New Trends and Issues Proceedings on Humanities and Social Sciences

Volume 6, Issue 1 (2019) 205-212

www.prosoc.eu

Selected Paper of 11th World Conference on Educational Sciences (WCES-2019) 07-10 February 2019, Milano Novotel Milano Nord Ca' Granda Convention Center, Italy

Misconceptions in radical numbers in secondary school mathematics

Ayten Ozkan*, Department of Mathematics, Yildiz Technical University, 34210 Istanbul, Turkey

Suggested Citation:

Ozkan, A. (2019). Misconceptions in radical numbers in secondary school mathematics. *New Trends and Issues Proceedings on Humanities and Social Sciences.* [Online]. *6*(1), pp 205–212. Available from: www.prosoc.eu

Selection and peer review under responsibility of Prof. Dr. Jesus Garcia Laborda, University of Alcala, Spain.

Abstract

The aim of this study was to determine misconceptions of the radicals of the high school students that attend ninth class. The samples of study consist of the students of a Secondary School in Istanbul, Turkey. Some sample questions are asked to related students to understand the misconceptions. According to the result of the study, it is seen that the students have misconceptions about radicals. It is observed that the students have superficial information about deep-rooted numbers and memorise the definition of deep-rooted numbers and try to use them in their mistakes. Some solutions are recommended to those students to overcome such difficulties.

Keywords: Mathematics education, radicals, misconceptions.

* ADDRESS FOR CORRESPONDENCE: **Ayten Ozkan,** Department of Mathematics, Yildiz Technical University, Esenler, 34210 Istanbul, Turkey. *E-mail address*: aytenustozkan@gmail.com

1. Introduction

Mathematical concepts are concepts that are misunderstood because of the abstract structures within them. When learning these concepts, if there is no relational understanding which means knowing what to do, the students may have misconceptions or difficulties related to the concept (Skemp, 1978). Rather than developing a learning system on which a meaningful learning can take place, instead of creating a suitable environment for learning, it becomes difficult to comprehend the concepts, and therefore misconceptions may arise. Significant learning occurs when the learner establishes a relationship between the existing knowledge and the new knowledge (Ausubel, 1960) and is only formed by the connection of new knowledge with schemas in the mind of the learner. The foundation of the basic concepts on a solid structure will ensure the foundation of the concepts to be built on this structure.

There are several studies investigating the causes of misconceptions in mathematics learning and suggesting solutions to eliminate misconceptions. In order to overcome the misunderstandings in these studies, the following suggestions were briefly given (Aygor & Ozdag, 2012; Booth & Koedinger, 2008; Chang, 1995; Cutugno & Spagnolo, 2002; Kazemi & Ghoraishi, 2012; Lai & Wong, 2017; Luchins & Luchins, 1985; Mason, 1989; Mikkila-Erdmann, 2001; Oberdorf & Taylor-Cox, 1999; Ozkan & Ozkan, 2012a; 2012b; Vlassis, 2004).

- The concepts acquired in daily life experiences and at school should not contradict each other.
- Students should be encouraged to link the concepts in the previously acquired knowledge with new information.
- Sampling should be used frequently and examples that the students may encounter in their daily lives should be selected.
- Textbooks and teacher books focusing on the concept education should be prepared.
- Misconceptions learned by the students in the past must be corrected. New concepts should not be taught before correcting old misconceptions.
- Teachers should guide students at every stage of the concept teaching.
- The teacher should take note of the experiences of the teachers as feedback of possible misconceptions in the future.
- In order to determine the pedagogical reasons, teachers' opinions about the misconceptions of other mathematics subjects can be taken.

Radicals are one of the most fundamental issues in which students have difficulty in learning and have some conceptual misconceptions. Although radical numbers are used in many areas of mathematics, they are often defined as difficult, unnecessary, complex processes and concepts that are not related to everyday life by students (Senay, 2002). Although this subject has been used in many disciplines, the fact that it is not used or used in daily life may have been a reason for students to be negatively approached and not to become concrete in the eyes of students. In addition, since the rooted numbers are not independent of the other subjects in mathematics, the missing and incorrect information of the students on some other subjects may be difficult for students to understand this issue and to solve the problems easily. Some difficulties can be caused by the fact that radicals do not perceive and disseminate each rule to a natural, integer and rational number of radicals (Duatepe Paksu, 2008). To master some basic mathematical subjects, such as rational numbers, absolute value, exponential numbers, identities and factoring, will enable the student to learn the subject of deeper numbers more quickly, and hence the operational and conceptual misconceptions will be reduced as this subject is built on a solid foundation.

The basic problem of the mathematics course we encounter basic concepts are not to teach the students. Therefore, misconceptions caused by the lack of concept in primary education are transferred to secondary education. In this study, it is aimed to determine and correct the misconceptions of radicals for ninth grade students.

Mathematics education has a great place and importance in the education process. Because of its structure, it influences as a result of economic and social life as a result of mathematics, scientific studies, technology and methods. However, we still face many problems in mathematics education. As long as there are problems in learning basic concepts and misconceptions, learning new concepts becomes impossible and it becomes difficult to understand them. One of the most important concepts in mathematics is the rooted number; misconceptions about this subject can cause many problems to students on different topics.

Before examining students' mistakes in the questions asked, let us remind you the information that ninth grade students should have in terms of numbers:

- To be able to determine the root number of real numbers
- To be able to say that the rooted number is also an exponential number
- To be able to decide how an expression in the root will come out

2. Methodology

As the purpose of this study was to reveal a current situation without changing it, screening model among quantitative research models was utilised. The study group consisted of 90 students attending ninth grade at a public school in Istanbul in the academic year 2017–2018. The students were asked 12 open-ended questions under 7 topics. The questions are based on the level of knowledge and achievements that should be related to the rooted numbers.

3. Findings

In order to achieve the purpose of the study, we have created sub-problems and examined the questions given below:

- The value of the rooted number and its place in the number line
- Writing the rooted number as an exponential number
- Examination of $\sqrt{a^2}$ = a
- $\sqrt{a^2+b^2}=a+b$ and $\sqrt{a\mathrm{m}b}=\sqrt{a}\mathrm{m}\sqrt{b}$ misconception
- Adding and subtracting in rooted numbers
- Multiplication and division in rooted numbers
- The power of rooted term

Question 1. Show that $\sqrt{23}$ which is located between two integers?

Table 1. Question 1

Groups	True	False	Empty	Σ
Number of Students	72	6	12	90
%	80	7	13	100

According to the answers in this question, there are two cases as follows:

1—Errors in the number line.

2—Conceptual errors in expressing the interval.

Question 2. Show that $\sqrt{7}$ in the number line.

Table 2. Question 2

Groups	True	False	Empty	Σ
Number of Students	50	26	14	90
%	56	29	15	100

According to the answers in this question, there are two cases as follows:

1—Errors based on lack of the knowledge about expressing the concept of number line.

2—Errors in understanding the value of the rooted number and transferring it to the real axis.

Question 3. Convert $\sqrt{125},\sqrt[3]{7},\sqrt{32},\sqrt{16}$ rooted numbers to exponential numbers.

Table 3. Question 3

Groups	True	False	Empty	Σ
Number of Students	24	52	14	90
%	27	58	15	100

According to the answers in this question, there are three cases as follows:

1—Errors obtained as a result of writing the expression in the root in the form of exponential numbers without adding the root degree.

2—Conceptual errors made with the thought of removing the degree of root from the force of the number in the root.

3—Writing the degree of the root to the numerator of exponent and writing the power of the number within the root to the denominator.

Question 4. Find the solution of $\sqrt{(-2)^2}+\sqrt{3^2}+\sqrt{(-1)^2}$.

Table 4. Question 4

Groups	True	False	Empty	Σ
Number of Students	52	32	6	90
%	58	36	6	100

According to the answers in this question, there are three cases as follows:

1—Taking the square of the numbers within the root and applying the collection process in real numbers to these radical expressions.

2—Simplicity of the number in the root with the degree of the root and processing the number in the same way.

3—Considering that the degrees of the roots under common root are the same.

Question 5. Find the solution of $\sqrt{a^2}-\sqrt{b^2}+\sqrt{c^2}\ (a,b<0;c>0)$.

Table 5. Question 5

Groups	True	False	Empty	Σ
Number of Students	2	40	48	90
%	3	44	53	100

According to the answers in this question, there are two cases as follows:

1- Errors due to the use of variables instead of numbers.

2- Operational errors resulting from processing by assigning some values to variables.

Question 6. Find the solution of $\sqrt{6^2+8^2}$.

Table 6. Question 6

Groups	True	False	Empty	Σ
Number of Students	76	8	6	90
%	84	9	7	100

According to the answers in this question, there are two cases as follows:

1—Extract the sum of the squares of two numbers in the root as the sum of the numbers out of the root.

2—Errors in taking square of any given two integers.

Question 7. Find the solution of $\sqrt{20+16}$.

Table 7. Question 7

Groups	True	False	Empty	Σ
Number of Students	80	4	6	90
%	89	4	7	100

According to the answers in this question, there is one case as follows:

1—Process the square root of the sum of the two numbers as the sum of the square roots.

Question 8. Find the solution of $(\sqrt{2})^4+(\sqrt[6]{3})^6+(\sqrt{5})^2$.

Table 8. Question 8

Groups	True	False	Empty	Σ
Number of Students	38	12	40	90
%	42	13	45	100

According to the answers in this question, there is two cases as follows:

1—Try to simplify the power with the degree of the root.

2—Written error when solving the force with the degree of the root.

Question 9. Find the solution of $(\sqrt[6]{x})^6+(\sqrt[3]{y})^6+(\sqrt{z})^4 \quad (x,y,z>0)$.

Table 9. Question 9

Groups	True	False	Empty	Σ
Number of Students	16	14	60	90
%	18	16	66	100

According to the answers in this question, there is four cases as follows:

1—Errors due to the use of variables instead of numbers.

2—Multiplying the root power of the rooted number by this number.

3—Misconceptions due to superficial information, regardless of the power of the number and the degree of root.

4—Operational errors resulting from processing by assigning some values to variables.

Question 10. Find the solution of $\sqrt{24}-\sqrt{54}+\sqrt{150}$.

Table 10. Question 10

Groups	True	False	Empty	Σ
Number of Students	40	36	14	90
%	44	40	16	100

According to the answers in this question, there is one case as follows:

1—Application of addition, subtraction between roots as in real numbers

Question 11. Find the solution of $\sqrt{a^2x}+\sqrt{b^2x}+\sqrt{c^2x}\ (a,b,c>0)$.

Table 11. Question 11

Groups	True	False	Empty	Σ
Number of Students	10	18	62	90
%	12	20	68	100

According to the answers in this question, there is two cases as follows:

1—Errors due to the use of variables instead of numbers.

2—Falling into error on getting parenthesis.

Question 12. Find the solution of . $\frac{2\sqrt{3}\times 3\sqrt{3}}{15}=\frac{2\sqrt{7}}{5\sqrt{7}}$

Table 12. Question 12

Groups	True	False	Empty	Σ
Number of Students	24	40	26	90
%	27	44	29	100

According to the answers in this question, there is two cases as follows:

1- Processing only the numbers outside the root, without adding the inside of the root.

2- Adding the multiplier in front of the root to the root.

4. Conclusion

When the obtained results are examined in detail, it is seen that the students have superficial information about the rooted numbers in general and that a large part of them memorise the definition of rooted number and try to use them in a deficient way. Besides, some of the students are equipped with incomplete or inaccurate information in some mathematical subjects related to radical numbers, and open-ended questions instead of test techniques have brought more striking results to our students. Because some students were focused on solving the questions with the test technique, they thought about going to the conclusion immediately, so that they did not understand the question well, and without any good analysis, they have fallen into operational and conceptual misconceptions. If we want to talk about some of the basic reasons of these misconceptions, the following issues are particularly remarkable:

- Some basic mathematical issues related to radical numbers are caused by shortcomings.
- Students do not have full control over the characteristics of rooted expressions, so they are mistaken for taking any expression within the root out of the root.
- Some students cannot express the rooted numbers as exponential numbers because they are far from the gain that the rooted number is also an exponential number.

Because of the reasons arising from both the education system and the examination system, the students prefer the multiple-choice exam technique to the other exam types. However, it is very difficult to measure high-level behaviours of students with a multiple-choice exam. Considering all these disadvantages, the main reason for preferring the written exam in this study; the aim of this is to provide the students with a suitable technique to measure cognitive steps (knowledge, comprehension, practice, analysis, synthesis and evaluation). It is a very effective method to determine if students have an in-depth understanding of the subject, to determine which subjects are lacking and to identify their misinformation. It enables the students to freely organise and present what they knows. In addition, the absence of written examinations in the multiple-choice tests positively affects the reliability of the exam.

References

Aygor, N. & Ozdag, H. (2012). Misconceptions in linear algebra: the case of undergraduate students. *Procedia-Social and Be-havioral Sciences, 46*, 2989–2994.

Booth, J. L. & Koedinger, K. R. (2008, January). *Key misconceptions in algebraic problem solving*. Proceedings of the Cognitive Science Society (vol. 30, No. 30).

Chang, C. Y. (1995). A study of the way of students' constructing geometry concept and the evaluation of the effects of geometry teaching strategies with integrated cooperative learning. *Bulletin of Educational Psychology, 28*, 144–174.

Cutugno, P. & Spagnolo, F. (2002). Misconception about triangle in elementary school. Retrieved September, 23, 2017, from http://math.math.unipa.it/~grim/SiCutugnoSpa.PDF

Duatepe-Paksu, A. (2008). Uslu Sayılar Ve Koklu Sayilar Konusundaki Ogrenme Guclukleri. In H. Akkoc, M. F. Ozmantar & E. Bingolbali (Eds.), *Matematiksel Kavram Yanılgılari ve Cozum Onerileri*. Ankara, Turkey: Pegem Akademi Yayincilik.

Kazemi, F. & Ghoraishi, M. (2012). Comparison of problem-based learning approach and traditional teaching on attitude, misconceptions and mathematics performance of University Students. *Procedia-Social and Behavioral Sciences, 46*, 3852–3856.

Lai, M. Y. & Wong, J. P. (2017). Revisiting decimal misconceptions from a new perspective: the significance of whole number bias in the Chinese culture. *The Journal of Mathematical Behavior, 47*, 96–108.

Luchins, A. S. & Luchins, E. H. (1985). Student's misconceptions in geometric problem solving. *Gestalt Theory, 7*(2), 66–77.

Mason, M. M. (1989). *Geometric understanding and misconceptions among gifted fourth-eighth graders*. Paper presented at the Annual Meeting of the American Educational Research Association, SanFrancisco, CA, March 27–31.

Mikkila-Erdmann, M. (2001). Improving conceptual change concerning photosynthesis through text design. *Learning and Instruction, 11*(3), 241–257.

Oberdorf, C. D. & Taylor-Cox, J. (1999). Shape up! *Teaching Children Mathematics, 5*(6), 340.

Ozkan, E. M. & Ozkan, A. (2012a). Misconception in exponential numbers in IST and IIND level primary school mathematics. *Procedia-Social and Behavioral Sciences, 46*, 65–69.

Ozkan, A. & Ozkan, E. M. (2012b). Misconceptions and learning difficulties in radical numbers. *Procedia-Social and Behav-ioral Sciences, 46*, 462–467.

Skemp, R. (1978). Relation understanding and instrumental understanding. *Arithmetic Teacher, 26*(3), 9–15.

Senay, S. C. (2002). Uslu ve Koklu Sayilarin Ogretiminde Ogrencilerin Yaptiklari Hatalar ve Yanilgilari Uzerine Bir Arastırma (Yayinlanmamis Yuksek Lisans Tezi). Selcuk Universitesi Fen Bilimleri Enstitusu, Konya.

Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in 'negativity'. *Learning and instruction, 14*(5), 469–484.

Van der Geer, J., Hanraads, J. A. J. & Lupton R. A. (2000). The art of writing a scientific article. *Journal of Scientific Communications, 163*, 51–59.