

Investigation of middle school seventh- and eighth-grade students' modelling skills

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Abstract

This study seeks to examine the mathematical modelling skills of middle school seventh- and eighth-grade students. Participants were formed by eight secondary school students studying at a state secondary school in a medium-sized district of a medium-sized province in the Eastern Black Sea Region of Turkey in the 2020–2021 academic year. It is a qualitative research and is designed as a case study. The whole process was recorded by video by having the students work on the mathematical modelling activity named 'Team Fiction' prepared by the researcher. In the collection of data, the group work form and report form prepared by the researcher, along with student solution papers and researcher's observation notes were used. All the data obtained were subjected to descriptive analysis and the following results were obtained: it is thought that regular modelling studies in schools will contribute to the development and change of modelling competencies of students.

Keywords: Mathematical modelling, modelling skills, modelling activities, school students, middle school.

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1. Introduction

The National Council of Teachers of Mathematics (NCTM) (2000, p. 4), an organisation of mathematics educators and teachers, emphasises that students should use mathematics to solve the problems in the world that surrounds them. For this reason, the aims of mathematics education have changed towards raising individuals who are capable of responding to real-life needs and solving real-life problems (National Council of Teachers of Mathematics [NCTM], 2000). In parallel with the international curricula, the mathematics curriculum in our country has been organised in a way that will enable individuals who can access the information they need, who can solve problems, have developed decision-making skills, are entrepreneurial, able to empathise, communicate well and think critically and innovatively (Ministry of National Education, 2005, 2009, 2017) because mathematics is of great importance in raising individuals who solve the problems they encounter by producing creative thoughts and transfer what they learn to daily life (Tutak & Guder, 2014). However, in today's world where mathematical thinking and understanding are becoming increasingly complex, learning to hypothesise mathematical concepts and conceptual systems, to understand, explain, reconstruct and analyse relationships has become a critical situation for students (Thomas & Hart, 2010). Mathematics has gotten out of that stereotypical traditional framework where the correct answer was found by replacing the formulas after memorising them; it has become flexible, where skills and practices that form the core of mathematical thinking gain importance.

Although mathematics lesson is expressed as a very important lesson, it is one of the lessons that students have the most difficulty with and fear the most (Celikoz & Duran, 2017; Tasdemir, 2009). The fact that mathematics, which has gained great importance in the modern age, is one of the subjects that students have the most difficulty with, 'How can it best ensure that students establish a connection between mathematics and real life?' It has revealed the necessity of seeking the answer to the question. Because if students are individuals who are able to produce solutions to real problems, use the mathematical knowledge they have learned in their daily lives and be aware of the unity between mathematics and the world, they may become afraid of mathematics, stop perceiving it as difficult and enjoy doing mathematics (Doruk, 2010). In this sense, mathematics educators conduct various researches and develop methods to enable students to establish a connection between mathematics and real life, to learn mathematics in the best way and to use this knowledge in extraordinary problem situations (Ev Cimen, 2008; Kececi et al., 2017; Yasar Kazu & Mertoglu, 2016). It is emphasised that in each period, depending on the characteristics and needs of the period, students should be trained in this direction by using different and appropriate methods and techniques, and at the same time, the problems of teaching mathematics can be reduced to the lowest level (Erdogan, 2018). 'Mathematical modelling', which is one of these studies, has become an increasingly important subject in mathematics education in recent years for exactly these reasons (Blum & Ferri, 2009; Lesh & Doerr, 2003).

Lingefjard (2006) stated that the modelling approach in mathematics education emerged due to the fact that traditional mathematics teaching did not develop students' ability to think and apply in different contexts. In traditional problem-solving, students reach a single answer that is accepted as correct by making placements on formulas on situations that are far from reality they read. For this reason, students may not be able or have difficulty in establishing a connection between mathematics and real life in such problems. However, mathematical modelling problems and activities are an excellent way for students to learn and understand many different aspects of mathematics in real life as well as learning mathematics (Kertil, 2008, p. 23 as cited in Lingefjard and Holmquist). Therefore, it is very important for students to acquire and apply mathematical modelling skills.

When the literature is examined, Keskin (2008) revealed a new mathematical modelling process consisting of five stages.

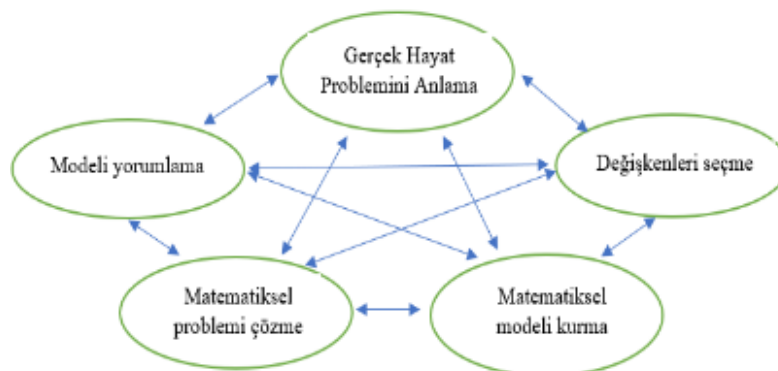


Figure 1. Keskin's (2008, p. 20) mathematical modelling diagram

Stating that it is not necessary for the stages to follow a linear order in the mathematical modelling process, Keskin emphasises that when necessary, the desired step can be returned. The mathematical modelling process shown in Figure 1 consists of the steps of understanding real-life problems, choosing variables, setting up the mathematical model, solving mathematical problems and interpreting the model; this process includes a cyclical relationship where the transitions between the steps can be achieved easily.

Mathematical modelling covers a process that includes metacognitive activities. Therefore, students sometimes encounter some difficulties while performing the stages in this cognitive process. Some competencies are required to successfully overcome these difficulties. These competencies appear as 'mathematical modelling competencies' in the literature.

It is seen that different definitions are made by many researchers about mathematical modelling competencies. While Blomhoj and Jensen (2003, p. 126) define mathematical modelling competence as performing all stages of mathematical modelling with a unique understanding, Niss, Blum and Galbraith (2007, p. 12) determine that it is the ability to identify appropriate questions, variables, correlations or assumptions in the presented real-world situation, translate them into mathematics and interpret and show the correctness of the solution of the mathematical problem associated with the given situation, compare or analyse given models, while investigating assumptions and checking the scope and properties the model has.

Adding the motivation dimension, Maaß (2006, p. 116) states that mathematical modelling competence includes skills and abilities as well as the desire to put these skills and abilities into action. Similarly, Ural (2018, p. 12) states that modelling competencies include the skills and abilities to carry out the modelling process in accordance with its purpose, as well as the willingness to do these activities. In this sense, it can be said that mathematical modelling competencies also include mathematical modelling skills. In addition, action and desire, i.e., personal effort, form the basis of mathematical modelling competence.

Blum and Kaiser (1997) mention the competences and sub-competencies that provide the transitions between the steps in order to complete the mathematical modelling process successfully in the mathematical modelling cycle that they have explained.

When the relevant literature in Turkey is examined, there are studies that examine students' modelling skills (Bal & Doganay, 2014; Dede & Yilmaz, 2013; Eraslan, 2012; Kal, 2013; Kertil, 2008; Olkun, Sahin, Akkurt, Dikkartin & Gulbagci, 2009; Tuna et al., 2013; Turker, Saglam & Umay, 2010) which state that most of the mathematical modelling studies are carried out with pre-service teachers and this situation can be considered as an indicator that researchers cannot benefit from the schools, which are the application area (Aztekin & Taspinar Sener, 2015). In addition, mathematical modelling, as a field-specific skill, took its place in the middle school mathematics curriculum for the first time in 2017. Therefore, after the introduction of mathematical modelling into the programme, it is thought

that studies on the examination of mathematical modelling skills of middle school students are also important. However, when looking at these studies (Ciltas, Demirci & Guler, 2018; Cora, 2018; Inan, 2018; Kaya, 2019; Mengi, 2019; Zihar, 2018), there was no study involving study groups at different class levels. The selection of the study group from seventh- and eighth-grade students in this study makes the study unique in terms of filling this gap in the literature.

Table 1. Mathematical modelling qualifications and sub-qualifications

A. Their competence to understand the real problem and create a model based on reality
A1. Competence to make assumptions about the problem and simplify the situation
A2. Competence to determine quantities, nomenclature and base variables that affect the situation
A3. Competence to relate variables
A4. The competence to investigate appropriate/accessible information and to separate necessary/unnecessary information in order to solve the problem
B. Qualifications for creating a mathematical model from the actual model
B1. Ability to mathematicalise related quantities and relationships between them
B2. Ability to simplify relevant quantities and relationships when necessary, reducing their number and complexity
B3. Ability to select appropriate mathematical representations and graphically represent states
A. Their ability to solve mathematical problems with the help of the mathematical model created
C1. Ability to break down the problem into smaller pieces, relate to similar problems, re-express and examine the problem, look at the problem from a different perspective, and use heuristic strategies such as diversifying quantities and available data
C2. Competence to use mathematical knowledge to solve the problem
D. Competence to interpret mathematical results in real situations
D1. Ability to interpret mathematical results in non-mathematical contexts
D2. Competence to generalise solutions developed for an exception for other situations
D3. Competence to review solutions using appropriate mathematical language or by communicating about solutions
E. Solution validation qualifications
E1. Make reflections on and critically control the solutions obtained
E2. Competence to review parts of the model or repeat the modelling process if it is understood that the solutions do not provide the situation
E3. Competence to explore different solutions and think about how the solution can be developed
E4. Competence to query the model in general

Adapted from Blum and Kaiser (1997) and Maaß (2006).

2. Method

This study is qualitative and designed as a case study. According to Creswell (2013, p. 89), qualitative studies in which the researcher collects detailed information about a situation with multiple data collection tools and describes the current situation are defined as case studies. Yin (2014, p. 16), on the other hand, explains the case studies, which he named as case studies, as an empirical investigation that investigates a situation in depth and in a real-world context. In this study, two focus groups of four people were considered as situations, and all stages of the studies that the students carried out during the modelling process were examined in depth. For this reason, the method of the research has been determined as a case study.

2.1. Workgroup

This research was conducted with seventh- and eighth-grade students studying in a state secondary school in a medium-sized district of a medium-sized province in the Eastern Black Sea region of Turkey in the 2020–2021 academic year. Criterion sampling, one of the purposeful sampling methods, was

used in the study. As a criterion, it was decided to choose the students with high levels of mathematics achievement, and in line with this purpose, the students' math course grade for the previous term was determined. Students with a math class average above 80 in the previous semester are considered to have a high level of achievement. *Fifteen* students meeting this criterion were identified. A total of eight students, four seventh grade and four eighth grade, randomly selected among these students, formed the research group. Throughout the application, these students worked in two groups of four, each being two seventh-grade and two eighth-grade students.

2.2. Data collection tools and data collection process

A mathematical modelling problem (Team Fiction) developed by the researcher was applied to the students in order to examine the modelling skills of the students. This mathematical modelling problem has been developed according to the interests of the students in the study group. At the point of determining the interests of the students, it was decided to hold individual interviews and then focus group interviews. The questions in the form used in the interviews were prepared by the researcher.

The questions in the interview form were examined by an expert lecturer and the following changes were made in order to provide more comfortable data collection in line with the expert's opinions:

- 'Do you like to travel, if your answer is yes, where would you like to travel?' and 'Do you like to research, if your answer is yes, what do you research?' 'What other occupations are you interested in and do you like to do?' questions were added.
- 'How do you spend your free time?' was changed to 'How do you spend your day, can you tell?'.
- 'If there is a traditional game or video game that you like to play, what is it(s)?' question was added.

As a result of the changes made, a semi-structured interview form consisting of nine open-ended questions was used first in individual and then in focus group interviews to determine the interests of the students.

Görüşme Soruları:

1. Hangi tür kitapları okumaktan zevk alırsınız?
2. Oynamaktan hoşlandığınız geleneksel bir oyun veya bilgisayar oyunu varsa ne(ler)dir?
3. Genelde ne tür filmler izlersiniz?
4. En çok sevdiğiniz ders hangisidir?
5. Araştırma yapmayı sever misiniz, cevabınız evet ise neleri araştırırsınız?
6. Hangi sporları yapmaktan keyif alırsınız?
7. Haber programlarında dikkatinizi en çok çeken hangi tür haberlerdir?
8. Gelecekte yapmak istediğiniz meslek nedir, neden?
9. Bir gününüzü nasıl geçirirsiniz, anlatır mısınız?

Figure 2. Interview form

When the data obtained from the individual and focus group interviews were examined, the following were obtained:

Students' answers to the 'What kind of books do you enjoy reading?' question were adventure, horror, fantasy, fairy tale and fable-type books. As a result of the focus group interviews, it was determined that students generally enjoyed reading fantasy books about adventure and fairy tales.

At the end of individual interviews, 'If there is a traditional game or computer game you like to play, what is it (s)?' was asked. In response to the question, only two students stated that they played computer games. Mucahit stated that he played Driving School and Minecraft games as a computer game and also liked quiz games, such as Who Wants to be a Millionaire. Saying that she played computer games, Zeynep stated that she played more childish games such as Talking Tom and Angela. Majority of the students stated that they played dodge ball, chess and hand pass from traditional games; and in the focus group interview, it was concluded that they liked team games more than individual games.

As the third question in the interviews, the students were asked 'What kind of movies do you usually watch?' The students answered this question by giving examples from the movies they liked most – scientific fiction films and animal documentaries, such as Abdurrahman Harry Potter; Humeyra stated that they liked watching adventure and action films such as Sherlock Holmes, The Avengers and Black Mirror, as well as drama films such as Last Hope, Wonder and Miracle. Similarly, other students stated that they liked comedy animated films such as Kral Sakir and Cheerful Feet and comedy films such as Kuzey Yildiz, Yahsi Cazibe and Hababam Class.

It can be said that students are generally more motivated to the lessons they are interested in and they like that lesson more than other lessons. In this sense, during the interviews, they were asked to answer the question 'What is your favourite lesson?' Six out of eight students said that their favourite subject was Science, as they found the subjects of space, microscope and living things interesting. Two students stated that they did not like to memorise at all, problem-solving and doing four operations were more enjoyable, and therefore their favourite subject was Mathematics.

To the question 'What sports do you enjoy doing?' all of the students answered volleyball and basketball. In addition to these answers, Hayrunnisa stated that they liked table tennis, Melisa swimming, Tuba walking, and Mucahit, and Abdurrahman stated that they liked playing football.

One of the questions asked to the students in the interviews is 'Which kinds of news are the most interesting in news programmes?' The students stated that the news about violence against women, accidents, discount tents, theft, sports, science and health discoveries were remarkable. In the focus group meeting, it was determined that the students generally found the news about accidents and theft attractive.

In the interview form, in order to determine the professional interests of students, the question 'What is the profession you want to do in the future, why?' was included. The students stated that they wanted to be two judges, two doctors, one nurse and one math teacher; the other two students stated that they have not yet decided on the profession they want to do in the future.

When the answers to the question 'What other activities are you interested in and enjoy doing?' were examined, it was determined that the students gave a common answer that they liked having a picnic and walking in nature. However, drawing pictures, picking mushrooms in the forest, listening to rap and pop music and going out for the first time are among the additional answers given to this question.

Considering that other data can be obtained in addition to the answers given to the questions asked to determine the interests of the students in the interviews – 'How would you spend your day, can you tell?' – a general question was asked in the form. The students gave usual answers to this question such as 'I am watching TV, doing homework'. Apart from these answers, it was determined that specific answers were given as 'I go to the mountain and graze the animals, we enter the kitchen with my brother and have a cleaning contest, we sit in the plateau in the summer and I grow vegetables in my garden'.

The interviews were effective in terms of increasing the effectiveness of the process and enabling students to work with the model creation activity prepared in line with their interests.

As a result of the interviews conducted to determine the common interests of the students, it was stated that majority of the students liked playing basketball and volleyball; they enjoy reading adventure, horror-type books; it was concluded that they found the news presented in the field of sports and health remarkable. Therefore, the modelling problem to be used in the research has been developed to be related to basketball.

The modelling activity named 'Team Fiction', developed in line with the common interests of the students, was evaluated by three instructors who are experts in mathematical modelling according to the design principles that should be in modelling activities. As a result of the evaluations made, it is understood that the modelling activity developed is in accordance with the design principles that should be in the modelling activities.

TAKIM KURMACA

Ergin ATAMAN, 2017 yılından itibaren "Anadolu Efes" isimli basketbol takımını antrenörlüğünü yapmaktadır. Antrenör 2019-2020 sezonu için takımı yeniden oluşturmak istemektedir. Bunun için yardımcı antrenör daha önceden belirlediği 12 tane oyuncunun boy uzunluklarını ve son üç maçtaki performans değerlerini gösteren listeyi Tablo 1'deki gibi hazırlayarak Ergin ATAMAN'a vermiştir. Bu takımın sezonun ilk maçını Banvit takımıyla oynayacaktır. Banvit takımının maçta oynayacağı 5 oyuncunun boy uzunlukları ve son üç maçtaki performans değerleri de Tablo 2'de gösterilmiştir. Ergin ATAMAN, bu maç için oynayacağı 5 oyuncuyu belirlemede ve bu oyuncuları sahaya neresinde oynatacağı konusunda ödüllü kararlıdır. Bu noktada antrenöre bir tavsiye mektubu yazarak hangi beş oyuncuyu maçta oynamasını gerektireceğini seçmesini ile açıklanan bir matematiksel model geliştirin. Ayrıca bu oyuncuları seçmesi durumunda Banvit takımını yenme ihtimali sizce ne kadar olur, bunu da mektubunuzda belirtin.



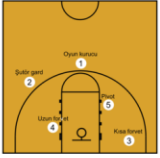
Tablo 1. Oyuncuların boy uzunlukları ve son üç maçtaki performans değerleri

Oyuncu İsmi	1.Maç			2.Maç			3.Maç							
	Ana	2	3	Ana	2	3	Ana	2	3					
Bany ERDAL	194	4	2	3	3	2	3	0	2	2	1	2	2	3
Yakup TAYGÖR	183	3	5	3	2	0	1	3	2	3	2	0	2	2
Mehmet AKILY	198	2	5	2	1	2	2	3	1	0	3	0	3	1
Özgür ERGİL	213	3	4	3	0	3	2	3	0	3	2	3	1	2
Barış KAR	206	3	1	0	3	2	3	1	3	1	2	2	2	2
Kaya YILMAZ	208	1	4	2	0	2	4	1	3	1	2	2	4	3
Emre ÖZGEN	209	0	1	3	1	2	2	2	2	1	2	0	3	2
Barış ÖZDEMİR	204	2	5	2	3	1	3	0	2	2	2	3	1	3
Barış ÖZCELİK	192	1	0	2	1	2	3	1	3	1	3	4	2	3
Yusuf YILMAZ	191	3	2	0	2	3	0	2	2	1	2	2	0	3
Burak SİLİNE	202	2	0	2	1	2	1	3	1	2	3	1	2	2
Cem HURF	203	4	3	1	3	1	3	2	3	0	1	4	2	4

Astı: Sayı atma yolunda gidilen takım arkadaşına pas verme anlamındadır.
Ribaunt: Atıktan sonra yemenden seken topun oyuncu tarafından yakalanması anlamındadır.

Tablo 2. Banvit takımının oyuncuları boy uzunlukları ve son üç maçtaki performans değerleri

Oyuncu İsmi	1.Maç			2.Maç			3.Maç							
	Ana	2	3	Ana	2	3	Ana	2	3					
Berke ATIL	210	1	3	2	2	1	3	2	1	4	2	2	3	3
Serdar GÖRÜKÇÜ	210	1	4	3	1	2	2	3	2	3	2	0	2	4
Mehmet AYVALI	206	1	3	2	2	0	0	4	1	3	1	2	4	0
Alihan PERİZ	194	4	2	3	0	1	1	0	2	2	0	1	3	2
Serkan ÇEM	182	3	3	2	2	1	4	2	3	1	1	0	2	2
MUĞLA														



Anadolu Efes takımına seçmiş olduğumuz 5 oyuncuyu hangi pozisyonlarda oynamalarını gerektireceği ilgili Ergin ATAMAN'a yardımcı olun. Yazacağımız tavsiye mektubunda oyuncuların yerlerini Astı belirlediğimiz açıkları ki antrenör daha fazla düşünmüş bu konuda.

Figure 3. Team fiction modelling activity

A calm environment has been prepared where students can do their studies comfortably. It is known that this prepared room was previously used as a school library and then emptied completely. This empty room was turned into a suitable environment for interview and modelling studies by placing suitable tables and chairs by the researcher. During this application, the students worked on this problem in two groups. The application was carried out during the hours of the Elective Mathematics Applications course, and students were given up to two lesson hours (60 minutes). While the students were working on the modelling activity, observation notes were taken by the researcher and no intervention was made to the students in any way. Important points, such as the questions that the students asked the researcher during their studies, the remarkable speeches they made in the group, the difficulties that the students faced, are included in the researcher's observation notes.

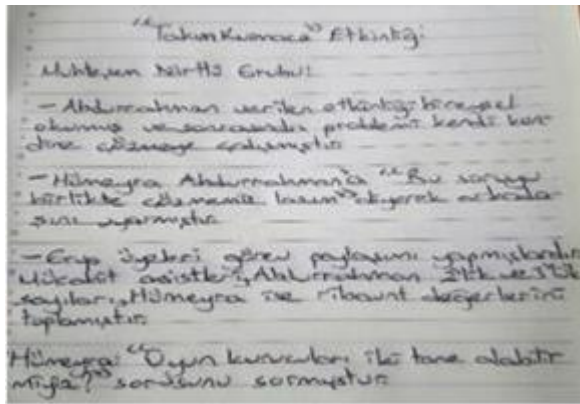


Figure 4. A section from the researcher's observation notes

After the activity sheets were distributed to the students, they were given as many blank papers as they wanted and were asked to write their solutions on these papers. In this study, video recordings were taken with the permission of the participants. The technological devices used in video recording were checked with regard to their defective status and charge status. Technological tools that make video recordings were positioned in a way that the participants were not disturbed, and the people who shot the video were selected from among the schoolmates who were familiar with the students in the study group and would not hesitate during the interview and modelling activities. The video recordings were analysed by transferring them to the computer environment and were very effective in analysing student works and obtaining research findings.

2.3. Analysis of data

The data obtained from the interviews conducted to determine the interests of the students were subjected to content analysis. Content analysis method, which is frequently used in studies conducted in the fields of communication, psychology, sociology and literature, is also frequently used in studies in the field of education. In content analysis, where it is aimed to reach the concepts and relationships that can explain the data, the data are subjected to a deeper process than descriptive analysis. Each of the interviews conducted in this study was videotaped, and then these recordings were transferred to the computer environment. By watching the video recordings, the answers given by each student to the questions in the interviews were written down and the interests of the students were tried to be determined. Thus, video recordings were transcribed from beginning to end and these raw data were made meaningful in accordance with the purpose of the research

Students' modelling skills were examined with the help of a rubric adapted from the studies of Tekin Dede and Bukova-Guzel (2014) and Sahin and Eraslan (2018). With the aforementioned scoring key, the data obtained from student solution papers, researcher's observation notes and video recordings were subjected to descriptive analysis within the scope of problem understanding, simplification, modelling, mathematical study, interpretation, verification and reporting skills; the levels at which students exhibited these skills were evaluated and scored.

In the stage of understanding the problem, students are expected to express what is given in the problem in their own words, determine what is given and what is desired and establish an appropriate relationship between them. The expected behaviour of students who move to the simplification step is to make realistic assumptions by determining the necessary and unnecessary variables. During the mathematical modelling phase, it is checked whether the students can correctly create the required mathematical model (s) according to the assumptions, and whether they associate these model (s) with each other by explaining them. It is expected that students will reach the correct mathematical solution by using the mathematical model (s) they have created in the mathematical study phase. What is expected from the students who come to the interpretation stage is that they interpret the correct mathematical solution correctly in the context of real life. In the verification phase, it is important to what extent students correct the errors determined by using the verification approach. In the reporting stage, which is the last step of the modelling, the important point is whether the report prepared by the students gives enough information about the solution process and whether it is clearly understood by the reader.

This scoring key used in evaluating modelling skills consists of three parts. In the first part, the work carried out by the groups is evaluated in terms of a total of six modelling skills: understanding the problem, simplifying the problem, creating a mathematical model (mathematisation), working mathematically, interpreting and verifying; the second part is from the part where each modelling skill is handled under different levels and the scoring takes place; and the third part consists of the part where the definitions of the levels are made. A section containing the levels and scoring of the scaled scoring key of simplification modelling skill and the definitions of these levels is given in Table 2.

Table 2. A section from the ranked scoring key

Modelling skills	Duzeyler points	Definition
Simplification	Level 1 0 points	Simplify the problem, identify necessary/unnecessary variables and make false assumptions
	Level 2 1 point	Simplify the problem to some extent and determine the necessary/unnecessary variables to some extent, but make false assumptions
	Level 3 2 points	Simplify the problem, identify required/unnecessary variables and make some degree acceptable assumptions
	Level 4 3 points	Simplify the problem, identify necessary/unnecessary variables and make realistic assumptions

For example, in this study, in the process of simplifying the problem, it was evaluated that the students of the Great Four groups determined the necessary/unnecessary variables to simplify the problem but made some acceptable assumptions, and these skills were given 2 points out of 3 points.

2.4. Validity and reliability

The two most common criteria used for the credibility of the results obtained from a study are validity and reliability (Baskale, 2016, p. 23).

Internal validity (credibility) is how accurate the results of a study are **and** whether the purpose of the research and the social reality of the participants are reflected correctly (Yagar & Dokme, 2018, p. 7). Accordingly, internal validity is actually – ‘Do the obtained findings reflect the real situation?’ – to seek an answer to the question. The ‘triangulation’ technique, which is one of the most known and applied strategies, was used to increase the internal validity of this study. This technique is used in qualitative research to minimise misperception and the invalidity of the results (Stake, 1995, p. 134). According to Denzin (1978, p. 301), there are four types of triangulation: the use of multiple methods in data collection, the use of multiple data collection sources, the participation of more than one researcher **and** the use of multiple theories to compare, check and validate the findings. In this study, using interview, observation and document analysis as multiple methods in data collection **and** diversification was made in the data collection stage, and the weakness in one was compensated by the counterbalancing power of the other. At the same time, a theoretical framework that determines the working method has been created by conducting a wide literature review. Thus, while **analysing** the transcripts obtained from the video recordings of the students during the **modelling** studies, within the scope of the theoretical framework, the student solution papers were also examined and the data obtained were presented to the reader in a way to support and complement each other.

One of the methods used to ensure internal validity is that the researcher interacts with the study for a long time. By long-term interaction, we mean to further increase the credibility of the data obtained by establishing a trust environment between the study group and the researcher. Since the researcher conducted the study with the students he had taken his course, it was seen that this trust environment already existed and this effect was reflected positively in the studies.

Another way to increase the credibility of the study is to take advantage of the recommendations of an expert on various stages of research (Lightning & Lightning, 2005, p. 268). In the process of preparing the modelling activity to be used in the study, structural control of the activity was ensured by a faculty member who is an expert and mathematical solutions of modelling activity were carried out by the researcher before application. Thus, it was tested whether the problem situation in the modelling activity provided the principle of modelling. At the same time, other design principles were observed when preparing the modelling activity. At this point, the modelling activity was prepared by

the researcher and examined by an expert, and then it was examined in terms of design principles by three instructors who also specialised in mathematical modelling.

External validity (transferability) relates to the extent to which certain findings of a study can be adapted to similar situations, provided that they preserve meanings and inferences (Arastaman, Fidan & Fidan, 2018, p. 59). Accordingly, the external validity is actually to look for the answer to the question 'Can research findings be tested in similar situations?' The method of studying the provision of external validity and the process of developing them with the data collection tools used has been clearly demonstrated.

Internal reliability (consistency) is based on whether other researchers will achieve the same results using the same data. With the rubric prepared in this study, it is aimed to quantify the modelling skills of the groups and to examine the development of modelling skills of secondary school students in the process by making comparisons between these scores. According to Yildirim and Simsek (2005, p. 242), one of the main objectives of reducing qualitative data to numbers is to increase reliability. From this point on, one of the ways to increase internal reliability is to obtain analysis support from experts in the analysis of the data and to ensure scoring reliability with different scorers (Baltaci, 2019, p. 381). Based on this, all the modelling skills exhibited by a randomly selected group (The Magnificent Four) were independently rated by a researcher and a lecturer who specialises in mathematical modelling to check whether the rubric is capable of improving the internal reliability of the study. As a result of these independent ratings, a difference of opinion and six consensuses were achieved. The reliability coefficient $(6/[6 + 1]*100)$ was determined as 86%. This indicates that the ratings are reliable.

In order for the study to be ethically appropriate, it was stated that the real names of the students who formed the study group would not be used, and different names were used instead of the real names of the students in the study.

After the students' interests were determined and after modelling, the students who formed the research working group were asked 'Why? How'd you do it? What did you mean here?'

3. Results

In this section, the findings of the data collected during the study of the students who make up the working group on the modelling activity called 'Team Building' are presented in detail.

3.1. Findings from the Magnificent Quartet group

3.1.1. Ability to understand the problem

Group members who prefer to read the given problem text quietly individually were seen to start discussing how to help Ergin ATAMAN in case of problems before starting to read the data presented about the performance of the players. Meanwhile, conversations between participants were recorded as follows:

Tuba: We're going to make the team here, what's his name? Ergin ATAMAN. This guy's struggling, or rather, he can't make up his mind. We're going to form the team and write a letter instead.

Melike: Look not only to form a team but to look (by showing the event) at the end says how likely you think it would be to beat Banvit team if he chose these players, write this down.

Hayrunnisa: Tables 1 and 2 say here. Ergin ATAMAN's was in Table 1. What was against Banvit's in Table 2?

Tuba: Well... Ok. We will form the team of Ergin ATAMAN from Table 1. Likewise, we're going to look at the other team from Table 2. Let me take a good look at these paintings.

Melissa: Tuba here is what says: Rebound with assist. Does anyone know what these are?

Tuba: Look below (showing the activity) it's written there.

Melissa: Okay, it says, but now we're going to look at what we're looking at, which is a lot of things here: 2s, 3s, height, rebounds, and an assist.

Tuba: On this last page, the teacher gives a picture, says positions. So we're going to have to do it here. Does this say anything?

Melissa: No, she didn't write them down... The teacher said you could look into what you didn't know. Let's go check these out.

With Melissa's suggestion, the students went to the computer class and researched basketball positions online and determined the characteristics that each player should have.

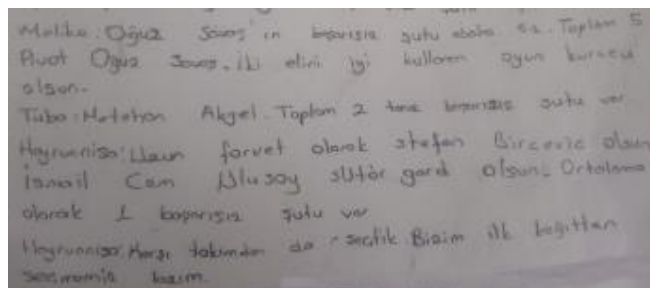


Figure 5. An excerpt from the form of group work of the magnificent quartet

Although they stated which table data students should use when creating individual teams, it was noted that they formed a single team by looking at the data in both tables in Figure 5. After a while, Hayrunnisa, a member of the group, told his friends that they should only benefit from the data in the first table when forming the Anadolu Efes team. The students then continued their studies in this way by remembering that the data in Table 1 belonged to Anadolu Efes and the data in Table 2 belonged to the Banvit team.

As can be seen from the above speeches and the presented section, the students read the problem text thoroughly and focused on what is requested of them before looking at the data presented in the problem. In this sense, although the group members had problems for a while, they determined that they should use the data presented in Table 2 when determining the players who make up the Anadolu Efes team and the players who make up the Banvit team. However, Melike told his friends that the team players they had chosen for Ergin ATAMAN should express in a letter that they would also write about the possibility of defeating the opposing team. Therefore, the students of the Magnificent Quartet verbally stated their determinations of the problem after their conversation and explained in detail what was given and what was desired. It is understood that the students who determine the characteristics of the player in each position by conducting research can establish an appropriate relationship with the performance values presented in the tables in determining the players. For these reasons, the problem understanding skills of the Magnificent Quartet were evaluated at level 5 and these skills were given a full score, i.e., 4 points.

3.1.2. Problem simplification skills

Here is how they talk about the assumptions they have created to make the problem even more understandable among students who determine which traits they should choose after doing research.

Tuba: Oguz Battle as a pivot, height above average. And he's got a lot of rebounds. His rebound will also be high, as the pivot will catch the ball better under the basket.

Melissa: Mykal Riley, according to the quarterback. He's going to be tall, and he's going to have a lot of failed shots.

Hayrunnisa: Don't let the short striker be too tall. It's short on the name.

Melike: Let's find a shooting guard.

Hayrunnisa: The shooting guard must be Cole Huff. His failed shots are few, he's very good in rebounds and assists.

Melike: Could the quarterback be Peace Melted? Three of these is pretty good.

Hayrunnisa: Mykal Riyel. The other one's failed shot is eight. We're going to look at the failed shot.

Tuba: The tall forward will be quick.

Melissa: We can't tell if it's fast. It's going to be long once, that's for sure. Kyle or Sovereign.

Hayrunnisa: Egemen's rebounds don't seem to be that good.

Melissa: Ege will be a tall striker.

Tuba: Berk will be a shortsy forward. Because the tall forward is short, has good assists and rebounds. Haa... And the shot is longer than the guard.

Tuba stated that the tall striker should be fast, while Melisa abandoned this assumption, saying that the players could not reach the conclusion of whether they were fast or not. Another point that is remarkable in the simplification of the problem is that no assumption was created except that the tall striker was tall in determining. Thus, they noted that the pivot is tall and has high rebounds in the team they will form the Magnificent Four group, the shooting guard's unsuccessful shots are low and their assists are high with their rebounds, the quarterback's unsuccessful shots are few and assists are high, the tall forward is tall, the short forward is taller than the shooting guard and his rebounds are high with his assists.

The students, who determined the players of both teams according to their positions in line with the determined assumptions, considered all the performance values given for each player they determined in terms of finding the possibility of the team they determined for Ergin ATAMAN to beat the opponent team.

According to these findings, in the process of simplifying the problem, it was evaluated that the students of the Great Four group simplified the problem by determining the necessary / unnecessary variables but made some acceptable assumptions, and these skills were given 2 points out of 3 points.

3.2. Mathematical modelling (mathematisation) ability

Noticing that the performance values of the players in three matches were given in both tables, they thought that it would be wrong to determine the player by looking at the rebound value in a single match, based on the assumption that their rebounds were high while determining the pivot, and they collected the rebound values in three matches and made their selections according to this result. Similarly, they created their mathematical models according to the criteria they determined when comparing performance for other players. In Figure 6, there is a mathematical model formed by the students based on the assumption that they have few unsuccessful shots in determining the playmaker.

When the students reached the stage of determining the long forward for both teams, they were found not to be able to create a valid mathematical model simply because they looked at the length of the height.

The students who reach the stage of finding the probability of defeating the Banvit team of the Anadolu Efes team they have formed, assists and rebounds, 2 and 3 points performance values for each match of the players they have chosen for both teams are positive integers; they associated the failed shot values with negative integers. Later, they thought the sum of these integers they obtained as points they got from the matches. In the last stage, they reached the total scores of the players by adding the points they got from all three matches for each player. The students who obtained the total score of the team by adding up the total points of the players in the teams in three matches,

tried to calculate the probability of Ergin ATAMAN's team beating the opponent as a percentage. However, while the students had to multiply the ratio of Anadolu Efes team's score to the total score of both teams by 100, they divided the team points they found by 100 directly, thinking that they calculated the probability of defeating as a percentage in this way and finished their solutions.

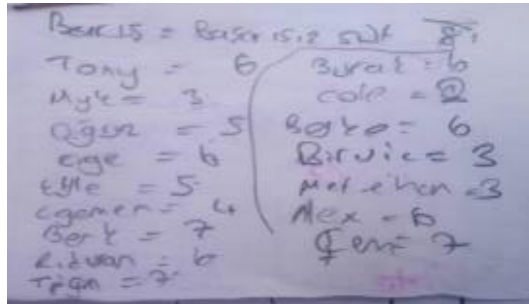


Figure 6. An excerpt of the Great Quartet's mathematicalisation of what is given

It was evaluated that the Magnificent Quartet created incorrect mathematical models based on some acceptable assumptions and demonstrated the ability to mathematicalise the problem at level 2, which was given a score of 1 out of 4 points.

3.3. Mathematical ability to work

The students of the Magnificent Quartet correctly carried out the solution of the mathematical models they created as a separate sum of the performance values given for each player and wrote the results they found as in Figure 7.

	First	Second	Third	Fourth	Sum
Burak	6	6	4	5	21
Tony	6	6	4	5	21
Myt	5	5	3	4	17
Qge	4	4	3	3	14
ege	4	4	4	3	15
Eyle	3	3	4	3	13
cgenen	3	3	5	6	17
Serk	5	4	2	7	18
E.ivan	2	1	5	6	14
Tigen	1	6	5	4	16

Figure 7. Some of the mathematical work carried out by the Magnificent Quartet

It was determined that the students correctly carried out their mathematical solutions despite creating an incorrect mathematical model at the point of calculating the probability of Anadolu Efes team beating Banvit team after selecting the players in the teams. Based on these findings, it was determined that the Magnificent Quartet correctly solved the incorrectly formed mathematical models, and their skills were evaluated at level 3 and these skills were given a score of 2 out of 4 points.

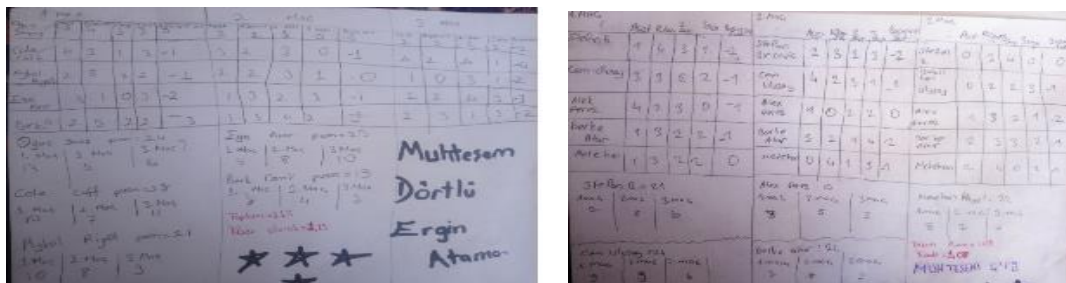


Figure 8. Mathematical studies carried out by the Magnificent Quartet in calculating the probability of Anadolu Efes' team beating Banvit's team

3.4. Interpreting skill

The students who carried out their solutions on their mathematical models correctly interpreted the mathematical results they found within the framework of the assumptions they determined and determined the team players according to their positions as in Figure 9.

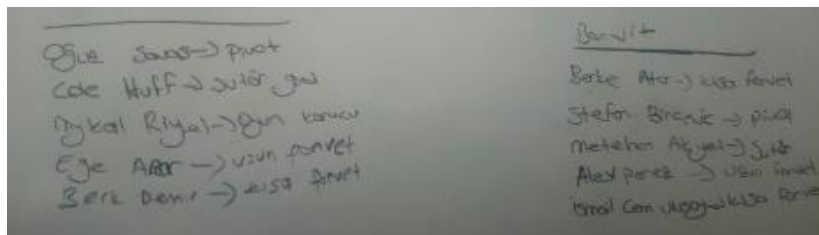


Figure 9. Teams of students of the Magnificent Quartet

Although they correctly carried out their solutions on the mathematical model that Anadolu Efes' team had created in error in the possibility of defeating Banvit's team, the students made incomplete interpretations of the possibility of defeat. Examples include the following conversations:

Tuba: How many results did we find Hayrunnisa?

Hayrunnisa: One says 1.15 and the other says 1.08.

Abdurrahman: Tuba, are you going to write this? I don't know where the odds are here, now Anadolu Efes has more points. Then he'll win for sure. Then there's no chance. I don't get it.

Tuba: Then let's not write it down.

Hayrunnisa: Hi... It's complicated, never mind that.

Melissa: Then let's write that our team wins..

According to the above conversations, the students found that they did not get the results in accordance with the word 'probability', so they chose to comment incompletely at the point of explanation of the possibility of defeat.

Based on the aforementioned citations and all these findings, it was decided that the students' ability to interpret the correct mathematical solution incompletely in the context of real life was at Level 4 level. Therefore, the skills of the Magnificent Quartet to interpret the Team Building problem were given a score of 3 out of 4 points.

3.5. Verification skill

The students created an erroneous mathematical model to find the possibility of the Anatolian Ephesus' team beating the Banvit's team. Although they correctly solved this model, this also caused students to underinterpret the possibility of defeat. It is understood that from the conversations between the students, they realised that the students interpreted the possibility of being defeated as incomplete. However, the students did not choose to control the assumptions they created, the mathematical models or the mathematical solutions they carried out in any way. Therefore, the students did not take a verification approach; these skills were evaluated as level 1 and given a score of 0.

3.6. Reporting skill

After their research, the students explained which of the players in different positions they used to determine which ones they used in a letter to Ergin ATAMAN, as in Figure 10.

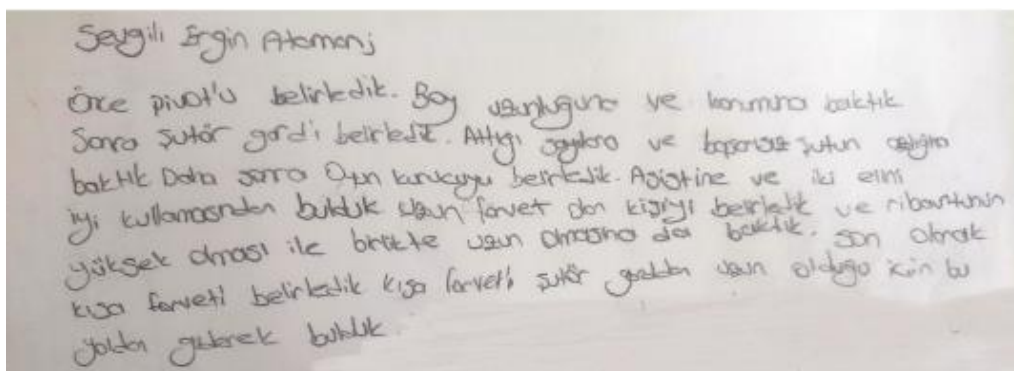


Figure 10. Assumptions made by the Magnificent Four after their research

It is noted that the students who have reached the stage of reporting their work during the modelling period write incompletely the assumptions they create when selecting players in all positions. No information was given about the mathematical models created in the report and the solution process of these models. In addition, in the process of interpreting the possibility of defeat, a superficial interpretation was made by saying that only Anadolu Efes' team would be defeated. Therefore, the letter written by the students to Ergin ATAMAN was considered very weak in terms of credibility. Therefore, students' ability to report was given a score of 0, considering that the report did not contain any information about the resolution process.

3.7. Findings from the shining stars of the future group

3.7.1. Ability to understand the problem

Conversations between the students of the Shining Stars of the Future group, who read the entire event individually, were as follows:

Humayra: Guys, we need to find them first, pivot, shooting guard.

Abdurrahman: What do you mean?

Humayra: We will investigate, so does anyone know about this? Mujahid, you like basketball, you know that?

Mujahid: Hi... I don't know, I don't know.

Zeynep: Then let's go check it out.

After Humayra said that they needed to find out what player positions were, the group members went to the computer class and investigated the player positions with Zeynep's suggestion.

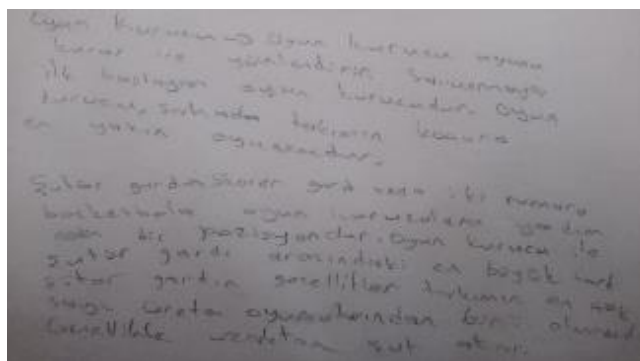


Figure 11. Some of the data obtained by the Shining Stars of the Future group after research to understand the problem

According to the data obtained after the research, the discussion between the students continued as follows:

Mujahid: Girls are like in football, for example, the striker is in front, he has to score well and then he has to pass well. So we're going to put together a team based on what we've found. We'll create Banvit. We're going to calculate the chances of the team we last created beating Banvit, so we're going to compare the teams, that's all.

Zeynep: Everybody understood, didn't they?

According to the above excerpt, Mujahid expressed to his friends what he understood about the problem in his own words, and Zeynep questioned whether his friends understood the problem. Therefore, it was observed that the students did the necessary research to understand the problem, they were able to interpret what they understood from the problem, they were able to establish a suitable relationship between them by determining what was given and what was desired and it was determined that their ability to understand the problem was at Level 5 level and these skills were given 4 points, i.e., full score.

3.8. Ability to simplify the problem

The students of the Shining Stars of the Future group drew, as in Figure 12, and determined the criteria by which they would choose the players. In line with this drawing, they started to produce ideas and form their assumptions in order to simplify the problem even more.

Humayra: Can we get two quarterbacks?

Abdurrahman: One person stands in the castle. We'll take one of them.

Humayra: Let's start with the most open missions. Let's start with the pivot. Berke Atar or Metehan Akyel can play in the pivot. Because his rebounds are good.

Abdurrahman: Tony looks like a quarterback, but his assists and rebounds are the same.

Mujahid: The quarterback has more 3s, more assists, few rebounds.

Abdurrahman: Why are they short of rebounds?

Mujahid: The furthest from the basket or less of the ball.

Zeynep: Rebounding and assists can be the same.

Zeynep: Peace will be melted.

Mujahid: Ridvan, let the shooting guard first. He's a 3-pointer, and he's got a good assist.

Humeyra: Pivot should be Ege Arar in our team. He's tall, he's got high ribs, and his 2s are good.

Mujahid: He's tall and he's got to be 3-pointers, he's got to have good rebounds.

Abdurrahman: What's left? Shorty striker. Short rebound, tall and 2-foot-2. That's short for long or better than a 2.

Zeynep: They're all long. Look, even the short striker is going to be tall.

Mujahid: Have you ever seen a short basketball player? It'll be stronger if they're all long.

In line with the conversations that took place to simplify the problem and the realistic assumptions generated after the research, the quarterback's 3 points and assists were high but their rebounds were low, the shooting guard was tall 3 and his assists were high, the tall rebounds of the pivot were high and the 2-point points were high, the tall forward had a high 3-point and rebounds and the short forward had a high 3-pointer and rebounds, rebounding and 2 points. In addition, Zeynep, one of the

members of the group, realised that all players should be tall and stated this situation to his friends. In response, Mujahid asked Zeynep, 'Have you ever seen a short basketball player? It has to be long anyway', he said, stressing that they make their assumptions based on fact.

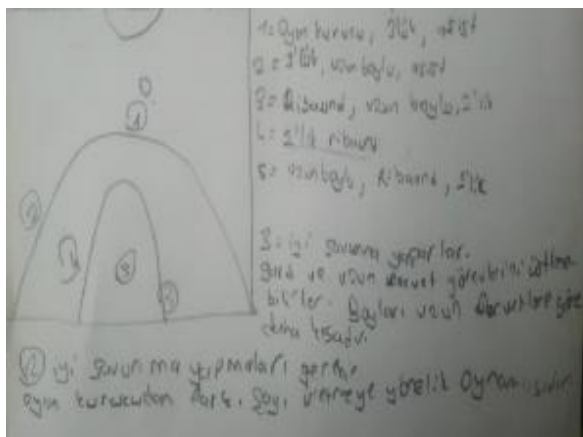


Figure 12. Assumptions made by the Shining Stars of the Future group after their research

After forming their assumptions about how to select players, the students began arguing as follows to determine what they would pay attention to in calculating the probability that the team they had determined would beat the opposing team.

Humayra: How do we make the possibility of defeat?

Mujahid: Humayra, we have determined the players according to the characteristics, we will look at those characteristics again, which is better. I told you, we're going to compare them.

Considering the conversations between the students, it was determined that the problem was simplified enough and the students made realistic assumptions thanks to the assumptions in the proposal for a possible solution for the problem. From this point on, a full score was given on 3 points for this skill, considering that they exhibited the ability to simplify the problem at level 4.

3.9. Mathematical modelling (mathematisation) skill

It was recorded that the students who made their assumptions expressed the variables mathematically and carried out the following conversation:

Zeynep: All players have three games, now it would be unfair if we looked at a match and chose it that way.

Mujahid: Let's find the average numbers, but the number will come out commas. Then let's find the sum of the numbers.

Abdurrahman: I agree.

According to the aforementioned speeches, the first idea came from Zeynep during the mathematicalisation of the variables and stated to his friends that it would be unfair if he chose the players based on the performance values in a single match. Mujahid then said that the players could find the average for each performance value. After a while, they stopped using this mathematical model, thinking that these averages could be comma-free. Instead, he proposed the model of finding the total value for each performance value, and his band mates decided to use it.

While the students created their mathematical models and continued to work on this model, Abdurrahman started to calculate on his own how many points each player scored in three games. After a while, there was a dialogue between Abdurrahman and his friends as follows:

Abdurrahman: Friends, look what I did here (by showing his actions)? I found the points that the top scorers scored.

Mujahid: What do you mean?

Abdurrahman: Now they're scoring a two-pointer and a three-pointer, or the team's scoring points.

Zeynep: But what good is that going to do us, in some, two is important, in others, three is important. We're not going to use their total points to determine players.

Hayrunnisa: Zeynep is right, and it's even harder. Let's not involve him now.

The students who discussed Abdurrahman's proposal for a new mathematical model in the form of the total points earned by the players from the two and three numbers did not accept this model on the assumptions they had already set, thinking that it would not work or even make it more difficult for them to work.

The image shows a handwritten table with the title 'En çok gol atan oyuncular (Üç maç için)' (Top scorers (3 matches)). The table has columns for player names and calculations for three different categories: '2' points, '3' points, and '4' points. The rows list three players: Mykal Riley, Kyle Wiltzer, and Cole Hufc.

Player	2 points	3 points	4 points
Mykal Riley	$2+3+3=8$	$2+1+1=4$	$3 \times 2 = 16 \text{ puan}$
Kyle Wiltzer	$2+3+3=8$	$0+1+2=3$	$3 \times 2 = 9 \text{ puan}$
Cole Hufc	$1+3+4=8$	$3+0+1=4$	$3 \times 2 = 16 \text{ puan}$

Figure 13. The mathematical model proposed by Abdurrahman and rejected by the Shining Stars of the Future group

The following dialogue was held to create the mathematical model among the students who reached the stage of calculating the probability of Anadolu Efes' team beating Banvit's team:

Humayra: Now let's write down the individual voters. Mujahid said we'd look at those traits again and compare them.

Zeynep: We mark the better student.

Mujahid: You're good, you're great.

At the point of calculating the probability of Anadolu Efes's team beating Banvit's team, the students put forward a mathematical model in the form of specifying the ratio of the number of players who are better in the Anadolu Efes team to the number of players in the teams as a percentage after comparing them according to the relevant characteristics of the players in line with the assumption they accepted at this stage, this model was accepted by the group members.

Based on all the excerpts and findings mentioned above, it was evaluated that the students' mathematical modelling skills were at Level 5 level and their ability to create mathematical models was evaluated as level 4 points, i.e., full score, with the idea that they were able to mathematically establish the relationship between the variables and distinguish those that are not in accordance with their assumptions from the mathematical models they have created, both in the process of selecting the players and in the calculation of the probability of the team they determine to beat the opposing team.

3.10. Mathematical ability to work

The students who determined the mathematical model based on the assumptions accepted during the determination of the players as the sum of the performance values in the three matches carried out their mathematical solutions and wrote their results as in Figure 14.

Player	Match 1	Match 2	Match 3	Match 4	Match 5
Yigit Arslan	8	6	4	7	8
Yigit Arslan	6	6	4	8	6
Yigit Arslan	5	7	8	4	9
Yigit Arslan	8	3	4	5	3
Yigit Arslan	4	6	4	8	6
Yigit Arslan	7	9	8	9	5
Yigit Arslan	9	5	5	6	4
Yigit Arslan	5	9	3	7	2
Yigit Arslan	7	9	5	6	6
Yigit Arslan	3	6	5	4	4

Player	Anadolu Efes	Banvit	Match 1	Match 2	Match 3	Match 4
metehan Akcel	206	3	14	3	7	2
ismet Cem Uslan	182	7	7	7	6	3
Berkte Akar	210	6	8	6	8	4
Stefan Bricovic	210	3	9	9	4	4
Alex Perez	191	6	5	7	3	3

Figure 14. Mathematical calculations carried out by the Shining Stars of the Future group

In order to explain in a letter to Ergin ATAMAN, the possibility of Anadolu Efes' team beating Banvit's team after determining the players, they wrote separate sums of the performance values of the players in the three matches and compared the players in the same position according to their relevant characteristics. Students who determined which player was more successful according to their position concluded that they should indicate the result as a percentage based on the word 'probability'. In this sense, they associated the percentage concept of the result they found correctly by proportioning the number of players who were better in Anadolu Efes than the opposing team.

Player	Match 1	Match 2	Match 3	Match 4	Match 5	Match 6	Match 7	Match 8
Yigit Arslan	3	8	3	6	2	4	4	0
Yigit Arslan	3	4	6	6	4	4	4	0
Yigit Arslan	4	4	4	5	3	8	6	4
Yigit Arslan	5	6	4	4	4	4	4	4
Yigit Arslan	6	6	4	4	4	4	4	4
Yigit Arslan	7	4	4	4	4	4	4	4
Yigit Arslan	4	4	4	4	4	4	4	4
Yigit Arslan	5	4	4	4	4	4	4	4
Yigit Arslan	4	4	4	4	4	4	4	4
Yigit Arslan	4	4	4	4	4	4	4	4

Player	Team	Result
Yigit Arslan	Banvit	✓
Sutan Bard	Ridvan	✓
Kiso Forvet	Mujal	✓
Yigit Arslan	Ber	✓
Piet	Ege	✓
Alex Perez	Banvit	3/3 = 100%
ismet Cem Uslan	Banvit	3/3 = 100%
Stefan Bricovic	Banvit	3/3 = 100%
Berkte Akar	Banvit	3/3 = 100%
metehan Akcel	Banvit	3/3 = 100%

Figure 15. Mathematical solutions carried out by the Shining Stars of the Future students in the process of finding the possibility of Anadolu Efes' team beating Banvit's team

Based on all these findings, it was determined that the students of the Shining Stars of the Future group reached the correct mathematical solution using the correctly formed mathematical models, and their skills were evaluated as level 5 level mathematically and these skills were given a score of 4 points..

3.11. Interpreting skill

The students who carried out their solutions on the mathematical models they created first determined the players and their positions, and then compared the players in the same positions to indicate which player was better. In this sense, it is understood that students can accurately interpret the results of mathematical models in the context of real life in accordance with the assumptions they have chosen.

	<u>Bla'im takım</u>	<u>Rakip Takip</u>
Oyun kurucu	Boris Ermiş ✓	Alex pere 2
Sütar Oard	Ridwan ✓	Ismail
Kısa forvet	Muhal	Metenen ✓
uzun forvet	Berk ✓	Stefon
Pivot	Ege	Berk ✓

Figure 16. The results of the students in the stages of determining the players according to their positions and finding the possibility of Anadolu Efes' team beating the opposing team

Considering these findings, it was observed that the students were able to interpret the numerical results they obtained correctly, and these skills were given a score of 4 points, i.e., a full score, considering that they realised the skills to interpret the problem at level 5.

3.12. Verification skill

When the students reached the stage of writing letters to Ergin ATAMAN by filling out the report form, the conversations between them were as follows:

Abdurrahman: Look again, Mujahid, is there anything we've done wrong? We're going to write it down in the letter.

Humayra: Me and Mujahideen are checking now.

Mujahid and Humayra confirmed the assumptions they created, the mathematical models they created based on these assumptions, the mathematical solutions they carried out by checking, and after making sure that they had no mistakes at these points, they proceeded to the reporting stage. Therefore, students were evaluated as being at Level 7 level during the verification phase and these skills were given 6 points, i.e. full score.

3.13. Reporting skill

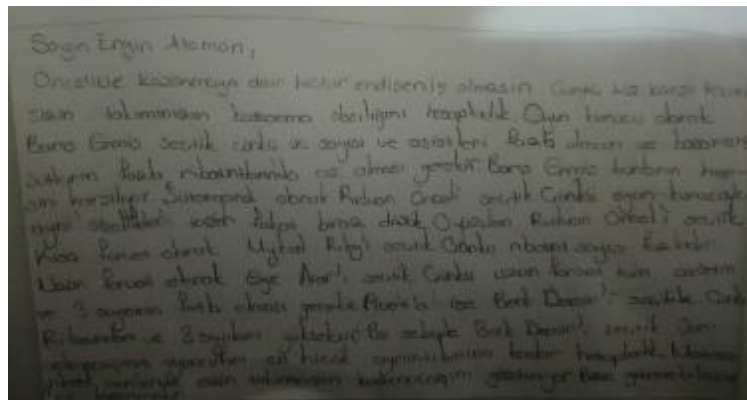


Figure 17. Report by Shining Stars of the Future group

When the report form filled out by the students was examined, it was noted that while they correctly and completely wrote down the assumptions they used to select quarterback, pivot and tall forward

players, they did not write down which assumptions they determined the shooting guard based on, but they incompletely wrote the assumptions they used to identify the shortsta forward. However, it is understood that they did not inform Ergin ATAMAN about what assumptions they created, what mathematical model they created, and any mathematical solution they realised in the process of calculating the possibility of selecting and defeating both players, although they stated that their team would win the opposing team and should have no concerns in this sense. Therefore, due to the fact that the report does not contain any information about the resolution process, it is evaluated that the students have achieved the reporting skills at Level 1 level and these skills are given a score of 0.

4. Discussion, conclusion and recommendations

In this research, it was aimed to examine the modelling skills of the students and a mathematical modelling situation developed for this purpose was applied. In the light of the findings, some results have been reached. The results were evaluated separately in terms of each mathematical modelling competence.

The students underlined the places they found important to understand the problem, used technology (Internet) to investigate concepts they did not know, verbally expressed what they understood about the problem, and asked each other questions from time to time. In this context, it can be said that students use many strategies in the process of understanding the problem.

It was noted that after reading the problem, the students made assumptions by focusing on the characteristics that each player should have and what data is required in which players before talking about what they understand about the problem. During their creation of their assumptions, it was found that the students made many assumptions and stopped using them by distinguishing those that were not suitable for the problem. However, it has also been observed that students create some acceptable assumptions from time to time. This can be exemplified by the fact that the Magnificent Four group is only tall in selecting the tall forward. Throughout the process, students in both groups successfully used tables and shapes to simplify the problem. It has been observed that students write on the basketball court by drawing the positions of the players on the shape, explaining the assumptions they have set for each player, thus trying to make the complex problem situation more meaningful to them. In this context, it was determined that both teams investigated what characteristics the players had online and made mostly realistic assumptions according to the results of the research.

It was determined that the Magnificent Four group chose the tall forward only by looking at the length, so they could not mathematicalise this situation, and in the process of calculating the probability of defeat as a percentage, they incorrectly created their models.

It has been observed that the team formed by the students of the Shining Stars of the Future group, who carried out their mathematical operations, successfully found the possibility of defeating the opposing team by taking advantage of the probability. In this sense, it was determined that the students used their mathematical work skills effectively and were successful in transferring the mathematical concepts and characteristics they had learned formally to real-life situations. It was determined that the students of the Magnificent Quartet group struggled at this stage when they tried to use the percentages topic and could not fully correlate the result with the concept of percentage. Therefore, it was determined that they incorrectly created their models during the calculation phase as a percentage of the probability of defeat. The only thing the students successfully completed here was that they found the total scores of the teams by associating the concept of failed shots with negative integers and other performance values with positive integers and calculating the scores of both the team they created and all the players in the opposing team. However, students who wanted to express these results with the concept of percentage were found to have failed. Despite the lack seen at this stage, it was observed that the students reached the conclusion of which team was more successful but could not express numerically the desired probability of defeat. This is thought to be

due to learning deficiencies in percentages of students. Thus, thanks to modelling studies, students' learning deficiencies can be identified and these deficiencies can be eliminated. However, it was noted that the students used many mathematical concepts such as probability, negative and positive integers, ratio, percentage subjects at this stage when they carried out their mathematical procedures. In this context, it can be said that students have the opportunity to transfer their mathematical concept knowledge to real-life situations through mathematical modelling. Similarly, Yildirim and Light (2015) stated that as a result of their work with 5th grade students, they had the opportunity to create mathematical concepts and features practically using their formal knowledge through mathematical modelling.

It has been concluded that the students of the Shining Stars of the Future group have created correct mathematical models in line with the realistic assumptions they have determined and have successfully interpreted these mathematical models because they have not made mistakes in their solution. Although the Magnificent Quartet made their solution correctly in finding the possibility of Anadolu Efes' team beating Banvit's team, their interpretation was also incomplete because they created their mathematical models incorrectly. In line with these results, it can be said that an error made in the previous steps of the modelling process negatively affected the next steps. In addition, during the interpretation phase, the students verbally expressed the mathematical results they obtained by transferring them to their real-life situations. This result supports the conclusion that students mainly benefit from verbal representations in the interpretation step of Ozaltun, Khederoglu, Kula and Bukova-Guzel's (2013) study.

During the verification phase, the Shining Stars of the Future group checked their assumptions, models, and solutions and made no corrections, despite realising that the model they created in calculating the probability of defeating the Magnificent Four group was flawed. There are many studies in the literature that have come to the conclusions that students do not control the mathematical models they have created and the accuracy of their solutions (Blum & Ferri, 2009; Cora, 2018; Duran, Doruk & Kaplan, 2016; Galbraith & Stillman, 2006; Sen-Zeytun, 2013). In addition, both groups did not write in their reports the models they created during the modelling process and the mathematical solutions they carried out. Therefore, it was found that students had more difficulty in the verification and reporting stages than in other stages. Hdiroglu et al. (2014) stated that this can be due to the nature of the problem, deficiencies in real-life experiences and students' unfamiliarity with modelling-style practices.

After reading the problem, it was noted that Abdurrahman started to solve the problem alone, while Hayrunnisa, one of the members of group, said that they should solve the problem together. Students were also surprised when the researcher said they could do research online when they needed it, and many said, 'So we're going to cheat?' This indicates that students are accustomed to traditional learning environments. In this context, student-centred studies should be included frequently in the courses and the social skills of the students should be allowed to be improved with the idea that peer teaching is also important.

Although the researcher stated that he would not give any guidance or intervention to the students during the modelling period, the students said from time to time, 'Are we going right or did we do it right?' and they wanted to get feedback.

It was noted that students who had difficulty expressing themselves and were timid at the beginning of the application were able to express their opinions without hesitation towards the end of the study and discussed the problem situations presented with their group mates. In this context, it has been seen that there are improvements in the communication skills of students to ask questions, discuss and question. This result coincides with the conclusion that there was an increase in students' social skills during the modelling activity (Gunalp, 2019).

In this study, which was carried out with a single modelling activity, it was determined that the students' mathematical modelling, validation and interpretation skills were poor. Therefore, modelling

studies should be carried out at regular intervals in schools and it can be examined whether modelling skills have improved by including researches in which students are allowed to work on multiple modelling activities.

The Team Fiction event is designed by researchers for students' interests. In this context, it has been observed that the students have attracted the attention of the problem situation presented and actively participated in the modelling process. The main purpose of using modelling activities is to show students the close bonds of mathematics with life and to enable them to use mathematics effectively in case of problems they encounter in their lives (Doruk & Umay, 2011). Therefore, it is thought that it is important for educators and researchers to use or design modelling activities that have features that will interest students, aiming to enable them to interact between real life and mathematics, to give students a modelling experience and to improve their modelling skills.

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