International Journal of Learning and Teaching

Volume 08, Issue 2, (2016) 156-162

http://sproc.org/ojs/index.php/ijlt

An attempt to reduce learning difficulties in Linear Algebra courses

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Suggested Citation:

Caligaris, M., G., Schivo, M., E. & Romiti, M., R. (2016). An attempt to reduce learning difficulties in Linear Algebra courses. *International Journal of Learning and Teaching*. *8*(2), 156-162.

Received February 17, 2016; revised March 11, 2016; accepted April 02, 2016;

Selection and peer review under responsibility of Prof. Dr. Hafize Keser, Ankara University, Ankara, Turkey.

Abstract

In engineering careers, the study of Linear Algebra begins in a student's first course. Some topics included in this subject are systems of linear equations and vector spaces. Linear Algebra is very useful but can be very abstract for teaching and learning.

In an attempt to reduce learning difficulties, different approaches of teaching activities supported by interactive tools were analysed. This paper presents these tools, which were designed with GeoGebra for the Algebra and Analytic Geometry course at the Facultad Regional San Nicolás at the Universidad Tecnológica Nacional in Argentina.

Keywords: Linear Algebra, GeoGebra, learning difficulties.

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1. Introduction

The teachers' daily work makes them consider how hard it is for students to understand the concepts of Linear Algebra. With regard to difficulties encountered by students, first-year teachers of FRSN share their diagnosis through their own classroom experiences. Students face difficulties caused by ignorance of certain mathematical symbols, either because they do not know or do not remember them. It is not uncommon for students to manipulate symbols without knowledge of a meaningful basis that is grounded in the context in which the symbols arise (Harel, Fuller & Rabin, 2008). From the beginning of the course, studying vectors—the passage of the geometric form to the component expression—causes difficulties in students that become errors. These difficulties are particularly exacerbated when the concepts of linear combination, linearly independent or dependent sets, subspaces and the basis and dimension of a vector space are introduced. Other common difficulties arise when students try to justify the value of truth of mathematical propositions; there are only a few students who manage to express themselves in the correct way.

Many of the studies found in the literature report students' learning difficulties concerning basic linear algebra concepts. Many studies discuss students who experience problems with the abstraction level of linear algebra materials. The high level of formalism in linear algebra seems to produce in students a lack of connection to what they already know in mathematics. Furthermore, the axiomatic approach to linear algebra appears to promote in students the feeling of learning a topic that does not seem necessary for their majors (Dogan-Dunlap, 2010).

Given these difficulties, a proposal for teaching activities incorporating interactive tools prepared with GeoGebra 5.0 beta software in order to reduce the obstacles, both in learning and teaching, was carried out. Many concepts of linear algebra are intimately related to geometry and can be reformulated with vector geometry—namely, they have geometrical counterpart. Some mathematical concepts can often be formulated as geometric concepts. This means that students' visual intuition can serve as a guide to understanding such concepts (Konyalıoğlu et al., 2011).

According to Arbain and Shukor (2015) GeoGebra software has a positive impact on students' achievement in the topic they analysed, Statistics. According to the authors, students had positive perceptions of GeoGebra software in terms of enthusiasm, confidence and motivation.

This paper is intended to show different Applets designed for the Algebra and Analytic Geometry course at the Facultad Regional San Nicolás at the Universidad Tecnológica Nacional in Argentina. All the tools were developed in Spanish; the Applets presented in this work were translated into English. The activities that can be performed with these tools are primarily intended to reduce the difficulties caused by the abstraction of certain contents of Linear Algebra, taking into account the work of conversion between semiotic registers of representation while providing elements to problematise the acquisition of the necessary knowledge of the studied topics.

The design of teaching and learning situations is a complex process, since the elements that influence the construction of personal meaning for students—teaching methodology, mathematical content, the role that students play in class and the available teaching materials—must be considered. With regard to the latter, in practically all learning situations materials of all kinds and media are used. Learning processes are mediated by the use of certain materials or technologies, which affect the way students learn.

2. GeoGebra Applets description

Diverse GeoGebra Applets were designed for the course. GeoGebra is a free software that brings together geometry, algebra and calculus allowing diverse representations of mathematical objects. In a previous work some animations of the fundamental concepts of calculus that emphasise the analysis of their geometric interpretations were discussed (Caligaris et al., 2015). GeoGebra 5.0 Beta also allows the easy creation of interactive applications with three-dimensional graphics. Fig. 1 shows the Applet prepared to work with the cross product.

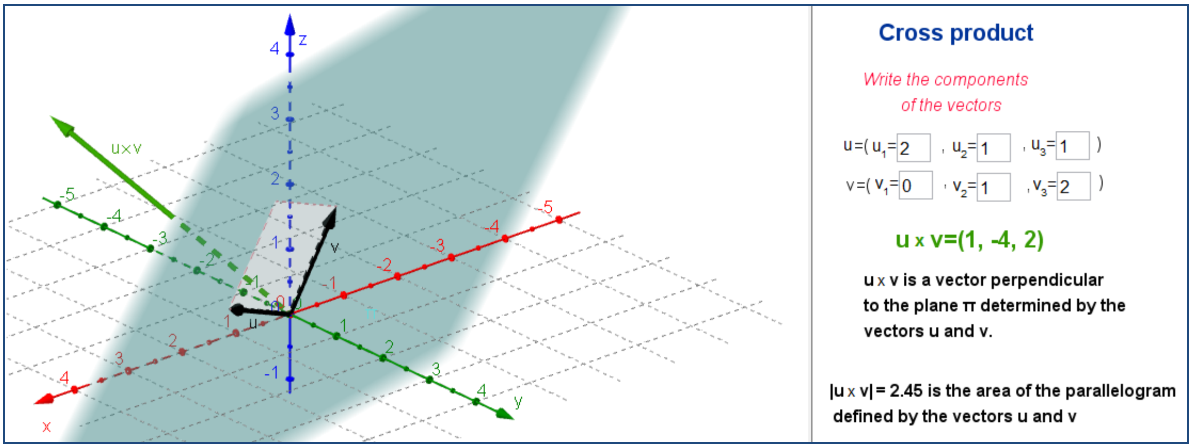

Figure 1. The cross product

The components of two vectors, **u** and **v**, are loaded in the respective input boxes. On the same side of the Applet, the resulting vector (**u** x **v**) and the value of the area of the parallelogram determined by the vectors **u** and **v** are displayed. When coefficients are loaded, vectors **u** and **v**, the plane containing them, the determined parallelogram and the resulting vector **u** x **v**, are shown.

Fig. 2 shows a similar Applet, prepared to work with the triple scalar product, some of its properties and the geometric interpretation of its absolute value. The components of three vectors, u, v and w, are loaded in the respective input boxes. On the same side of the Applet, the resulting product and the value of the volume of the parallelepiped determined by the vectors u, v and w are displayed. When coefficients are loaded, the vectors and the determined parallelepiped are shown.

Both Applets, cross product and scalar triple product, allow students to analyse, for example, what happens when one vector is null o when two vectors are collinear.

Fig. 3 shows an Applet that displays the various subspaces that can be generated by a set of 3D vectors.

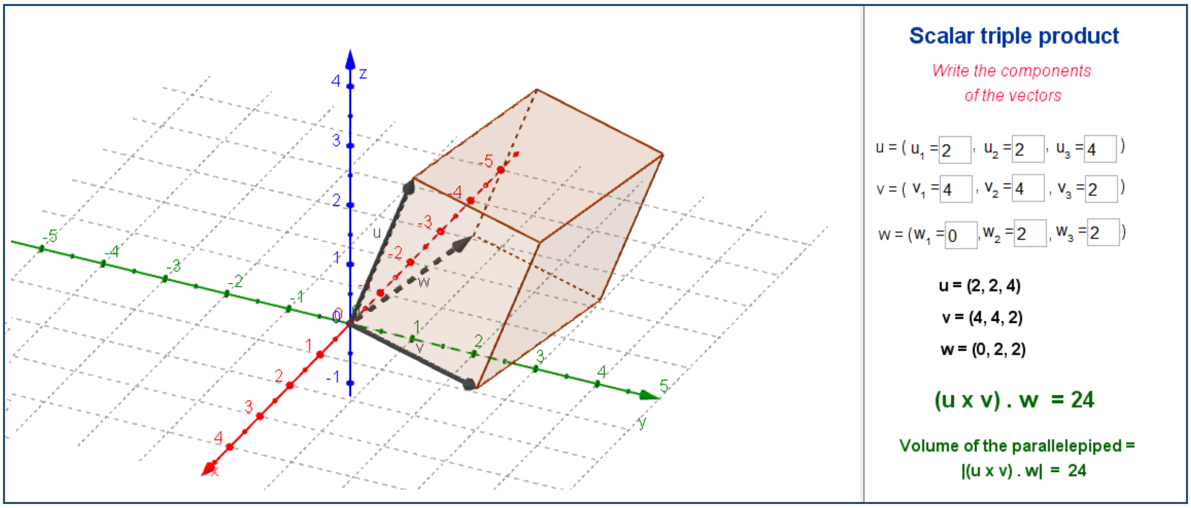

Figure 2. The scalar triple product

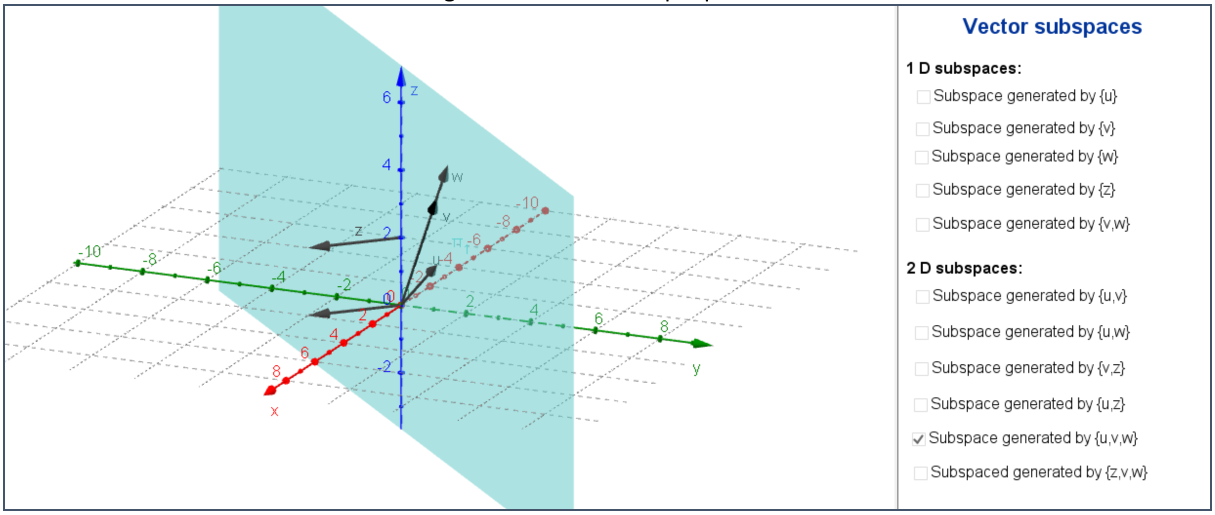

Figure 3. 1D and 2D vector subspaces

The Applet in Fig. 3 can also be used to work in class concepts such as linear independence, basis and dimension. When the set is selected on the right side of the Applet, the generated subspace is shown on the left. The use of this tool in the classroom allows students to first approach the concepts mentioned above in the graphic register.

The treatment of these issues deepens using the Applet shown in Fig. 4, including the equation corresponding to the generated subspace, thereby incorporating conversion work between graphic and symbolic registers. The components of three vectors, **u**, **v** and **w**, are loaded in the respective input boxes and, on the same side of the Applet, the set A = {**u**, **v**, **w**} and the equation of the subspace generated by A are shown. For trivial cases different texts appear: "A is a set of three vectors and is linearly independent, so it generates R^3" or "A generates the null subspace, which is a trivial case" as the case may be.

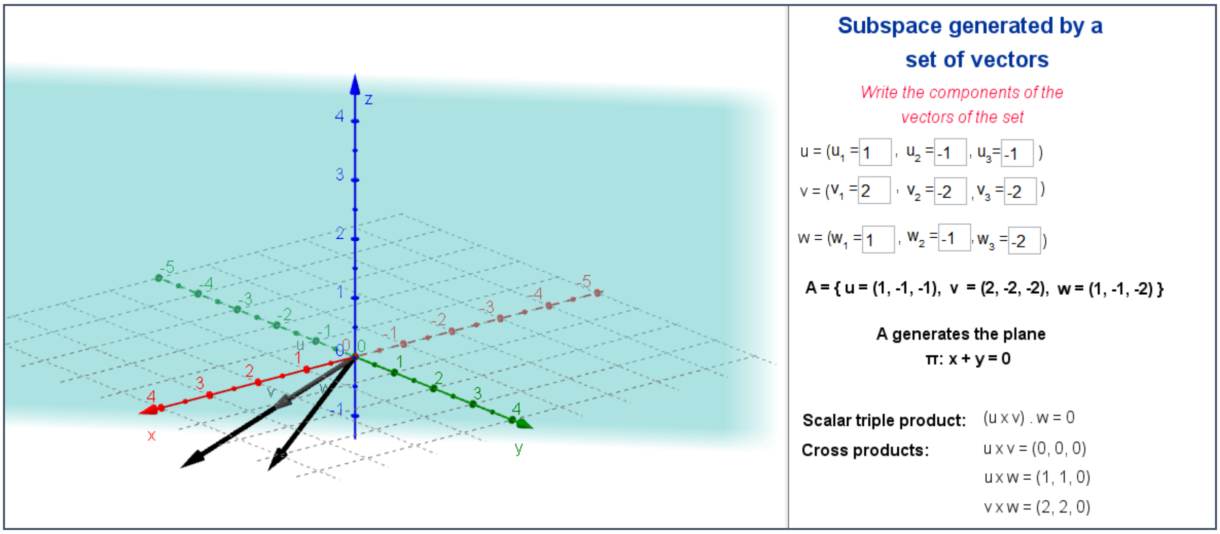

Figure 4. Subspace generated by a set of vectors

The different cross products and the scalar triple product are also shown in Fig. 4. They are used in the classroom to analyse the connection between these products, the linear dependence of the vectors and the dimension of the generated subspace. In this Applet the symbolic computation view can be hidden, as in the example, or can be shown, depending on the subject being studied. Here, the system to obtain the equation of the generated subspace is presented and solved. In this view, the values of the x, y, z coordinates of the points that form the subspace are expressed in terms of different coefficients, unlike the equation on the right side that is expressed in a different way. This allows teachers to propose an interesting classroom activity to move from one form of expression to another.

Beginning with a geometric and algebraic analysis of concepts as generated subspace, as well as basis and dimension, which emphasises the visual approach, later helps to generalise and study other vector spaces.

Two other tools were prepared, which were used in class to conduct a review of the contents for the midterm. The purpose of these latter activities is to work on the justification of true or false propositions. This way of working generates an interesting and productive discussion in class and allows us to intensify the conversion between the graphic register and the natural language. One of these tools is shown in Fig. 5. The figure exhibits some answers.

When systems of linear equations with three unknowns are solved in classroom, analytical resolution is usually favoured, leaving the geometric interpretation of the solutions to be expressed in natural language or to be represented by informal sketches instead of using a Cartesian coordinate system.

Even though the importance of the informal representation is valued, using visual tools allows students to visualize concepts and reduce difficulties without requiring extra time, both in lectures and practical classes.

Fig. 6 shows the Applet prepared to display graphical representations of the solutions in a 3D Cartesian coordinate system and the rank of the matrices A and A|B. The figure shows a system with unique solution.

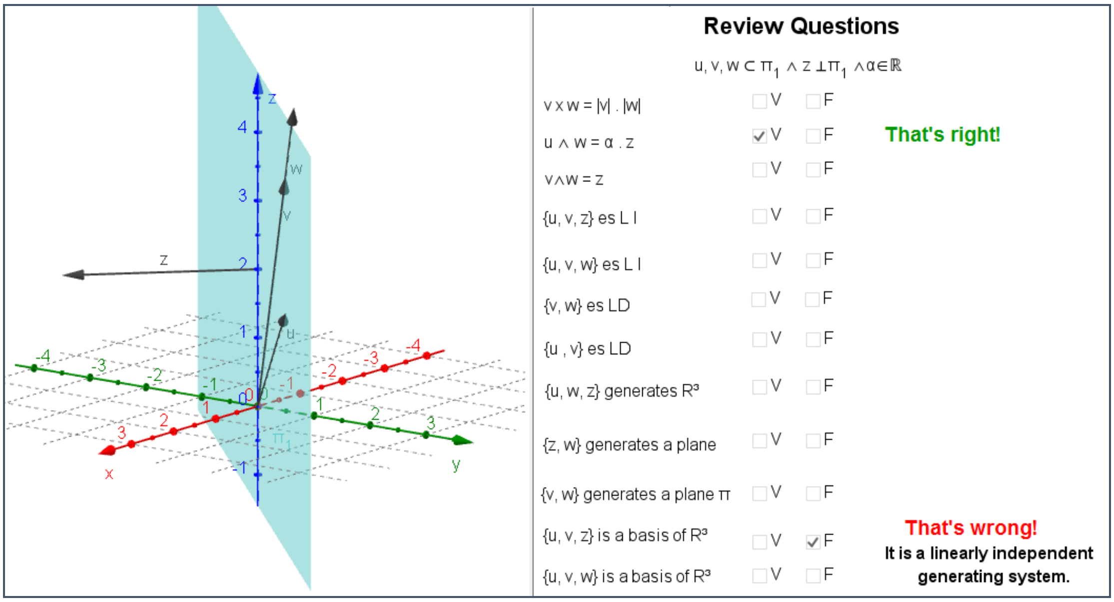

Figure 5. A review for vectors products

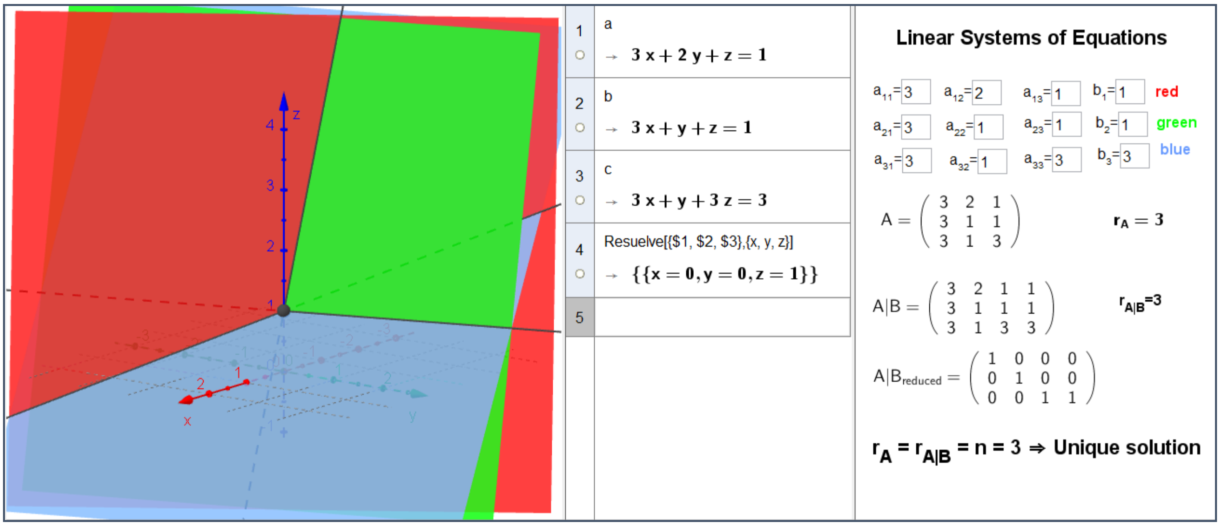

Figure 6. Applet to solve linear systems of equations

The coefficients of the system and the independent terms are loaded in the respective input boxes and, on the same side of the Applet, the coefficient matrix A, the augmented matrix A|B, and the reduced form are shown. The relationship between their ranks and the number of unknowns are also exhibited and it is indicated whether the system is inconsistent or consistent, even if, in the latter case, it is determinate or indeterminate. In the symbolic computation view the equations that make up the system and its solution appear. On the left side of the Applet, the geometric interpretation of the problem is shown. The planes are displayed in red, green and blue as indicated in each line of the corresponding coefficients.

3. Use of the tools in the classroom

The presented tools have been well considered by students. The use of these tools in the classroom allowed us to not only improve the visualization of the theoretical explanations given on the board but also helped to increase students' motivation.

Properly selected different examples are initially shown and then the students suggest other ones. The fact that they could test results with different examples that they proposed resulted in a more dynamic class without loss of mathematical rigour. For example, in the representation of the absolute value of the scalar triple product students clearly saw how, according to the choice of the vectors, the parallelepiped "disappeared".

4. Conclusions

It is important that teachers employ the contribution of technology in the classroom to help students visualize concepts and verify the results they obtain when carrying out their activities.

The difficulties of graphing on the board when working on 3D geometry makes teachers prefer the algebraic register, being the graphic one that allows students to assert, reinterpret, and elaborate the topics presented in the classroom and discover characteristics or properties thereof.

Finally the authors think that classroom activities that can be planned with the presented tools can reduce the difficulties identified by teachers that are related to the abstraction of the contents and the conversion activities between registers of representation. Moreover, incorporating these applications into teaching and learning processes allows us to design teaching strategies that cannot be applied if we work only with chalk and blackboard as teaching resources.

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