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The problem-based approach in mathematics teaching and its impact on mathematical problem-solving performance

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Abstract

Problem-solving is a key aspect of mathematics instruction. Although it is part of several national curricula, its implementation is still being vastly explored. This research aimed to investigate and update the existing educational process in mathematics teaching in primary education by designing and evaluating an experimental model of problem-based mathematics education. The study used a pedagogical experiment, with a sample of 240 pupils in Grade 3 from four selected Slovenian primary schools. The results showed that the use of an experimental model of problem-based learning had a statistically significant effect on pupils' mathematics achievement in all the domains of mathematics in terms of numbers, geometry, logic, and set theory. The study suggests that modern school reforms should focus on problem-based mathematics instruction as a means of achieving meaningful and lasting knowledge in students.

Keywords: Achievement; experiment; mathematics; problem-solving; problem-based instruction.

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1. Introduction

Problem-solving is essential to mathematics education (Doorman et al., 2007). It provides students with opportunities to engage with mathematical concepts and ideas in meaningful and authentic contexts (English & Gainsburg, 2015; Jurdak, 2006). Through problem-solving, students can develop their mathematical reasoning, critical thinking, and analytical skills (Belecina & Ocampo Jr., 2018). They learn how to formulate problems, identify relevant information, and apply appropriate strategies and techniques to solve them. Moreover, problem-solving promotes active self-regulated learning (Muis, 2008; Huang et al., 2024), as students take responsibility for their learning and monitor their progress and performance. By using problem-solving in mathematics education, teachers can create a student-centered (Saragih & Napitupulu, 2015) and inquiry-based learning environment that fosters creativity, curiosity, and collaboration (Divrik et al., 2020; Gómez-Chacón et al., 2023). Furthermore, problem-solving can help students develop a positive attitude toward mathematics (Funkhouser, 1993) and appreciate its relevance and applicability in various domains of life.

Teaching students to solve problems in mathematics means promoting students' learning and understanding of mathematics (Ali et al., 2010). This approach involves presenting students with problems that require them to engage in mathematical thinking and reasoning to arrive at a solution (Francisco & Maher, 2005). Problem-solving teaching encourages students to use higher-order thinking skills, such as analysis, synthesis, evaluation, and creativity, to explore and investigate mathematical concepts and relationships (Collins, 2014; Widana et al., 2018). It also promotes the development of metacognitive and affective competencies (Lai et al., 2015), as students learn how to monitor their thinking and emotions and how to persist and persevere in the face of challenges and setbacks.

Problem-solving is particularly important in the early stages of mathematics learning (Tarim, 2009; van Bommel & Palmér, 2018; Li et al., 2024). Through problem-solving, children develop their critical thinking skills and acquire mathematical knowledge that applies to real-life situations (Lambert, 2000). Research shows that incorporating problem-solving activities in early math education can enhance children's conceptual understanding, improve their problem-solving abilities, and increase their motivation and interest in mathematics (Cheung & Kwan, 2021). Thus, it is essential to incorporate problem-solving in early math education to help children develop the necessary skills and knowledge needed for future academic success (van Bommel & Palmér, 2018).

Despite the importance of using problem-solving-based instruction, several studies have found that teachers lack competencies and knowledge about problem-solving in mathematics (Bahram, 2020; Chapman, 2015). In particular, teachers' math anxiety and math self-efficacy are correlated to teachers' ability to solve problems in mathematics (Akinsola, 2008). Moreover, teachers' beliefs about mathematics and problem-solving also affect the extent to which they use this strategy in class (Saadati et al., 2019). Teachers' attitudes toward problem-solving may also influence students' beliefs about it (Yorulmaz et al., 2021).

Considering the Slovenian context, it might be noticed that in the national mathematics curriculum for primary school (Žakelj et al., 2011), several goals regarding problem-solving are mentioned. In particular, it is stated that students need to develop reasoning, generalizing, abstracting, investigating, and problem-solving. Considering, for instance, arithmetic, students should be able to solve arithmetic problems that are also related to real life. Nevertheless, the model of teaching is still traditional, that is, behavioristic. Teachers do include elements of the national curriculum involving problem-solving, but the teaching model has not yet shifted from a behavioristic one to a problem-based approach. Therefore, it is unclear whether students exposed to a problem-based approach would be better problem solvers than those exposed to traditional lessons.

1.1. Literature review

1.1.1. Mathematical problems

A mathematical problem is one in which a person attempts to overcome a problem situation by using mathematical tools (Phonopichat et al., 2014; Zhu, 2007). In the broadest sense, a mathematical problem is defined by Magajna (2003) as an (inner) feeling of discomfort because one is unable to explain a certain fact to oneself or is unable to achieve a desired goal. Problems in math can be classified according to different criteria. In general, they can be distinguished by the visibility of the solution path. Magajna (2003) distinguishes between (Kolovou et al., 2011; Mwei, 2017):

1. routine problems (which are not even problems in the usual sense, but rather exercises), where the solution path is clear to the solver in advance, either because he has already been through it several times or because the path is evident from the formulation of the problem or the context;

2. non-routine problems where the solution path is unclear and the problem is encountered for the first time. The state of the solution routine depends to a large extent on the knowledge and experience of the solver.

Problems in mathematics are also distinguished by the specificity of the goal. When the goal of the problem is well defined, we speak of a closed problem, and when the goal is not defined, we speak of an open problem, where the solver sets his own goals and tries to achieve them by investigating them (Bahar & Maker, 2015). Cotič and Valenčič Zuljan (2009) distinguish mathematical problems at the classroom level mainly according to the path and the goal and classify them into:

1. problems with a closed path and a closed goal;
2. problems with an open path and a closed goal;
3. open-path and open-goal problems, noting that closed-path and closed-goal problems dominate mathematics teaching at this level.

This can give students the misconception that most mathematical problems have a single path and a single solution. If we want to achieve higher levels of proficiency in students, then we need to offer them other types of problems that stimulate their thinking in finding paths and different solution strategies.

Vec and Kompare (2006) note that most problems (that students encounter in school mathematics, for instance arithmetic problems, and problems in science) are closed (structured). When solving closed problems in school, we usually use domain-specific strategies to solve a particular type of problem within that domain. However, in contrast to school problems, most life problems are open-ended and at least partly unstructured. Teaching, therefore, requires the design of learning situations that are characterized by similarities to life problem situations. As a rule, these problems are open-ended problems in which students go through all the stages of problem-solving, from identifying and interpreting the problem to finding, testing, and judging solutions. These types of problems can be used at all stages of the learning process: they can provide a realistic context in which to build and deepen knowledge or to build on it.

1.1.2. Problem-Solving in Mathematics

Mathematical problem-solving is the process of employing mathematical ideas, know-how, and techniques to resolve a mathematical issue (Schoenfeld, 1985). It entails locating the issue, comprehending it, coming up with a solution plan or strategy, putting the plan into action, and evaluating the outcome (Polya, 1945; Schoenfeld, 1987; Voskoglou, 2011). To foster critical thinking, reasoning, and mathematical literacy, problem-solving is a crucial component of mathematics education (Daulay & Ruhaimah, 2019; Lee, 2016). Additionally, it assists students in gaining a deeper comprehension of mathematical ideas and techniques as well as in applying what they have learned in practical settings (Carson, 2007; Stacey, 2005).

Problem-solving, therefore, emphasizes independent solving problems and overcoming the learned constraints that make it difficult to move from the initial to the final state. Thus, we solve a problem when we want to answer a question or achieve a goal, and we cannot easily recall the answer from long-term memory or encounter obstacles to solving it. According to Schoenfeld (1985), to effectively solve problems, individuals must possess and proficiently use appropriate resources (such as mathematical concepts and procedures), heuristic strategies (both general and specific), metacognitive control (the ability to oversee and monitor the entire problem-solving process), and appropriate beliefs (in terms of one's perspective, motivation, and confidence).

A closer examination of current teaching practices reveals that opportunities for students to develop essential problem-solving thought processes are notably limited. This scarcity in practice underscores a need for instructional strategies that actively foster analytical and critical thinking skills within real-world contexts. Schoenfeld (1985) mentions several times that it is necessary to change the conception of mathematics in general from learning formulas to finding different solutions, from memorizing formulas to exploring patterns, from solving problems to formulating research questions, making connections, and solving problems. If teachers' pay more attention to this, then students will have the opportunity to experience mathematics as a changing, dynamic, evolving discipline, rather than the rigid, absolute, rigid, and closed discipline that is currently imposed.

1.1.3. Problem-based learning

Problem-based learning is defined as a way to foster more creative forms of thinking, experiencing, and evaluating (Seibert, 2021), which is also the basis for problem-solving (Merritt et al., 2017; Nurlaily et al., 2019). It should therefore not be seen only from a narrow methodological perspective as a teaching method. Nevertheless, we can agree that this principle cannot be subordinated to others, because it alone cannot be used to realize all the curriculum objectives and at the same time to observe all the principles of teaching, but learning is more effective if we introduce problem-orientation and problem logic (Strmčnik & Lavtižar 2001). In particular, problem-based learning focuses on problems in which students can construct their knowledge (Mulyanto et al., 2018).

Strmčnik & Lavtižar (2001) classifies problem-based learning as a teaching principle. The principle of problem-oriented teaching is applied when problem orientation is present in all phases of the learning process. On the other hand, problem-solving is a teaching method. Problem-based learning extends to the whole classroom, to all its content and process dimensions, while problem-solving has a narrower meaning and covers only part of the learning activity. Nevertheless, problem-solving is where problem-based learning innovations are most directly expressed (Aslan, 2021; Zhou, 2020).

Problem-based learning is oriented towards both exploring the problem situation and acquiring new knowledge, as well as investigating and reflecting on one's way of learning (Downing et al., 2009). In doing so, students build on and acquire both contextual and procedural knowledge as well as metacognitive knowledge (Downing et al., 2009; Downing et al., 2011; Siagan et al., 2019). Problem-based learning is characterized by a problem situation, which can be defined by the teacher or by the teacher together with the pupils. In doing so, pupils acquire new knowledge through exploration, argumentation, verification, and taking a standpoint while being mentally active. In this way, problem-based learning to a large extent balances the relationship between reproductive and creative knowledge acquisition (Strmčnik & Lavtižar 2001).

A slightly different view of problem-based learning is offered by Wood (2003), who specifically analyses both the role of the learner and the role of the teacher, as both are involved in problem-based learning. From the learner's perspective, problem-based learning (PBL) offers several key advantages that contribute to a deeper and more integrated educational experience, as highlighted by Wood (2003). First, PBL encourages learners to take an active role in solving problems independently, enhancing not only their understanding of the acquired knowledge but also fostering collaborative skills essential for continued learning. Additionally, this approach facilitates connections across disciplines, promoting knowledge transfer and integration between subjects. Students also experience heightened motivation as they engage in formulating and resolving problems, making learning more meaningful and self-directed. Furthermore, PBL activates prior knowledge, allowing students to connect new information with existing frameworks, ultimately developing essential cognitive and problem-solving skills.

The effectiveness of problem-based learning can be evaluated by: the teaching style or teacher guidance/orientation, the characteristics of the learners and classroom interaction, the individual learning goals of the learners, as well as the problem itself. It is therefore a rather complex teaching method with several interrelated success factors. Problem-based learning is learner-centered. It assumes that learners will explore, and link theory to practice, apply experience and knowledge to formulate the problem, and find solutions (Savery, 2006; 2015; 2019). Poorly structured problems that relate to an authentic life situation and adequate support from the teacher are crucial for the effectiveness of problem-based learning.

1.2. Purpose of study

Modern school reforms around the world aim to achieve meaningful and lasting knowledge in students, and this is the reason why curricula, syllabuses, and, consequently, mathematics teaching are being updated. We decided to carry out this research to investigate and update the existing educational process in mathematics teaching in primary education. The research problem was focused on the design and evaluation of an experimental model of problem-based mathematics education.

The aim of the present research is, therefore, to evaluate the efficacy of the problem-based approach to the more traditional also known as the behaviorist model of teaching mathematics in primary school. In particular, we expect students who are exposed to a more problem-solving-oriented mathematics lesson to have higher achievements in tasks of all taxonomic levels and from all mathematics domains (arithmetic, geometry, and logic).

2. METHODS AND MATERIALS

An experiment was carried out in school practice with Grade 3 pupils to determine whether the use of an experimental model of problem-based learning had a statistically significant effect on pupils' mathematics achievement. Our general research hypothesis is therefore the following:

GH: Students receiving an experimental model of problem-based mathematics instruction will be more successful in solving mathematical problems in all mathematical domains (arithmetic and algebra, geometry with measurement, and logic and language) than students receiving classical (behavioristic) mathematics instruction.

2.1. Research design

In the present research, the experimental method of pedagogical research was used. The experiment was carried out in existing primary school departments. This means that the sections were not equated to chance differences before the experiment. Pupils in the experimental group were exposed to a problem-based learning environment, while students from the control group continued studying in a traditional model. The experimental and control groups included female teachers, equal in terms of educational level.

2.2. Participants

The study was carried out on a sample of 240 pupils in the 3rd grade in four randomly selected Slovenian primary schools from the Coastal Region, 100 pupils were included in the experimental group (57.0% females) and 140 pupils in the control group (52.1% females). All the selected primary schools were urban schools with equally solid working conditions. The socioeconomic status of students of both groups has shown that pupils are mainly from middle-class families.

2.3. Data collection instruments

The study used a pedagogical experiment as part of the empirical research approach. Quantitatively, we analyzed the data using knowledge tests (two knowledge tests in mathematical problems).

2.3.1. Test of knowledge

For the research, two knowledge tests were designed. The initial and final mathematical knowledge of the experimental and control groups was assessed using an initial and final knowledge test. The tasks in both knowledge tests were designed taking into account Gagné's taxonomy (Cotič & Žakelj, 2004) and the mathematical content of grade 3 according to the mathematics curriculum (Žakelj et al., 2011).

The test was composed of three parts, each of which was composed of 12 tasks. For each part, students had 1 school hour (50 minutes) to solve it; they took the test on three different days. The majority of the tasks were open-ended, in some exercises the pupils needed to circle one of the four given answers. Each exercise was worth from 1 to 3 points: students got the points only if they solved the problem correctly; otherwise, the task was evaluated with 0 points.

Regarding the taxonomic levels of the test, tasks regarded (1) the understanding of the concepts and definitions (conceptual knowledge), (2) the solving of simple problems (procedural knowledge), and (3) the solving of complex problems (problem-solving). In the latter, students had to decide the most suitable procedure to apply to solve the problem and needed to think about whether the tasks could be solved. For the present research, problems from levels I and II are considered "elementary" problems, while problems from level III are considered "complex" problems.

The instrument developed tests students' knowledge of (1) arithmetic (A), (2) geometry and measuring (G), and (3) logic and set theory (L). In the test, problems concerning arithmetic were preferred, since it is the mathematical topic that pupils are the most used to in the first grades of elementary school. The characteristics of both knowledge tests (objectivity, reliability, and validity) were demonstrated on a pilot sample of 102 Grade 3 pupils from two randomly selected Coastal schools.

The validity of the test was determined through qualitative analysis. The contents of the test, which aimed to measure students' mathematical knowledge (and, specifically, problem-solving abilities) of grade 3 pupils, were analyzed by a group of experts (professors of mathematics education, mathematicians, and elementary school teachers). Based on their critical analysis of the instrument, we improved its contents and structure. Moreover, the test achievements were positively and statistically significantly correlated with students' mathematics grades ($r = .89$; $p < .001$).

The objectivity of the instrument regarded two factors, (1) the objectivity of test-taking, and (2) the objectivity of the evaluation of pupils' answers. Regarding the first, researchers gave pupils detailed information about the test itself, thus limiting the possible researchers' influence on pupils. Concerning the second, the test was structured in such a way that there were many closed-type questions. Moreover, the tests were corrected and graded by three independent people: a researcher (R) and two mathematics teachers (T1, T2). The inter-raters' correlations were 1.00 (T1, T2), .99 (T2, R), and .99 (T1, R).

The reliability of the instrument was assessed using the method of parallel tests (Mueller & Knapp, 2018). The same students were tested two times; the second test was not the same as the first, as it was a parallel version of it, that is, a test that was composed in such a way that it tested the same contents and taxonomic levels as the first. The measure of reliability was obtained considering the correlation between the two results. The correlation is positive, high, and statistically significant ($r = .97$; $p < .001$), indicating excellent reliability of the instrument. Moreover, the research also considered the difficulty of the test and its items (Mahjabeen et al., 2017). In particular, we maintained only the test items that had the index of difficulty, thus the ratio between correct answers and all answers, between 10% ("difficult" questions) and 90% ("easy" questions).

2.3.2. Procedure

The initial knowledge test was administered to the control (CG) and experimental (EG) groups before the start of the experiment, and the final knowledge test was administered after the experiment under the same conditions and with the same tester. The study was carried out over 8 months. The year before the start of the experiment, teachers from the EG were instructed to experiment. During these sessions, teachers acquired theoretical and practical knowledge about problem-solving, received additional material (workbooks, worksheets, etc.), and were instructed not to use a problem-oriented model of teaching during the current year. During the study itself, the EG was taught mathematics in grade 3 using the experimental model of problem-based learning, while the CG was taught mathematics in grade 3 using the (traditional) behavioral model.

2.4. Analysis of data

The data in the empirical part were analyzed using the statistical analysis software SPSS 26.0. The following statistical procedures were used:

- analysis of covariance (ANCOVA);
- descriptive statistics: mean (*M*), standard deviation (*SD*), frequencies, minimum (*min*), and maximum (*max*);
- Kolmogorov-Smirnov test for normality of distribution;
- *T*-test (Cochran-Cox approximate t-test method) to determine the differences in knowledge of mathematical problems in mathematical content (arithmetic; geometry with measurement; logic and language) between the experimental and control group students at the beginning and the end of the experiment.

3. RESULTS

3.1. Pre-test

In Table 1 we present the descriptive statistics for the pre-test. In addition, we report the results of the t-test, which was used to check for possible differences between the experimental and control groups. For the research, we calculated the statistics for easier (I) and more complex problems (II). Easier problems are those that are from the second Gagné taxonomy scale, while more complex problems are those from the third Gagné taxonomy. A graphic representation of the test achievements is in Figure 1. Differences between the EG and CG might be found in the pre-test, in favor of the EG.

Table 1

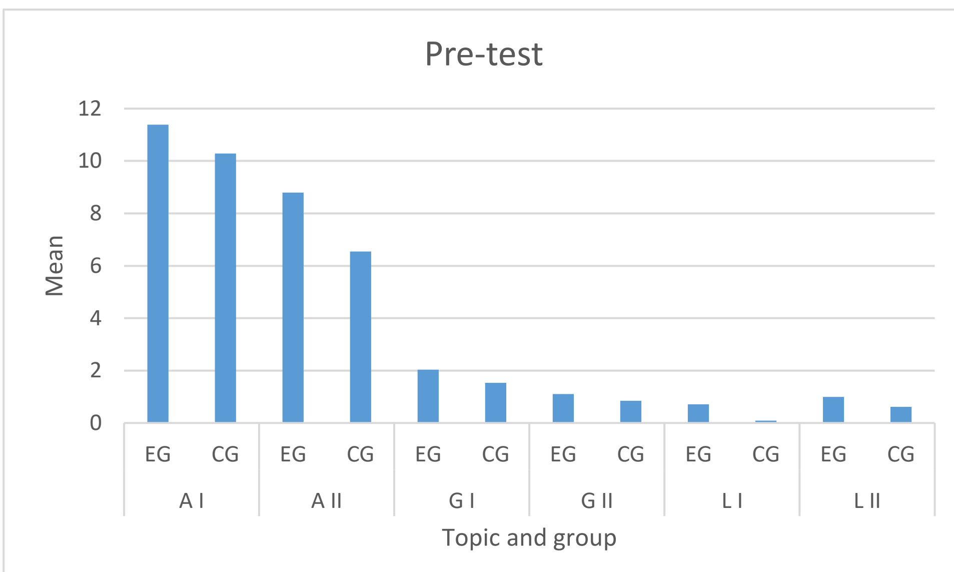
Descriptive statistics and the results of the t-test for the pre-test.

Test	Group	*M*	*SD*	*min*	*max*	*t*
A I	EG	3.34	1.46	0	5	3.27**
	CG	2.63	1.44	0	5	
An II	EG	2.94	1.40	0	5	3.11**
	CG	2.30	1.35	0	5	
G I	EG	1.43	.74	0	2	2.49*
	CG	1.16	.75	0	2	
G II	EG	.46	.50	0	1	.069
	CG	.46	.50	0	1	
L I	EG	.35	.55	0	2	-.76
	CG	.41	.56	0	2	
L II	EG	.62	.63	0	2	2.90**
	CG	.36	.61	0	2	

Note. A = arithmetic; G = geometry; L = logic; EG = experimental group; CG = control group. * $p < .05$; ** $p < 0.01$.

Figure 1

The means of the pre-test.



3.2. Post-test

In Table 2 we present the descriptive statistics for the post-test. In addition, we report the results of the ANCOVA test with one variable (the results of the initial tests), which was used to check for possible differences between the experimental and control groups about the initial test. For the research, we calculated the statistics for easier (I) and more complex problems (II). A graphic representation of the test achievements is in Figure 2.

From Table 2, we understand that the EG had higher achievements than the CG in solving both easier and more complex problems. The statistically higher achievements are present in the domain of geometry and logic.

Table 2

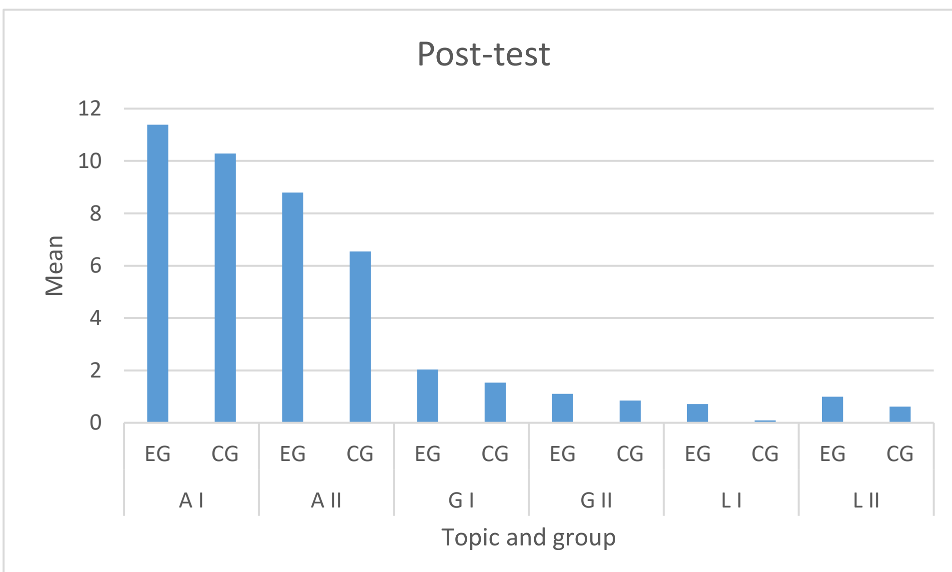
Descriptive statistics and results of the ANCOVA for the post-test.

Test	Group	M	SD	min	max	*F*
A I	EG	11.38	2.00	5	13	1.17
	CG	10.29	2.99	1	13	
A II	EG	8.79	4.88	0	17	1.50
	CG	6.54	4.86	0	16	
G I	EG	2.03	.96	0	3	6.02*
	CG	1.53	1.11	0	3	
G II	EG	1.10	.94	0	2	4.01*
	CG	.85	.96	0	2	
L I	EG	.71	.46	0	1	125.98***
	CG	.09	.29	0	1	
L II	EG	.99	.88	0	2	4.07*
	CG	.62	.82	0	2	

Note. A = arithmetic; G = geometry; L = logic; EG = experimental group; CG = control group. * $p < .05$; ** $p < .01$; *** $p < .001$.

Figure 2

The means in the post-test.



3.3. Comparison between the EG and CG

Students in both groups were good at solving simple arithmetic problems (for instance, "My mother is 31 years old. My daughter Anna is 20 years younger. How old is Ana?"), as these types of problems appear most frequently in our textbooks and workbooks, and are also the ones we teachers ask most often. These types of problems were solved correctly by 87.5% of EG students and 79.2% of CG students. Even simple problems in logic and geometry were solved more successfully by EG students. It was evident how narrow a range of problems was solved by CG students, as they rarely solved logic and geometry problems by measurement. In simple logic problems, pupils had to sort the elements of a given set according to two properties, using different tables (Carroll diagrams). 70.8% of pupils in EG were successful in these problems, while only 9% of pupils in CG were successful, even though such problems also appeared in the grade 3 prescribed mathematics textbook. In the EG, students solved these mathematical problems through three levels of presentation (that is, enactive, iconic, and symbolic). CG students solved the problem only in the workbook at the iconic or symbolic level. They did not do the concrete-experiential activity, which is, however, at the level of the first three grades, one of the obligatory steps towards the development of cognitive processes. At the same time, they did not integrate spreadsheets in a meaningful way in solving various problems, both mathematical and non-mathematical.

In simple geometry problems with measurement, 67.8% of EG pupils and 51.1% of CG pupils were successful. Although the problems were very simple, they were alien to the CG pupils, who themselves stated that they were different from the ones they solved at school. In our schools, pupils are used to solving problems in examinations that are very similar (or even identical) to those they solve in class. Since mathematics lessons do not develop different strategies for solving mathematical problems, students are then helpless in solving "different" problems.

Even with more challenging problems, the EG students were more successful. The average achievement in solving more difficult problems in arithmetic was 51.78% in EG and 38.50% in CG. In geometry problems, the average achievement was 55% in the EG and 42.70% in the CG. There was also a large difference in the knowledge of logic problems (EG achieved 49.43%, CG 31.11%). The more difficult problems included compound (guided and unguided) problems and:

- problems that do not have sufficient data to solve;
- problems with more data than needed for the solution;
- problems with multiple solutions;
- problems in which the given data are read from a spreadsheet.

In addition, the test asked students to solve the following problems:

- to formulate a meaningful problem for a given illustration and then solve it;
- to formulate a meaningful question to the text of the problem and then solve the problem;
- choose a mathematical problem that goes with the given calculation.

Of these problems, CG pupils in mathematics lessons solved mainly composite problems and fewer problems where they read the given data from a table and problems where they formulated a meaningful problem to a given illustration. In the EG, however, all of these problems were solved in the classroom. Interestingly, students from both groups performed the worst on the composite problems. In EG, 18.5% of students solved these problems correctly, while in CG 14.4% of students did so. Although these problems are often solved in mathematics lessons, only some students can decompose a compound problem into sub-problems on their own, without teacher guidance, asking intermediate questions that lead them to the solution.

In Table 3 the frequencies (in %) of how well students from EG and CG solved more challenging problems are presented.

Table 3

The frequencies of correctly solved problems in EG and CG.

Problems	EG	CG
Problems with more data than required	59.5	41.6
Problems with less data than required	82.0	63.3
Problems with multiple correct answers	20.8	10.0
Problems where data are presented in tables	80.5	71.1

Formulate a problem based on a picture	78.4	62.6
Formulate a question based on a text	54.3	37.4
Choose a mathematical problem that can be solved with a given operation	71.6	44.4

Table 3 shows that EG students outperformed CG students on all types of problems. Both did poorly on problems with multiple solutions (20.8% in EG and 10% in CG), as in most cases they settled for one solution, even though they were told in the task to write down all possible solutions. Most students believe that if you find one solution, there is no need to look for others because they have "solved" the problem correctly. The EG teachers also found that these were the problems that caused the most problems for the pupils. Only the ablest students searched for or found more solutions even if the teacher did not guide them during the problem-solving.

Pupils who were in the EG in third grade are now in fourth grade. In these classes, mathematics is still taught according to the concept we developed in our experiment. The teachers of these four fourth grades have been trained in the new concept of mathematics education through continuous training and study groups. On the final test for the fourth grade, 42.2% of the students solved these types of problems correctly, which confirms the findings of some contemporary research (Ali et al., 2010; Lessani et al., 2017) that the ability to encode the components of a problem and to plan the solution of different types of problems progresses with age and with appropriate teaching.

For problems with more data than needed for the solution, it was found that 59.5% of the EG students were able to read the problem text with comprehension and were able to find the data needed for the solution. In CG, 41.65% of students solved the problem correctly. The incorrect solutions were mainly because they used all the data, including unnecessary data, in the solution, as they always "had to" use all the data in the problems they encountered in class.

Students were very successful in solving problems that did not have enough data to solve, with 82% of EG students and 63.3% of CG students successfully solving this problem. Most students stated that they liked this type of problem the most, as they found it "very good" that they could determine the value of the missing data themselves. Teachers also confirmed that pupils were very motivated to solve these types of problems, but were surprised that pupils were able to find the missing data very quickly and then assign very meaningful values to it.

In addition to these problems, the favorite problem for students from both groups was to create a meaningful problem to go with a given illustration. 80.5% of the EG students and 62.6% of the CG students solved this problem correctly. We also analyzed how original the students were in formulating their problems. The EG pupils formulated a wide variety of problems:

- problems using multiplication, division, addition, and subtraction to solve them;
- problems in geometry;
- problems in the data processing.

4. DISCUSSION

In CG, students only designed problems that they solved using multiplication and addition. At the same time, the problems were constructed exclusively according to the model in the workbook. This points to the fact that our current mathematics teaching provides little opportunity for students to solve problems in their way and that creativity is not sufficiently encouraged. However, in the EG, the teachers developed flexible and creative thinking. The problems that the pupils constructed show that pupils at this age level can construct mathematical problems in a creative and meaningful way if properly encouraged by the teacher.

In the next problem, students were asked to add one or more meaningful questions to the text of the problem and then solve the problem. This problem was successfully solved by 54.3% of the EG pupils and only 37.4% of the CG pupils. The problem text was made "harder" by writing the given numerical data in words rather than numbers. Research (Moghdam et al., 2012) has shown that writing a quantity with a number or with a word has a great influence on the comprehension of the text. Therefore, the teacher needs to be aware of all the factors that influence the understanding of the problem and to guide the pupil skillfully in solving it. In the EG, teachers instructed students to underline information that is written in both number and word form.

Students in both groups did very well on problems in which they read the given data from a table. In the EG group, 80.5% of the students solved the problem correctly, and in the CG group, 71.1% of the students solved the problem correctly. In recent years, reading and writing in various forms of spreadsheets have been integrated more into classroom teaching in both mathematics and science (Isiksal & Askar, 2005). This is confirmed by the results obtained.

In the test, pupils were also given a problem where they had to find the problem that the given calculation solves among given problems. This was a twofold task: to understand the text and to have a good grasp of the basic operations of calculus. The same numbers "appeared" in all the given problems, so the students had to read all three problems very carefully and then decide on the correct solution. In most cases, CG students chose the first problem, as only 44.4% of the students solved the problem correctly. This confirms the findings of previous studies (Bernardo, 1999; Phonapichat et al., 2014) that students do not read the whole text, but in most cases rewrite the given data and perform the computational operation that is being discussed in the mathematics lesson at the time. The EG pupils solved the given problem very well, with 71.6% of the solutions being correct. In the EG mathematics lessons, we have been putting a lot of emphasis on the first stage of problem-solving: understanding the problem.

There are several possible limitations of our study. Firstly, the sample size is still small, so the results cannot be generalized to the whole population. Additionally, the study only focuses on students from one particular region. Future research could explore whether problem-based mathematics instruction is effective in other countries and with other curricula. This could help to determine whether the results of this study are generalizable to other contexts.

Secondly, the study only examines the impact of a specific teaching method on student performance in math, and it does not take into account other factors that may affect student performance, such as motivation (Gilbert et al., 2014), beliefs about mathematics (Mason, 2003), mathematics anxiety (Ashcraft & Ridley, 2005; Wu et al., 2012), and others.

Thirdly, the study only examines the short-term impact of the teaching method, and it is unclear whether the effects would persist over the long term. While this study showed positive results in the short term, it would be interesting to see if the effects of problem-based mathematics instruction persist over a longer period. Future research could follow up with the students from the experimental and control groups to see if the differences in achievement continue to hold in later grades or even in adulthood.

This study found that teacher training was an important factor in the success of problem-based mathematics instruction. Future research could explore different types of teacher training and their effectiveness in implementing problem-based mathematics instruction. This could help to identify the most effective ways to train teachers for this type of instruction.

While this study showed that problem-based mathematics instruction was effective in improving student achievement, it would be interesting to explore whether this type of instruction also increases student engagement and motivation. Future research could measure student engagement and motivation and compare them between the experimental and control groups. Moreover, problem-based mathematics instruction could potentially be enhanced through the use of technology (Kirabo et al., 2024; Jacinto & Carreira, 2017; Lee & Hollebrands, 2006; Weigand et al., 2024). Future research could explore how technology can be used to support problem-based mathematics instruction and whether this enhances student achievement and engagement.

5. CONCLUSION

The main purpose of our study was to test the hypothesis that EG students who received a new model of mathematics instruction, in which problem posing and problem-solving were the leading didactic-mathematical activity, would be more successful in solving all types of mathematical problems than CG students who received traditional mathematics instruction, in which the main emphasis is on training in arithmetic operations.

To check the validity of the hypothesis, a pedagogical experiment was developed. Students from the EG and CG took an initial (pre-)test to check for possible differences in the initial knowledge. Results have shown that students from EG had better achievements than students of CG in the majority of the mathematics domains. As a possible explanation for why this phenomenon occurred, teachers' experiences might be considered. For instance, the teachers from the EG were prepared for the experiment throughout the year through various forms of training (workshops, lectures, seminars, etc.) when they were teaching in the second grade. They were also given materials (method manual, task book, workbook, etc.) which they then used in the experiment when students were in the third grade. Although we agreed with the teachers not to introduce experimental factors in mathematics lessons in the second grade, it certainly changed their way of looking at mathematics teaching. They did not introduce new content into the classroom, but their concept of mathematics teaching changed. The new concept makes the problem the central content. In other words, mathematics is not taught just "for the numbers and the operations between them", but for the "problems". Thus, all fundamental mathematical concepts were built from problem situations that arose from the pupils' experience. They emphasized generalized knowledge and problem-based learning, in contrast to the prevailing concept of mathematics education in this country, which is based primarily on the transfer of skills and knowledge. Instruction based on such transfer, however, "implements" knowledge and techniques immediately after the introduction of new content, regardless of whether the mental connections have been made so that the learner understands the content and can apply it. This transfer can also include the transfer of rules, principles, and methods to similar tasks without the learners understanding and being able to apply them. The scope of this transfer is relatively narrow. For example, the technique of computational operations falls within this range.

After the experiment, which lasted 8 months (almost a school year), students from both groups were required to answer the final (post-)test. In both EG and CG, teachers taught mathematics according to the 2011 curriculum. However, the concept of teaching in the two groups was fundamentally different. In EG, the mathematical problem and the related problem-solving and problem-exploration methods took a leading and central place in all mathematical content. In CG, the teaching was more oriented toward mastering algorithms or teaching and learning specific recipes and calculation skills. Also, mathematical problems, which were mainly uniquely solvable and arithmetic in nature, were solved in the traditional way (Gestalt) according to the calculus, answer model; this lacked instruction in solving these problems at different levels of presentation and in a meaningfully guided analytical-synthetic process.

Based on all the results obtained and their analysis, we can conclude that the model of mathematics teaching implemented in the EG, in which problem posing and problem solving were the leading didactic-mathematical activity, is a success. This confirms our research hypothesis: EG will be more successful than

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CG in solving both simple and more complex problems in the context of arithmetic, geometry with measurement, and logic with sets.

Ethical Approval: Written informed consent was obtained from all participants' parents and all participants agreed to join the research. Researchers followed the European Code of Conduct for Research Integrity and the Helsinki Convention.

Conflict of Interest: The authors declare that there is no conflict of interest.

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