enhance mathematical communication abilities (Arcoverde et al., 2022). According to other studies, it is crucial to teach students mathematics to develop their understanding of the subject. Students' mathematical communication abilities also help clarify the innovative teaching approach by the development of students' mathematical competence, increasing student accountability, and activeness, being proactive in building mathematical understanding, building solid knowledge, and forming and developing the ability to connect learning mathematics with practice. For pupils to gain proficiency and quality in mathematics, efforts to improve their mathematical communication skills are required (Uyen et al., 2021). Meanwhile, this study will examine gender inequalities in the mathematical communication of low-ability high school pupils.

1.3 Purpose of the Study

Clear communication of ideas is facilitated by mathematics, which also serves as a social learning exercise. Furthermore, the significance of mathematical communication is consistent with the goals of studying mathematics. While there has never been any study that specifically looked at the mathematical communication of Palu City high school students with weak arithmetic skills. This study aims to provide a more thorough description of mathematical communication among high school students with weak mathematical skills. The researcher attempts to address the following study topics to reach this goal: How well do students in Palu, Indonesia, with soft math skills, communicate with one another?

2. Method and Materials

2.1. Research Model

This kind of research uses a qualitative descriptive methodology. To get a detailed description and understanding of the scenario, research is done by looking at an ideal situation. The subject of the study is the mathematical communication of pupils on constrained material with weak math skills.

2.2. Participants

Students in the senior high school in Palu, Indonesia, who were in the twelfth grade, served as the research subjects. The selection of students with low mathematics abilities is carried out by paying attention to the distribution of mathematical skills and gender aspects in social characteristics and behavior during school. The Mathematical Ability Test consists of 20 questions adapted from the National Examination. The grouping of students' mathematical ability levels refers to the opinion of which is high if the test score is $80 <$ score test ≤ 100, moderate if the test score is $60 \leq$ the score test ≤ 80, and low if the test score is < 60.

2.3. Data collection instruments

Instruments are utilized as the data collection tools. Both the supporting instruments and the primary instruments are under issue. The principal tool in qualitative research is the researcher, who also serves as the data collector, processor, analyst, and report writer—supporting tools in the form of interview instructions and restricted material assignments. This test is used to explore written and oral mathematical communication on research subjects. The material presented in this test is limited to the intuitive understanding of limits. Before being used as a supporting instrument, the material was first discussed with the supervisor and validated by a mathematics teacher at a high school in Palu City, Indonesia.

2.4. Data collection process

The data collection process was carried out by giving limited material assignments to the subject. Obtaining an overview of students' mathematical communication is continued by conducting in-depth interviews with the subject. The interview consists of a series of questions that have been designed as reference material or guidelines during the interview, but these questions can be developed or adapted to circumstances during the interview process (Kallio et al., 2016). Table 1 shows the aspect indicators used.

Table 1

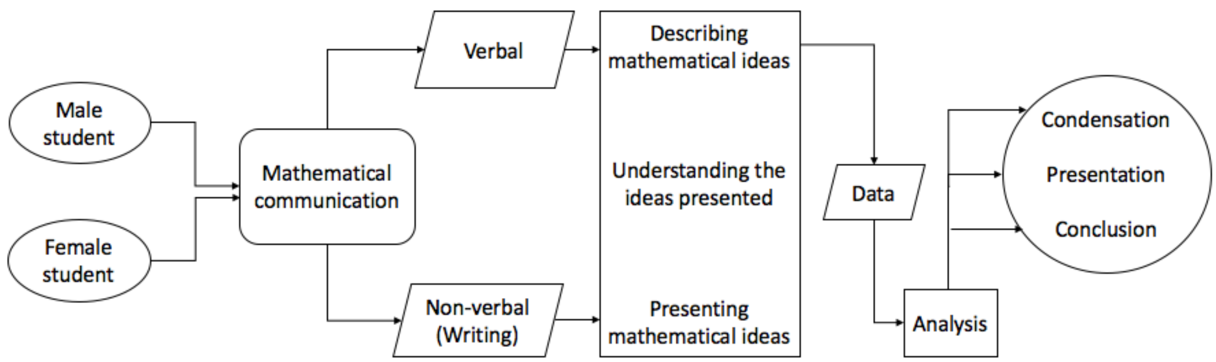
The aspect of math communication

Relation	Description
Describing mathematical ideas	Describing mathematics ideas by speech
	Describing mathematical ideas by writing
	Using mathematical terms also notation
	Express mathematical ideas visually in different types
Understanding the ideas presented	Understanding ideas presented in written form
	Expressing the meaning of ideas presented orally
	Reveal the relationship of various concepts
Explain/ present mathematical ideas	Explaining mathematical ideas verbally
	Explaining ideas accompanied by the use of terms and mathematical notation

2.5. Data analysis

Data analysis in this study refers to techniques developed by Milles et al., (2014), namely data condensation, data presentation, and conclusion (Figure 1). The method of selecting, concentrating, streamlining, abstracting, and changing data from observations, field notes, video recordings, and interviews is known as data condensation. The presentation of data in this study is a collection of information that has been organized which allows for conclusions and action. Next is the analysis activity, which includes drawing conclusions and verification.

Figure 1
Data Paring Design



2.6. Ethical consideration

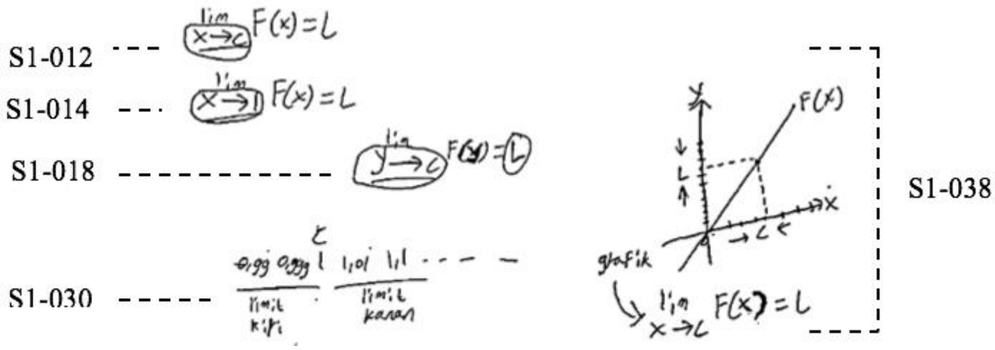
The study, its findings, and reporting conformed to the ethical standard of seeking informed consent, observing anonymity where necessary, and not harming people, organizations and environment.

3. Result

3.1. Male students with low mathematics ability (S1)

Figure 2 shows the results of the test and S1 interviews in describing mathematical ideas. Figure 2 shows an analysis of the data from the S1 interview in describing mathematical concepts orally, where S1 describes the ideas using language and without being long-winded. S1 describes limits as an approximation, presents the notation of limits, and pronounces them correctly. Then S1 describes several different approaches. S1 then describes the left and proper limits according to his understanding to reveal the relationship between the two. S1 can visually represent ideas through graphs and explain them verbally.

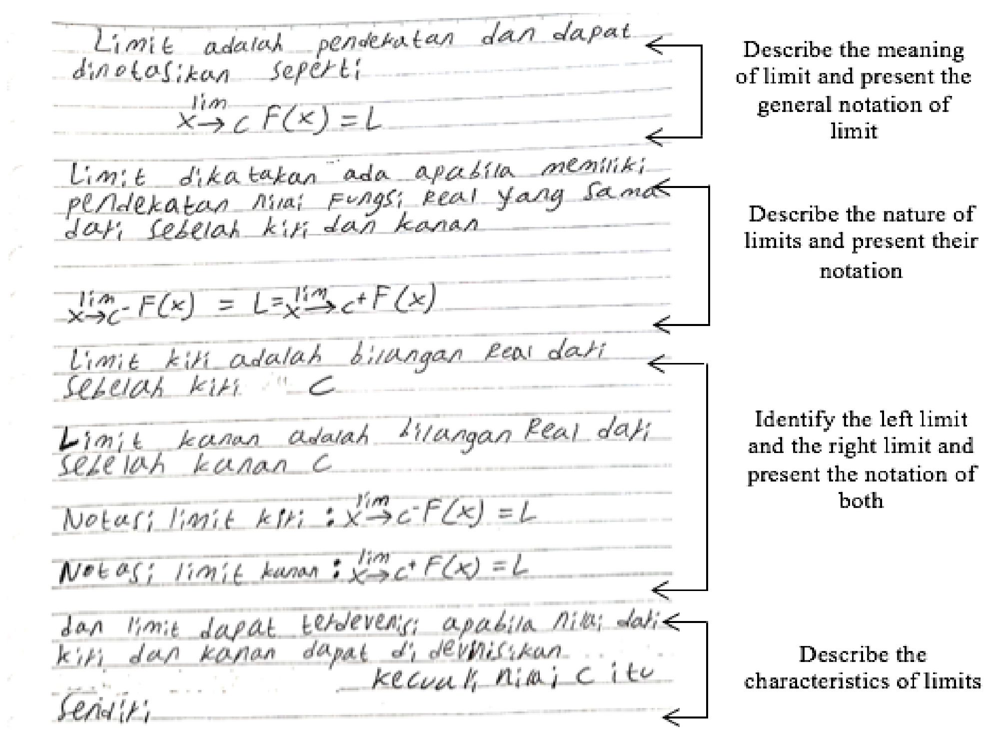
Figure 2
S1's doodles while describing the answer



The results of the S1 test when describing mathematical ideas in writing are presented in Figure 3. Figure 3 shows that S1 can describe the concept by presenting the essential points of the limited material. S1 can also present ideas using their language briefly and accompanied by appropriate mathematical notation, but the difficulty in describing ideas using words in writing is shown through an ambiguous presentation of ideas in describing the characteristics of limits.

Figure 3

Output of S1 limit material test



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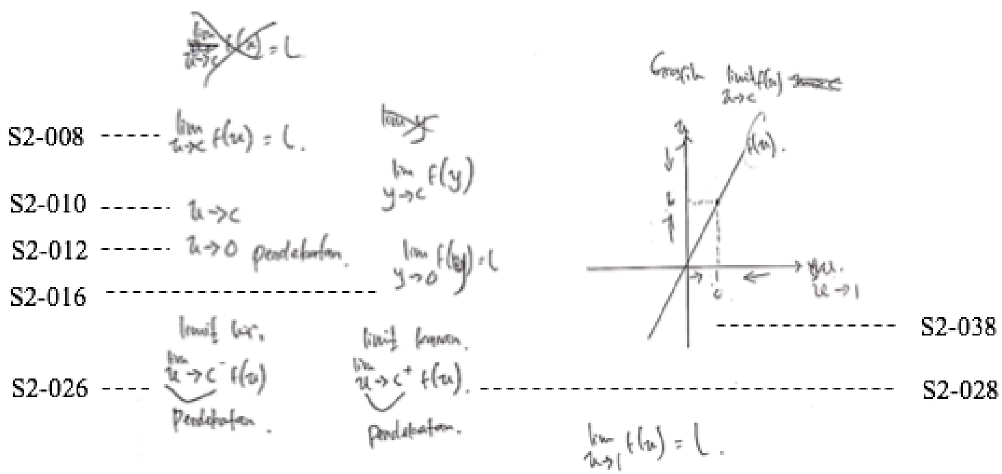
*3.2. **Female student with low mathematics ability** (S2)*

Figure 4 shows the results of material tests and S2 interviews in describing mathematical ideas. Figure 4 shows that S2 describes an idea in a long-winded manner and can present limited notation but is wrong in saying it verbally. S2 can also represent ideas visually in graphs and explain them but tends to be less confident in describing their understanding, which has resulted in several mistakes in the use of mathematical symbols.

Figure 4

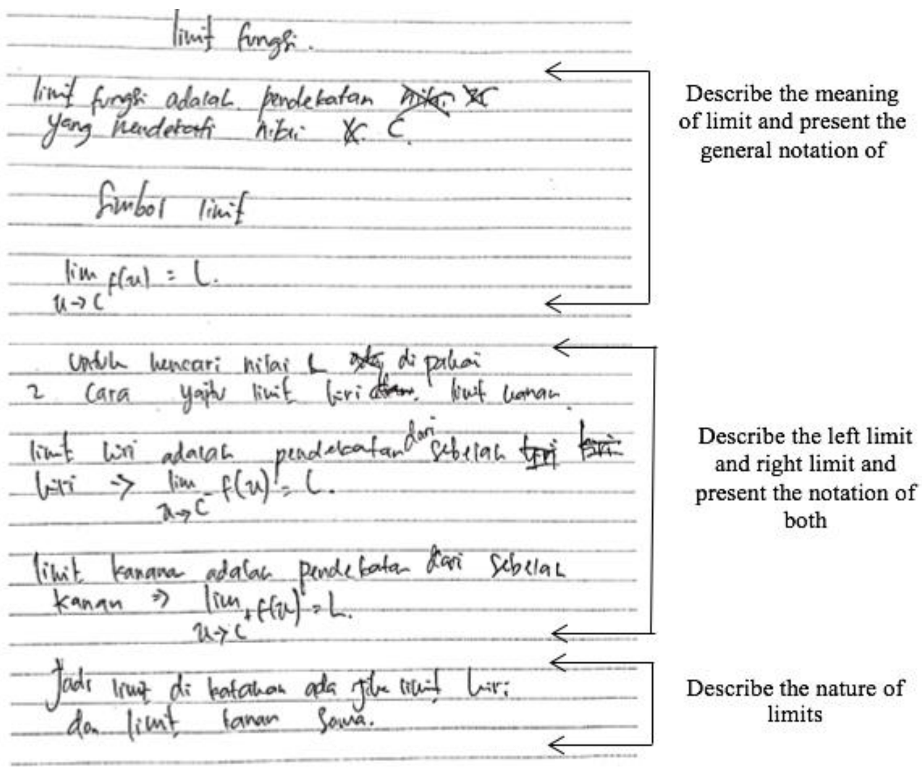
S2's doodles when describing



The results of the S2 limit material test when describing mathematical ideas in writing are presented in the following Figure 5. Figure 5 shows that S2 can give the essential points of the limited material and use short sentences accompanied by an appropriate mathematical notation. S2 describes the left limit and proper limit as a choice of methods to find a function's limit value. This ambiguous expression of ideas shows that S2 has difficulty describing ideas using written words.

Figure 5

Output of S2 limit material test



4. Discussion

When describing a mathematical idea orally, S1 stated the limit as an approach. It succeeded in presenting the $\lim\limits_{x\to c} f(x) = L$ notation correctly and mentioning it correctly. Then S1 shows that $x \to c$ is an approximation to the limit, presenting several examples of different approaches, namely $x \to 1$ and $y \to c$. Furthermore, S1 describes the nature of the limit, namely, "*a limit can be said to exist if it has the same left and right limits*" and reveals the left and proper limits one by one. S1 describes the left limit as an actual number to the left of point c and the appropriate limit as an actual number to the right of point c. In addition, S1 managed to represent $\lim\limits_{x\to c} f(x) = L$ in graphical form. Paroqi et al., (2020) found that male students can state problems in graphic form. Graphs can be used to communicate in mathematics (Lanya et al., 2020). After the observation was extended, S1 had no difficulty describing their mathematical ideas orally, and the subject could express their understanding well, firmly, and using clear sentences.

When describing mathematical ideas in writing, S1 presents mathematical ideas by writing essential points from the material. S1 presents the definition of limit accompanied by the correct notation. S1 describes the nature of the limit by presenting the notation $\lim\limits_{x\to c^-} f.(x) = L = \lim\limits_{x\to c^+} f.(x)$. Furthermore, S1 defines the left limit and proper limit and presents the notation of both correctly, namely $\lim\limits_{x\to c^-} f(x) = L$ and $\lim\limits_{x\to c^+} f(x) = L$. This is appropriate to (Aini et al., 2020) that the mathematics communication of male students can use mathematics notation. Until finally presenting the sentence "*limit can be defined if the values from the left and right can be defined except the value of c itself*," the presentation of unclear sentences and corrections on the test results may indicate that the subject has difficulty expressing mathematical ideas using words in the text—written form. The low ability of students to change and describe their understanding can impact low mathematical communication (Sari et al., 2018). Thus, it can be concluded that S1 can orally express mathematical ideas but has difficulty describing them in writing. In addition, S1 can use mathematical notation and represent it visually in graphs.

When S2 described a mathematical idea orally, S2 stated the limit as an approach. It presented the $\lim_{x \to c} f(x) = L$ notation correctly but incorrectly stated it verbally. S2 shows that $x \to c$ is an approximation to the finite limit and presents several examples of different approaches, namely, $x \to 0$ and $y \to 0$. Next, S2 describes the left and proper limits one by one. S2 reveals the left/right limit as an approximation from the left/right of the point and presents the notation of both $\lim_{x \to c^-} f(x)$ and $\lim_{x \to c^+} f(x)$. However, S2 had difficulty describing the process of finding the grade of the left and proper limits, and the subject could not express her understanding clearly. Finally, S2 connects the left and proper limits, namely, "*the limit is said to exist if the left and right limits are the same*." In addition, S2 represents $\lim_{x \to c} f(x) = L$ in graphic form. Graphs, diagrams, symbols, and equations can be used to communicate in mathematics (Aini, et al, 2020). After the extension of the observation, S2 had difficulty describing her mathematical ideas verbally, the subject tended to be in a hurry and was not confident, so there was an error in the use of variables to show the approach to the function $f(y)$, the subject also made a mistake when placing the x and y axes on the graph $\lim_{x \to c} f(x) = L$. However, the subject immediately identified and corrected errors when the researcher asked. (Kamid et al., 2020) found the issue of verbal and mathematics communication of female students was less confident.

When S2 described mathematical ideas in writing, S2 presented mathematical ideas by writing essential points from the limited material. S2 defined limit as an approach to the value of c and then presented the limit notation using the symbol term $\lim_{x \to c} f(x) = L$. S2 describes the left limit and proper limit. It presents both the notation correctly, namely $\lim_{x \to c^-} f(x) = L$ and $\lim_{x \to c^+} f(x) = L$, finally revealing that "*limits are said to exist if they have left and right limits. the right limit is the same*". This is appropriate with (Lanya et al., 2020) research that female students' mathematical communication can use mathematical notation. However, S2 revealed that the left limit and the proper limit are two ways to find a function's limit value. This sentence contains a double interpretation because it places the left limit and proper limit as a choice of methods that can be used to find the limit value. The test results presentation of ambiguous sentences and doodles indicated that the subject had difficulty expressing mathematical ideas using words. The soft skills of changing and describing their understanding can impact low mathematical communication (Budayasa & Lukito 2019; Sari et al., 2018). Thus, it can be summed up that S2 has difficulty describing mathematical ideas orally and in writing. The subject uses unclear sentences and lacks confidence in communicating their understanding. S2 can use mathematical notation and represent $\lim_{x \to c} f(x) = L$ in graphical form.

S1 can verbally express the entire meaning of mathematical ideas. S1 has no difficulty expressing it verbally, and the subject can say his understanding clearly and confidently. This shows that the issue understands the whole idea. S1 initially expresses an approximation to the limit and states, "*limit f(x) where x approaches variable c equals variable L*" The incorrect use of the term variable indicates that S1 cannot use mathematical terms correctly. After the observation extension, it is known that S1 understands the variables as all real numbers contained in an equation.

S1 then stated the nature of the limit, namely, "*a limit is said to exist if it has the same real function value approach from the left and right,*" and explained how to obtain its value. S1 further reveals the left limit and proper limit. Through extended observations, it is known that the subject understands the left boundary and proper limit only to the numbers around point c. Then, when these numbers are substituted into f(x), it will produce an L value called the approach. Finally, S1 confirms that the left/correct limit and the approach to the value of the objective function from the left/right have different meanings. However, S1 can show that $x \to c^-$ and $x \to c^+$ are approximations of the left and proper limits. The subject implies that it is possible that the limit of a function is not determined at point c itself and provides an example before attempting to describe the limitations that the value of f(x) approaches L when x approaches c and that are specified in an interval comprising c. However, the subject could not communicate the idea in writing, so the use of sentences that were unclear and tended to be meaningless was found.

Thus, it can be concluded that S1 can express the meaning of ideas presented orally in clear and confident sentences. This strengthens the findings of (Kamid et al., 2020) that the oral communication of male students tends to be straightforward, provides many explanations, and answers firmly and confidently. Through this process, it is known that S1 has misconceptions about the left and proper limits and cannot correctly use mathematical terms, namely variables and substitutions. (Lee, 2015) state that communication contributes to students' conceptual development.

S2 has difficulty expressing the meaning of the ideas presented orally, and the subject expresses her understanding convolutedly, so it is difficult to understand. S2 initially expresses "*limit as an approximation to the value of c*." S2 states that c is an actual number approximated by x, and L is the limit value of the f(x) function. Furthermore, S2 tries to reveal the process of determining the limit of f(x), but the subject has difficulty expressing the idea verbally. This is evidenced by the use of sentences which is not clear and the subject statement, "*I know, but I do not know how to say it*." In addition, S2 is wrong in mentioning $\lim_{x \to c} f(x) = L$ notation orally.

S2 then revealed the relationship between the left limit and the proper limit. A limit is said to exist if it has the same left and adequate limits, so it is necessary first to know the value of both. However, S2 has difficulty expressing the idea orally, and the subject uses convoluted sentences: "*If the right and left boundaries coincide, the limit is considered to exist. You must search for both to determine if they are present or absent.*" Through extended observations, it is known that S2 understands the limit intuitively and knows the relationship between the left boundary and the proper limit. S2 showed that $x \to c^-$ and $x \to c^+$ are approximations of the left and proper limits. Besides that, the subject also understands that *"- and +*" are only symbols that show the direction of the number approaching point c.

Thus, it can be concluded that S2 understands the limited material intuitively and understands the ideas presented in written form but has difficulty expressing their meaning orally. The subject states that her understanding is convoluted, difficult to understand, and lacks confidence. This is consistent with Kamid et al., (2020) results that female students frequently use long-winded, timid, and unconfident spoken mathematical communication.

5. Conclusion

The mathematical communication of male and female students with weak mathematics abilities differs. This difference does not appear in written mathematical communication but is significant in oral mathematical communication. In verbal and mathematical communication, male students can express their understanding using their language without being long-winded and confident. Female students express their understanding in a convoluted manner, lack confidence, and use sentences that are difficult to understand.

In written communication, both males and females present mathematical ideas by intuitively writing down essential points from the concept of limits, using mathematical notation correctly, and successfully representing ideas visually in graphs. However, both students were not able to present ideas in good mathematics. This is indicated by the presentation of ideas in writing using ambiguous sentences and doodles on the material test results. The recommendation for further research is to organize education by applying the right approach to develop and train students in mathematical communication, especially oral

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References

Aini, S. D., Zayyadi, M., & Saleh, K. A. (2020). Written mathematical communication skills on open-ended problems: is it different based on the level of mathematics ability? *MaPan: Jurnal matematika dan Pembelajaran*, *8*(2), 179-204. https://doi.org/10.24252/mapan.2020v8n2a2

Arcoverde, Â.R.d., Boruchovitch, E., Góes, N.M. et al., (2022). Self-regulated learning of Natural Sciences and Mathematics future teachers: Learning strategies, self-efficacy, and socio-demographic factors. *Psicol. Refl. Crít. 35*(1). https://doi.org/10.1186/s41155-021-00203-x

Asikin, M., & Junaedi, I. (2013). Kemampuan komunikasi matematika siswa SMP dalam setting pembelajaran RME (Realistic Mathematics Education). *Unnes Journal of Mathematics Education Research*, *2*(1). https://journal.unnes.ac.id/sju/ujmer/article/view/1483

Berlin, R., & Cohen, J. (2020). The convergence of emotionally supportive learning environments and college and career-ready mathematical engagement in upper elementary classrooms. *AERA Open*, *6*(3), 2332858420957612. https://journals.sagepub.com/doi/abs/10.1177/2332858420957612

Budayasa, I. K., & Lukito, A. (2019). Metacognitive activity of male students: difference field independent-dependent cognitive style. In *Journal of Physics: Conference Series, 1218*(1), 012025. https://iopscience.iop.org/article/10.1088/1742-6596/1218/1/012025/meta

Chasanah, C., & Usodo, B. (2020). The Effectiveness of Learning Models on Written Mathematical Communication Skills Viewed from Students' Cognitive Styles. *European Journal of Educational Research*, *9*(3), 979-994. https://eric.ed.gov/?id=EJ1262491

Chung, Y., Yoo, J., Kim, S. W., Lee, H., & Zeidler, D. L. (2016). Enhancing students' communication skills in the science classroom through socioscientific issues. *International Journal of Science and Mathematics Education*, *14*, 1-27. https://link.springer.com/article/10.1007/s10763-014-9557-6

Firdiani, N. H., Herman, T., & Hasanah, A. (2020). Gender and mathematical communication ability. *Journal of Physics: Conference Series*, *1521*(3), 032095. https://iopscience.iop.org/article/10.1088/1742-6596/1521/3/032095/meta

Halpern, D. F., Benbow, C. P., Geary, D. C., Gur, R. C., Hyde, J. S., & Gernsbacher, M. A. (2007). The science of sex differences in science and mathematics. *Psychological science in the public interest*, *8*(1), 1-51. https://journals.sagepub.com/doi/abs/10.1111/j.1529-1006.2007.00032.x

Harianja, J. K., Hernadi, S. L., & Indah, I. (2020). Students' Mathematical Conceptual Understanding and its Relation to the Mathematical Communication Skills. *Jurnal Penelitian Pendid.kan dan Pengaja.an Matematika*, *6*(1), 1–12. http://jurnal.unsil.ac.id/index.php/jp3m/article/download/JOK61/1032

Kallio, H., Pietilä, A. M., Johnson, M., & Kangasniemi, M. (2016). Systematic methodological review: developing a framework for a qualitative semi-structured interview guide. *Journal of Advanced Nursing*, *72*(12), 2954-2965. https://onlinelibrary.wiley.com/doi/abs/10.1111/jan.13031

Kamaliyah, Zulkardi, & Darmawijoyo. (2013). Developing the Sixth Level of PISA-Like Mathematics Problems for Secondary School Students. *Indo.MS-JME*, 4(1), 9–2.8. https://doi.org/10.22342/jme.4.1.559.9-28

Kamid, R. M., Fitaloka, O., Basuki, F. R., & Anwar, K. (2020). Mathematical communication skills based on cognitive styles and gender. *International Journal of Evaluation and Research in Education*, *9*(4), 847–856. https://doi.org/10.11591/ijere.v9i4.20497

Kaya, D., & Aydin, H. (2016). Elementary mathematics teachers' perceptions and lived experiences on mathematical communication. *Eurasia Journal of Mathematics, Science and Technology Education*, *12*(6), 1619–1629. https://doi.org/10.12973/eurasia.2014.1203a

Klang, N., Karlsson, N., Kilborn, W., Eriksson, P., & Karlberg, M. (2021). Mathematical problem-solving through cooperative learning—the importance of peer acceptance and friendships. *Frontiers in Education*, *6*, 710296. https://www.frontiersin.org/articles/10.3389/feduc.2021.710296/full

Lanya, H., Zayyadi, M., & Aini, S. D. (2020). Mathematic Communication Skills in Senior High School Based on Gender Differences. *Pi: Mathematics Education Journal*, *3*(2), 75-81. https://ejournal.unikama.ac.id/index.php/pmej/article/view/4718

Lee, J. (2015). "Oh, I just had it in my head": Promoting mathematical communications in early childhood. *Contemporary Issues in Early Childhood*, *16*(3), 284-287. https://journals.sagepub.com/doi/abs/10.1177/1463949115600054

Lestari, F., Saryantono, B., Syazali, M., Saregar, A., Madiyo, Jauhariyah, D., & Umam, R. (2019). Cooperative learning application with the method of network tree concept map: Based on Japanese learning system approach. *Journal for the Education of Gifted Young Scientists*, *7*(1), 15–32. https://doi.org/10.17478/jegys.471466

Lutfianto, M., Hartono, Y., & Zulkardi. (2013). Unfinished Student Answer in PISA Mathematics Contextual Problem. *Indo.MS-JME*, 4(2), 18.8–193. http://repository.unsri.ac.id/id/eprint/6016.

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). Fundamentals of Qualitative Data Analysis in Qualitative Data Analysis: A Methods Sourcebook. 69–104.

Morgan, C., Craig, T., Schütte, M., & Wagner, D. (2014). Language and communication in mathematics education: An overview of research in the field. *Zdm*, *46*, 843-853. https://link.springer.com/article/10.1007/s11858-014-0624-9

Mulawarman, W. G., Komariyah, L., & Suryaningsi. (2021). Women and leadership style in school management: Study of gender perspective. *Cypriot Journal of Educational Sciences*, *16*(2), 594–611. https://doi.org/10.18844/CJES.V16I2.5638

Oktaviyanthi, R., & Supriani, Y. (2015). Utilizing Microsoft Mathematics in Teaching and Learning Calculus. *Indonesian Mathematical Society Journal on Mathematics Education*, *6*(1), 63-76. https://eric.ed.gov/?id=EJ1079603

Ontario Ministry of Education. (2005). *The Ontario curriculum, grades 1-8. Mathematics.* Ontario, Ministry of Education.

Pantaleon, K. v., Juniati, D., & Lukito, A. (2018a). The oral mathematical communication profile of prospective mathematics teacher in mathematics proving. *Journal of Physics: Conference Series*, *1108*(1). https://doi.org/10.1088/1742-6596/1108/1/012008

Pantaleon, K. v., Juniati, D., Lukito, A., & Mandur, K. (2018b). The written mathematical communication profile of prospective math teacher in mathematical proving. *Journal of Physics: Conference Series*, *947*(1). https://doi.org/10.1088/1742-6596/947/1/012070

Paroqi, L. L., Mursalin, M., & Marhami, M. (2020). The Implementation of Realistic Mathematics Education Approach to Improve Students' Mathematical Communication Ability in Statistics Course. *International Journal for Educational and Vocational Studies*, *2*(10). https://doi.org/10.29103/ijevs.v2i10.3311

PISA, O. (2019). Results (Volume 1): what students know and can do. Paris: PISA.

Putri, R. I. I., & Aisyah, N. (2020). Learning Integers with Realistic Mathematics Education Approach Based on Islamic Values. *Journal on Mathematics Education*, *11*(3), 363-384. https://eric.ed.gov/?id=EJ1294680

Qohar, A., & Sumarmo, U. (2013). Improving Mathematical Communication Ability and Self-Regulation Learning of Junior High Students by Using Reciprocal Teaching. *Indonesian Mathematical Society Journal on Mathematics Education*, *4*(1), 59-74. https://eric.ed.gov/?id=EJ1078962

Ramazan, O., Danielson, R. W., Rougee, A., Ardasheva, Y., & Austin, B. W. (2023). Effects of classroom and school climate on language minority students' PISA mathematics self-concept and achievement scores. *Large-scale Assessments in Education*, *11*(1), 11. https://link.springer.com/article/10.1186/s40536-023-00156-w

Ratnasari, N., Tadjudin, N., Syazali, M., Mujib, M., & Andriani, S. (2018). Project Based Learning (PjBL) Model on the Mathematical Representation Ability. *Tadris: Jurnal Keguruan Dan Ilmu Tarbiyah*, *3*(1), 47. https://doi.org/10.24042/tadris.v3i1.2535

Rohid, N., Suryaman, S., & Rusmawati, R. D. (2019). Students' Mathematical Communication Skills (MCS) in Solving Mathematics Problems: A Case in Indonesian Context. *Anatolian Journal of Education*, *4*(2), 19–30. https://doi.org/10.29333/aje.2019.423a

Rusyda, N. A., Ahmad, D., Rusdinal, R., & Dwina, F. (2020). Analysis of students' mathematical communication skill in calculus course. *Journal of Physics: Conference Series*, *1554*(1). https://doi.org/10.1088/1742-6596/1554/1/012043

Sari, D. K., Supahar, & Ralmugiz, U. (2018). The influence of android-based isomorphic physics (Forfis) application on analogical transfer and self-diagnosis skill of students at SMA Negeri 3 Kupang. *Jurnal Pendidikan IPA Indonesia*, *7*(2), 154–161. https://doi.org/10.15294/jpii.v7i2.14268

Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Educational studies in mathematics*, *46*(1), 13-57. https://link.springer.com/article/10.1023/A:1014097416157

Sumaji, S., Sa'dijah, C., Susiswo, S., & Sisworo, S. (2020). Mathematical communication process of junior high school students in solving problems based on apos theory. *Journal for the Education of Gifted Young Scientists*, *8*(1), 197-221. https://dergipark.org.tr/tr/doi/10.17478/jegys.652055

Triana, M., & Zubainur, C. M. (2019). Students' Mathematical Communication Ability through the Brain-Based Learning Approach Using Autograph. *Journal of Research and Advances in Mathematics Education*, *4*(1), 1-10. https://eric.ed.gov/?id=EJ1267501

Uyen, B. P., Tong, D. H., & Tram, N. T. B. (2021). Developing mathematical communication skills for students in grade 8 in teaching congruent triangle topics. *European Journal of Educational Research*, *10*(3), 1287–1302. https://doi.org/10.12973/eu-jer.10.3.1287

Vale, I., & Barbosa, A. (2017). The importance of seeing in mathematics communication. http://repositorio.ipvc.pt/handle/20.500.11960/3762

Wang, M. T., & Degol, J. L. (2017). Gender gap in science, technology, engineering, and mathematics (STEM): Current knowledge, implications for practice, policy, and future directions. *Educational psychology review*, *29*, 119-140. https://link.springer.com/article/10.1007/s10648-015-9355-x

Xie, G., & Liu, X. (2023). Gender in mathematics: how gender role perception influences mathematical capability in junior high school. *The Journal of Chinese Sociology*, *10*(1), 10. https://link.springer.com/article/10.1186/s40711-023-00188-3

Yusra, D., & Saragih, S. (2016). The Profile of Communication Mathematics and Students' Motivation by Joyful Learning-based Learning Context Malay Culture. *British Journal of Education, Society & Behavioural Science*, *15*(4), 1–16. https://doi.org/10.9734/bjesbs/2016/25521

Zhao, T., & Perez-Felkner, L. (2022). Perceived abilities or academic interests? Longitudinal high school science and mathematics effects on postsecondary STEM outcomes by gender and race. *International Journal of STEM Education*, *9*(1), 42. https://link.springer.com/article/10.1186/s40594-022-00356-w

Zhu, Z. (2007). Gender differences in mathematical problem-solving patterns: a review of literature. *International Education Journal*, *8*(2), 187-203. https://eric.ed.gov/?id=EJ834219