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Learning Techniques for Solving Linear Equations Systems. Power Systems Case study

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Abstract

The use of matrix algebra has expanded considerably in the last 25-30 years, in parallel with computer technology evolution. Introducing of matrix notation leads to simple and concise formulation of highly complex applications. In the first instance, a linear model can be developed or if the model is nonlinear, it can be linearized in first approximation, once or every step of a solving iterative process. In this paper, the authors will present two methods used to solve linear equations systems. First methods will be solved by manual calculation and the second method will be solved using a computer program, SISLIN, developed in Power Systems Department of the Politehnica University Timisoara. Methods are presented to students who are asked to apply the methods for case studies. Volume calculation is large, for which the authors analyze student's concentration and attention degree.

Keywords: nonlinear equations; numerical coefficients; Newton-Raphson method; method Bailey

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1. Introduction

Many problems from technical field and also in terms of power engineering field lead to mathematical models which involve solving large dimension linear equation (Thangaraj, 2008), (Thangaraj, 2013). Moreover, for any problem which can be developed, in first instance, a linear model or, if the model is nonlinear, it can be linearized in first approximation, once or every step of a solving iterative process. In these paper linear equations systems can be solved by two methods: Gauss version elimination method and Southwell method. First method is part of direct methods while the second method is iterative. A system of n linear equation with coefficients $a_{ij} \in \Re$, i = 1, 2, ..., n , j = 1, 2, ..., n and free terms $b_i \in \Re$, i = 1, 2, ..., n , with n unknown, $x_i \in \Re$, i = 1, 2, ..., n , is defined by relation (1) (Kilyeni, 2014), (Kilyeni, Fülöp, & Dumitrescu, 1997). Matrix and vector notations for coefficients matrix A, free terms column vector b and unknown column vector x are defined by relation (2). It results matrix form of linear system (3).

$$\begin{cases} a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 + \cdots + a_{1n} \cdot x_n = b_1 \\ a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 + \cdots + a_{2n} \cdot x_n = b_2 \\ a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 + \cdots + a_{3n} \cdot x_n = b_3 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1} \cdot x_1 + a_{n2} \cdot x_2 + a_{n3} \cdot x_3 + \cdots + a_{nn} \cdot x_n = b_n \end{cases} \tag{1}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$

$$\mathbf{b} = [\, b_1 \;\; b_2 \;\; b_3 \;\; \cdots \;\; b_n \,]^t \tag{2}$$

$$\mathbf{x} = [\, x_1 \;\; x_2 \;\; x_3 \;\; \cdots \;\; x_n \,]^t$$

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b} \tag{3}$$

2. Presentation of analyzed methods

2.1. Gauss version elimination method (triangulation)

This method involves transforming the system (3) in an equivalent form (4),

$$\mathbf{A}^* \cdot \mathbf{x} = \mathbf{b}^* \tag{4}$$

Method's algorithm (Kilyeni, Barbulescu, & Simo, 2013), (Kilyeni, Negru, 1991) is the following:

a) Necessary initializations are made: A_0 = A, b_0 = b
b) At a certain step k, k = 1, 2, ..., n , element of matrix A^k and of vector b^k are calculated with relations (5) - (6), system reaching to the form (7);

$$a_{kj}^{k} = \frac{a_{kj}^{k-1}}{a_{kk}^{k-1}}, \qquad j = 1, 2, \cdots, n\sqrt{b^2 - 4ac}$$
$$b_{k}^{k} = \frac{b_{k}^{k-1}}{a_{kk}^{k-1}} \tag{5}$$

$$a_{ij}^{k} = a_{ij}^{k-1} - a_{ik}^{k-1} \cdot a_{kj}^{k}, \quad i = k+1, k+2, \cdots, n \quad , \quad j = 1, 2, \cdots, n$$
$$b_{i}^{k} = b_{i}^{k-1} - a_{ik}^{k-1} \cdot b_{k}^{k}, \quad i = k+1, k+2, \cdots, n \quad , \quad j = 1, 2, \cdots, n \tag{6}$$

$$\begin{cases} x_1 + a_{12}^1 \cdot x_2 + a_{13}^1 \cdot x_3 + \ldots + a_{1k}^1 \cdot x_k + a_{1,k+1}^1 \cdot x_{k+1} + \cdots + a_{1n}^1 \cdot x_n = b_1^1 \\ x_2 + a_{23}^2 \cdot x_3 + \cdots + a_{2k}^2 \cdot x_k + a_{2,k+1}^2 \cdot x_{k+1} + \cdots + a_{2n}^2 \cdot x_n = b_2^2 \\ \cdots\cdots \\ x_k + a_{k,k+1}^k \cdot x_{k+1} + \cdots + a_{kn}^k \cdot x_n = b_k^k \\ \cdots\cdots \\ a_{n,k+1}^k \cdot x_{k+1} + \cdots + a_{nn}^k \cdot x_n = b_n^k \end{cases} \tag{7}$$

c) Finally, system reaches to form (8). Solution results clearly.

$$\begin{cases} x_1 + a_{12}^1 \cdot x_2 + a_{13}^1 \cdot x_3 + \ldots + a_{1n}^1 \cdot x_n = b_1^1 \\ x_2 + a_{23}^2 \cdot x_3 + \cdots + a_{2n}^2 \cdot x_n = b_2^2 \\ \cdots\cdots \\ x_n = b_n^n \end{cases} \tag{8}$$

$$x_n = b_n^*, \; x_i = b_i^* - \sum_{j=i+1}^{n} a_{ij}^* \cdot x_j \quad , \qquad i = n-1, n-2, \cdots, 1 \tag{9}$$

2.2. Southwell method (residues)

System defined by relation (3) is written to form (10). Residue relationship is written as (11), residue column vector being $\mathbf{r}^k = [\, r_1^k \;\; r_2^k \;\; r_3^k \;\cdots\; r_n^k \,]^t$.

$$a_{ii} \cdot x_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} \cdot x_j = b_i \quad , \qquad i = 1, 2, \cdots, n \tag{10}$$

$$r_i^k = a_{ii} \cdot x_i^k + \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} \cdot x_j^k - b_i \neq 0 \quad , \qquad i = 1, 2, \cdots, n \tag{11}$$

Method's algorithm (Kilyeni, Barbulescu, & Simo, 2013), (Kilyeni, Negru, 1991) is the following:

a) x is initialized with x0 : $\mathbf{x}^0 = [\, x_1^0 \;\; x_2^0 \;\; x_3^0 \;\cdots\; x_n^0 \,]^t$
b) At a certain step k, k = 1, 2, 3, ... , the new values of variable are determined with relation (12),

$$x_m^k = x_m^{k-1} + h^{k-1}, \; x_i^k = x_i^{k-1} \quad , \qquad i = 1, 2, \cdots, n \quad , \quad i \neq m \tag{12}$$

where correction h^{k-1} and residues have the expressions:

$$h^{k-1} = -\frac{r_m^{k-1}}{a_{nn}} = -\underset{i}{Max}\left\{\left|\frac{r_i^{k-1}}{a_{ii}}\right|\right\}, \quad r_i^{k-1} = a_{ii}\cdot x_i^{k-1} + \sum_{\substack{j=1\\ j\neq i}}^{n} a_{ij}\cdot x_j^{k-1} - b_i \quad i=1,2,\cdots,n \tag{13}$$

c) Termination condition of calculation process is $\left|h^{k-1}\right| \le \varepsilon$.

Starting with the second iteration, practical relation to determine the residues can be used:

$$r_m^k = 0, \quad r_i^k = r_i^{k-1} - \frac{a_{im}}{a_{ii}}\cdot r_m^{k-1} \quad , \quad i=1,2,\cdots,n \quad , \quad i\neq m \tag{14}$$

$$x_m^k = x_m^{k-1} + \alpha\cdot h^{k-1} \tag{15}$$

3. Numerical results and discussions

Fourth order linear equation system is defined by relation (16).

$$\begin{cases} 8\cdot x_1 - 2\cdot x_2 + \quad x_3 \quad\quad = 18 \\ \quad x_1 + 6\cdot x_2 + 2\cdot x_3 - 3\cdot x_4 = 8 \\ \quad\quad x_2 + 4{,}3\cdot x_3 + 2\cdot x_4 = -4{,}6 \\ 3\cdot x_1 + 3\cdot x_2 - \quad x_3 - 10\cdot x_4 = 7 \end{cases}, \quad A = \begin{bmatrix} 8 & -2 & 1 & 0 \\ 1 & 6 & 2 & -3 \\ 0 & 1 & 4{,}3 & 2 \\ 3 & 3 & -1 & -10 \end{bmatrix}, \quad b = \begin{bmatrix} 18 \\ 8 \\ -4{,}6 \\ 7 \end{bmatrix} \tag{16}$$

3.1. Manual solving of application with Gauss version elimination method (triangulation)

Triangulation of system's coefficients matrix with appropriate processing free terms vectors, is performed in n = 4 steps:

a) Step 1

- First equation

$$(8/8)\cdot x_1 - (2/8)\cdot x_2 + (1/8)\cdot x_3 + (0/8)\cdot x_4 = 18/8$$
$$1{,}000\cdot x_1 - 0{,}250\cdot x_2 + 0{,}125\cdot x_3 - 0{,}000\cdot x_4 = 2{,}250$$

- Second equation

$$(1-1\cdot 1)\cdot x_1 + (6+1\cdot 0{,}25)\cdot x_2 + (2-1\cdot 0{,}125)\cdot x_3 + (-3-1\cdot 0)\cdot x_4 = 8-1\cdot 2{,}25$$
$$0{,}000\cdot x_1 + 6{,}250\cdot x_2 + 1{,}875\cdot x_3 - 3{,}000\cdot x_4 = 5{,}750$$

- The third equation is not modified, because $a_{31} = 0$
- Fourth equation

$$(3-3\cdot 1)\cdot x_1 + (3+3\cdot 0{,}25)\cdot x_2 + (-1-3\cdot 0{,}125)\cdot x_3 + (-10-3\cdot 0)\cdot x_4 = 7-3\cdot 2{,}25$$
$$0{,}000\cdot x_1 + 3{,}750\cdot x_2 - 1{,}375\cdot x_3 - 10{,}000\cdot x_4 = 0{,}250$$

- System after step 1

$$\begin{cases} 1\cdot x_1 - 0{,}250\cdot x_2 + 0{,}125\cdot x_3 + 0{,}000\cdot x_4 = 2{,}250 \\ 0\cdot x_1 + 6{,}250\cdot x_2 + 1{,}875\cdot x_3 - 3{,}000\cdot x_4 = 5{,}750 \\ 0\cdot x_1 + 1{,}000\cdot x_2 + 4{,}300\cdot x_3 + 2{,}000\cdot x_4 = -4{,}600 \\ 0\cdot x_1 + 3{,}750\cdot x_2 - 1{,}375\cdot x_3 - 10{,}00\cdot x_4 = 0{,}250 \end{cases}$$

b) Step 2. The manner of calculation is similar with de first step, the calculations are being performed for the second, third and fourth equations.

- System after step 2

$$\begin{cases} 1\cdot x_1 - 0,250\cdot x_2 + 0,125\cdot x_3 + 0,000\cdot x_4 = 2,250 \\ 0\cdot x_1 + 1\cdot x_2 + 0,300\cdot x_3 - 0,480\cdot x_4 = 0,920 \\ 0\cdot x_1 + 0\cdot x_2 + 4,000\cdot x_3 + 2,480\cdot x_4 = -5,520 \\ 0\cdot x_1 + 0\cdot x_2 - 2,500\cdot x_3 - 8,200\cdot x_4 = -3,200 \end{cases}$$

c) Step 3. The calculation is performed for third and fourth equations.

- System after step 3

$$\begin{cases} 1\cdot x_1 - 0,250\cdot x_2 + 0,125\cdot x_3 + 0,000\cdot x_4 = 2,250 \\ 0\cdot x_1 + 1\cdot x_2 + 0,300\cdot x_3 - 0,480\cdot x_4 = 0,920 \\ 0\cdot x_1 + 0\cdot x_2 + 1\cdot x_3 + 0,620\cdot x_4 = -1,380 \\ 0\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 - 6,650\cdot x_4 = -6,650 \end{cases}$$

d) Step 4

- Fourth equation

$$(6,650/6,650)\cdot x_4 = 6,650/6,650$$

$$0,000\cdot x_1 + 0,000\cdot x_2 + 0,000\cdot x_3 + 1,000\cdot x_4 = 1,000$$

- System after step 4

$$\begin{cases} 1\cdot x_1 - 0,250\cdot x_2 + 0,125\cdot x_3 + 0,000\cdot x_4 = 2,250 \\ 0\cdot x_1 + 1\cdot x_2 + 0,300\cdot x_3 - 0,480\cdot x_4 = 0,920 \\ 0\cdot x_1 + 0\cdot x_2 + 1\cdot x_3 + 0,620\cdot x_4 = -1,380 \\ 0\cdot x_1 + 0\cdot x_2 - 0\cdot x_3 + 1\cdot x_4 = 1,000 \end{cases}$$

Solving of triangular system is made from x_n and finishing with x_1.

$$\begin{cases} x_4 = 1,000 \\ x_3 = -1,380 - 0,620\cdot 1,000 = -2,000 \\ x_2 = 0,920 + 0,300\cdot 2,000 + 0,480\cdot 1,000 = 3,000 \\ x_1 = 2,250 + 0,250\cdot 2,000 + 0,125\cdot 2,000 + 0,000\cdot 1,000 = 3,000 \end{cases}$$

3.2. Solving of application with Southwell method (residues) using SISLIN program

Database is created and solved in program SISLIN. The main window is presented in figure 1. Database contains system order, error and maximum number of iterations, system coefficients matrix and free terms vector. Figure 2 presented window with database solved in triangulation program. In the next step used iterative method is selected and system solution is initialized.

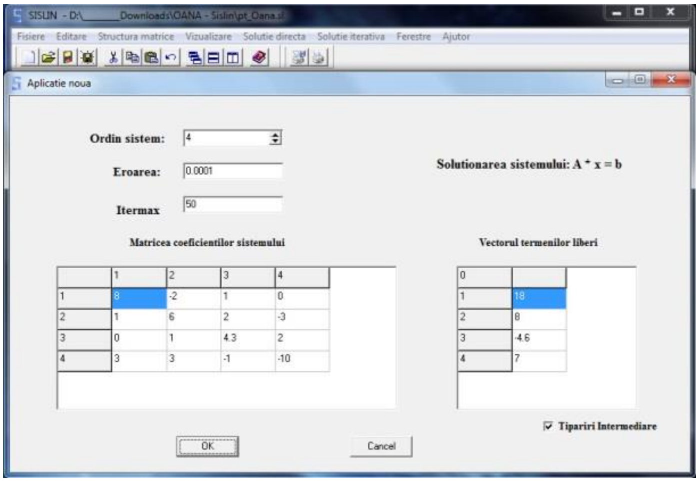

Figure 1. Database created in SISLIN program

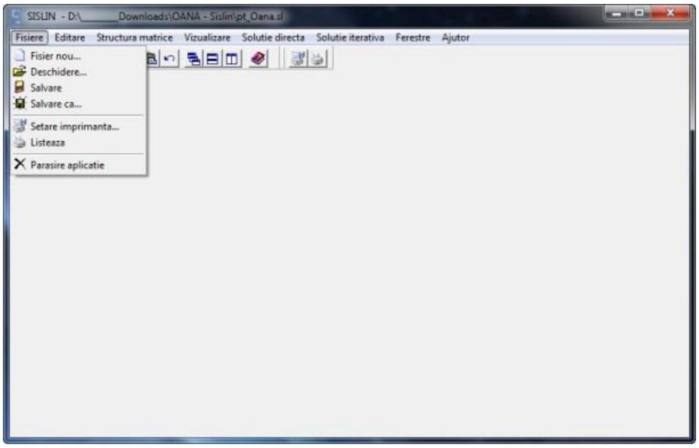

Figure 2. Solving database in SISLIN program

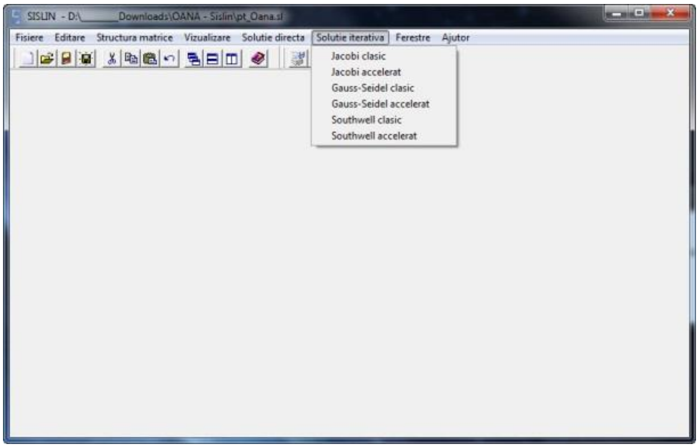

Figure 3. Selecting the method used for calculation

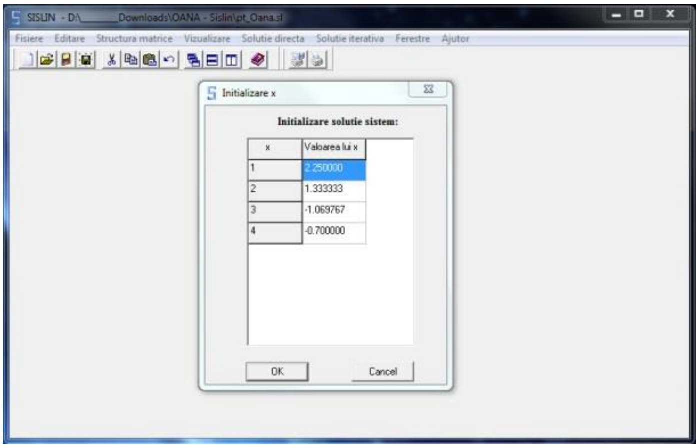

Figure 4. System solution initialized

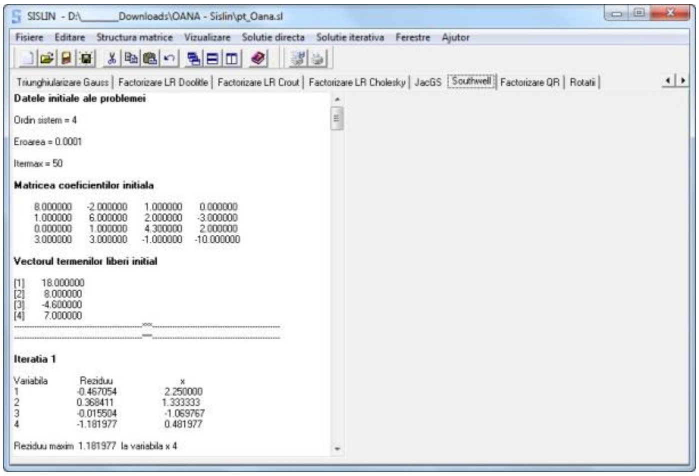

Figure 5. Values obtained after solving the application

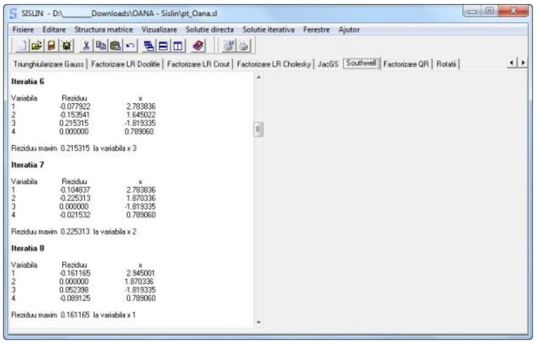

Figure 6. Values obtained after solving the application

The next figures (Figures 5-7) present values obtained after solving application using calculation program. Maximum number if iteration is Itermax = 50. In case of linear equation from relation (16) a number of 39 iterations were required. Values obtained for variable x and residues are presented for the first 8 iteration and for the last iteration. The results obtained using allowable error (in program, EpsX), ε = 10-4, are presented in Fig. 7.

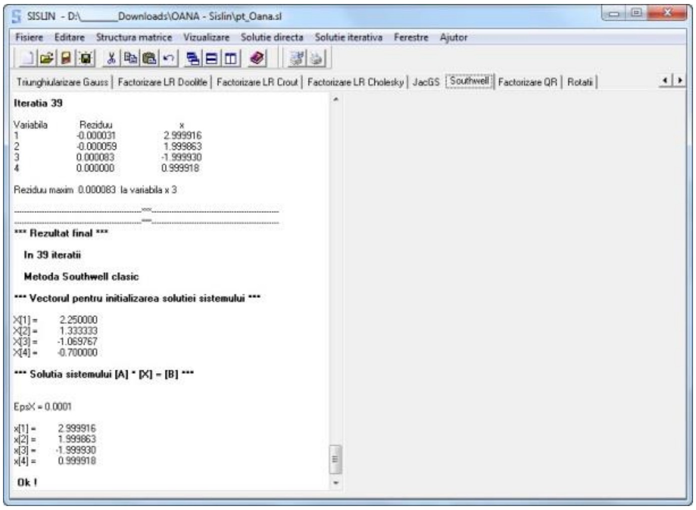

Figure 7. Values obtained after solving the application.

4. Conclusions

Triangulation method requires less computing time, because of the number of elementary arithmetic operations per step. If the system is solved repeatedly for different values of free terms, then triangulation of matrix A is made once for the first solving. For the following solving only the operations on free terms is repeated and the last step of algorithm is made. Solving through manual calculation using Southwell method requires greater computing time, which is why the use of computer program is the best solution. Software tool is easy to use and has the all facilities offered by Delphi environment. Also, the user is guided step-by-step to solve numerical application. In case of numerical application analyzed the exact solution is obtained, although it worked only with 4 significant digits.

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