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A practice for using Geogebra of pre-service mathematics teachers' mathematical thinking process

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Abstract

We aim to examine the pre-service mathematics teachers' mathematical problem-solving processes by using dynamic geometry software and to determine their evaluations based on experiences in this process. The design is document analysis, one of the qualitative research approaches. In the fall semester of the 2019–2020 academic year, a three-problem task was carried out in a classroom environment where everyone could use Geogebra individually. A total of 65 pre-service mathematics teachers enrolled in the course of educational technology. This task includes questions that they would use, their knowledge of basic geometric concepts to construct geometrical relations and evaluations related to this process. Besides the activity papers of the prospective teachers, Geogebra files were also examined. The result is pre-service mathematics teachers who are thought to have a certain level of mathematical background are found to have incorrect/incomplete information even in the most basic geometric concepts and difficulties with regard to generalisation.

Keywords: Dynamic geometry, Geogebra , instructional technologies, mathematical thinking, teacher education.

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1. Education, culture and intercultural encounters of individual potential

Visualisation and exploration of mathematical objects in multimedia environments can make learning concepts easier. In recent years, the use of software has increased, especially in geometry teaching (Lognoli, 2017). The dynamic geometry software gives the opportunity to learn concepts, their properties and mathematical relationships between concepts in interactive ways, allowing for dynamic manipulations on shapes, which allows inferences such as generalisation and seeing plenty of examples, deciding on specific situations. In other words, reasoning can be moved to another dimension, thanks to dynamic geometry software (Uygan & Bozkurt, 2019). The most common of this software is Geogebra . Geogebra is a dynamic geometry system that increases knowledge and skills in mathematics. The use of Geogebra facilitates completing and expanding teaching strategies based on manipulative activities. Markus Hohenwarter, the designer of Geogebra , named the programme 'dynamic mathematics software', as it contains computer algebra systems and dynamic geometry software features. Geogebra has input field, graphic window, spreadsheet window and algebra window interfaces. By transferring mathematical concepts, symbols, graphics and their values to the table, it provides fast transitions between different representations. It differs from other computer algebra systems and dynamic geometry software with these properties (Aktumen, Yildiz, Horzum & Ceylan, 2011).

The Geogebra software was first introduced in 2002 as part of a master's thesis prepared by Markus Hohenwarter. Later, Hohenwarter developed the Geogebra software as part of his doctoral thesis in mathematics education. Markus Hohenwarter continued his studies at Florida Atlantic University after 2006. Within the scope of a project carried out in the field of teacher education, he worked to ensure that Geogebra was used more effectively in mathematics learning environments. On the other hand, internet environments named Geogebra forum and Geogebra wiki were created in 2005. Afterwards, the International Geogebra Institute was established in 2007. The software has been translated into 70 languages. The Geogebra website is visited by millions of people from hundreds of countries annually. A total 140 national Geogebra institutes have been established in different countries. In many countries, applications related to Geogebra are included in textbooks (Hohenwarter & Hohenwarter, 2011, as cited in Simsek & Yasar, 2019). One of the features of Geogebra that makes it spread so fast worldwide is that it is free with all its components (software, help, applications, etc.) and even on mobile applications. Geogebra can work on many operating systems. The Geogebra interface generally includes menus, toolbars, an algebra window, a graphic window, a spreadsheet window and an input bar. These interfaces can be customised according to the users' request. In addition, Geogebra is constantly developed by software designers and new versions are available (Dogan & Lavicza; as cited in Simsek & Yasar, 2019).

Karaarslan, Boz and Yildirim (2013) compared a number of technology applications used in mathematics and geometry education. Among all the software in the study in the field of mathematics education, in Turkey Geogebra was found to be the most common software. The reason for the widespread use of this software in Turkey is the presence of the Turkish version and the presence of its institute in two big cities. It is observed that the use of Geogebra has increased with educational studies and some scientific meetings organised by the institutes.

On the other hand, researches around the world and Turkey emphasised that Geogebra 's use is becoming widespread with its features, such as visuality, concretisation, easy accessibility, attractive/easy menu, multiple representations and transitions between them, being frequently updated (Budinski, Lavicza & Fenyvesi, 2018; Hohenwarter & Hohenwarter, 2011; Hohenwarter & Lavicza, 2007; Kepceoglu & Yavuz, 2017; Ozgen, Apari & Zengin, 2019; Rohaeti & Bernard, 2018; Simsek & Yasar, 2019; Tatar, Akkaya & Kagizmanli, 2011; Zengin & Tatar, 2014). The common suggestion of the researches is that the dynamic geometry software should be included in mathematics learning and teaching environments. Researches have examined how students' reasoning is shaped in environments where dynamic geometry software is used and they recommend using the software for reasoning, thinking deeply, problem-solving, creativity and analysis, instead of it being used simply as a presentation and visualisation tool. In this context, the key point is to strengthen the interaction between the software and the user (Kukey, Gunes & Genc, 2019; Sengun & Kabaca, 2016).

As mentioned earlier, although the number of researches related to Geogebra has increased in recent years, researches related to geometric constructions are scarce (Alwahaibi, Al-Hadabi & Al-Kharousi, 2020; Sengun & Kabaca, 2016). For this reason, it is considered important to examine the process of constructing geometric objects using Geogebra . In this context, this study has two main objectives:

a) To examine pre-service mathematics teachers' processes of making a mathematical construction using Geogebra .
b) To examine the evaluations made by pre-service mathematics teachers based on their experiences in this process.

2. Method

The design of the research is document analysis, which is one of the qualitative research methods. Document analysis involves the review of written materials that contain information about the phenomenon or facts intended to be investigated. Documents are important sources of information that should be used effectively in a qualitative research. In such researches, the researcher can obtain the data he/she needs without observing and interviewing (Yildirim & Simsek, 2011).

The study group consisted of totally 65 pre-service mathematics teachers in 2 groups of 35 and 30 people. These individuals were pre-service mathematics teachers who registered for the 'instructional technologies' course in the fall semester of the 2019–2020 academic year. Within the scope of the course, tasks involving different mathematical concepts were given to the pre-service teachers by teaching Geogebra 2 hours a week during the fall semester. The data used in the research were obtained from the midterm exam of the 'instructional technologies' course. The exam was held in computer labs where courses were held during the semester. Similar to the tasks shown throughout the semester, questions were asked to the pre-service teachers on the basis of inductive and deductive logic from the basic concepts of triangle, quadrilateral and Euclidean geometry. These questions were taken from the tasks exhibited by Baki (2019, p. 477). During the exam, pre-service teachers were asked to answer the questions using Geogebra . In this process, we tried to determine how they think and where they have difficulties or mistakes, by examining their answer papers in writing, as well as the files with the extension of Geogebra (.ggb).

The concept of 'quadrilateral' is included in the fifth-grade mathematics programme, but the concept of 'regular polygon' is in the seventh-grade curriculum. Similarly, the subject of 'angles in polygons' is one of the seventh-grade topics. The subject of 'triangle' exists from the first stages of primary education to the eighth grade, and the topic of 'bisector' is also covered in the eighth grade. In this study, we aimed to realise pre-service teachers' construction as a result of a mathematical reasoning process that includes many concepts, such as triangles, diagonals, types of quadrilaterals, bisectors, centroid of a triangle and relationships between them. In this regard, we note the following:

a) In the first question, first, they were asked to draw a square and connect the corners of the square with any P point placed in the square. The features of the new quadrilateral formed by connecting the centroids of the four triangles formed in the square. Then, we asked them to draw the properties of the second quadrilateral by following the same steps inside this new quadrilateral.
b) In the second question, first, they were asked to draw a random quadrilateral. Similar to the first question, the properties of the quadrilateral that were formed as a result of connecting the centroids of the triangles formed after connecting any P point taken in the quadrilateral to the corners of the quadrilateral was asked. Here, we also examined whether students drew with irregular quadrilaterals and looked at the situations including irregular quadrilaterals.

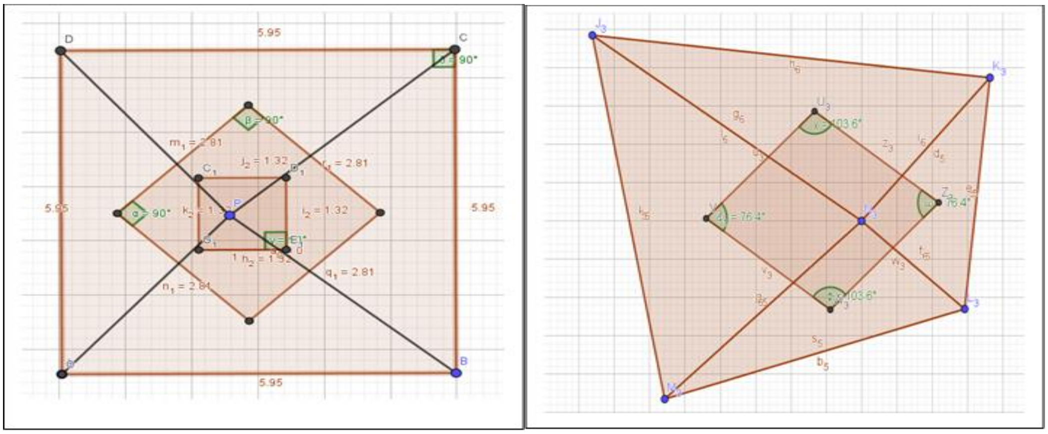

Figure 1. Drawings of question 1 and question 2

As seen from the first question, it was asked whether there is an inductive reasoning process (from a special case (square) to the general (quadrilaterals)). In the second question, it was asked whether there is a deductive reasoning process (starting from the quadrilateral to the square, rectangle. etc.). The drawings of the questions are shown in Figure 1. The data were analysed by content and descriptive analysis methods. Direct citations were made in the sections where pre-service teachers evaluated the effectiveness and the use of Geogebra in general. While the descriptive analysis method is mainly used in direct citations, content analysis is used in analysing their drawings and their papers. For the reliability study, the research authors examined each other's exam papers and agreed on the analysis.

3. Results

3.1. Examining reasoning processes

In this section, the reasoning processes of pre-service teachers are specified.

Table 1. General situation

		Incorrect answers											
Correct answers		**Considering that the triangle's centroid is the cut-off point of the bisectors**		**Terminological errors**		**Mistakes arising from misunderstanding the question**		**Generalisation cases**		**Reaching a conclusion without looking at the angles of the shape**		**Other answers**	
Frequency (person)	Percent (%)	Fre.	Per.	Fre.	Per.	Fre.	Per.	Fre.	Per.	Fre.	Per.	Fr.	Per.
23	35%	2	3%	8	12%	2	3%	39	60%	14	22%	17	26%

When examining Table 1, it can be seen that 23 out of 65 pre-service teachers (35%) answered the questions correctly and all the remaining pre-service teachers gave incorrect answers. Those who gave an incorrect answers were 39 (60%) people who made generalisations, 2 people (3%) who considered the centroid of the triangle as the cut-off point of the inner bisectors, 8 people (12%) who made terminological errors, 2 people (3%) who made mistakes due to misunderstanding the question, 14 people (22%) who reached a conclusion without looking at the angles of the shape and, finally, 17 people (26%) who gave different answers. Here, we see pre-service teachers who make more than one type of error and the percentage exceed 100.

3.1.1. Those who conclude the reasoning process correctly

In this section, teachers who reason and answer questions are correctly examined.

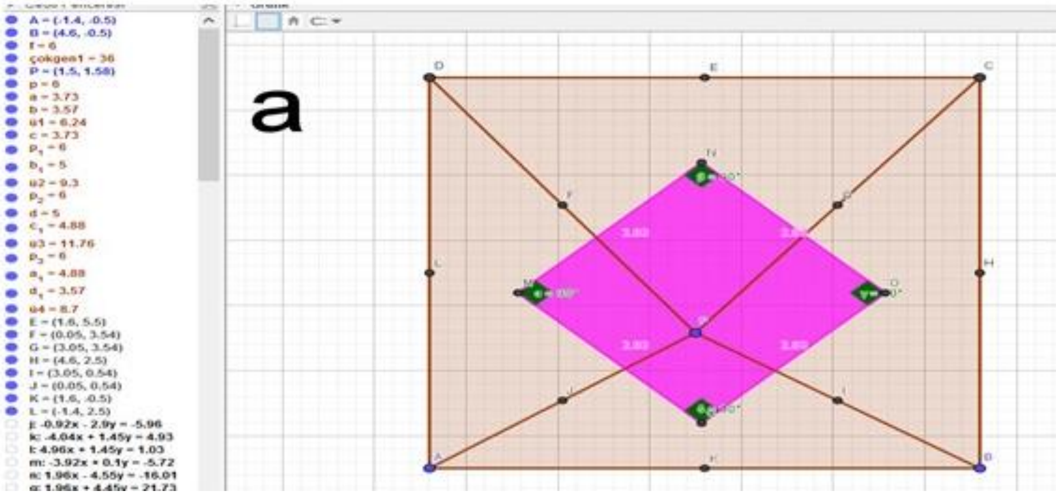

Figure 2. Geogebra file of P1 with the correct answer

In Figure 2, we see a screenshot of the Geogebra file of P1, who answered the first part of the first problem correctly. The process where P1 gains a square within the ABCD square from the instructions given is seen from both the algebra window and graphics window of the Geogebra . P1 made the drawing correctly with the instructions using Geogebra , and after proving that the innermost shape is a square, he transferred what he did in the process to his paper.

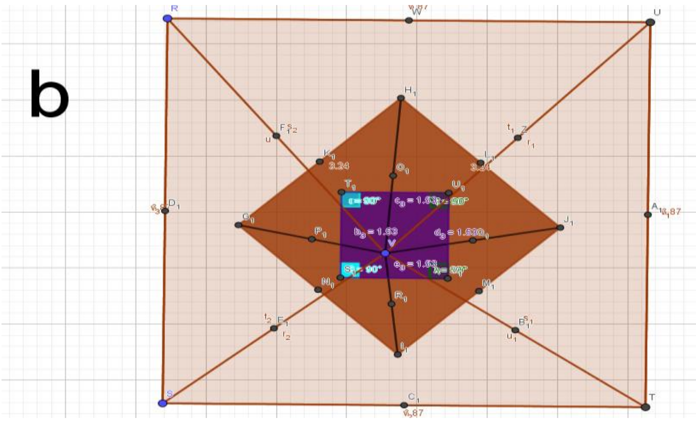

Figure 3. Geogebra file of P1 responding to the first question

In Figure 3, we see a screenshot of the Geogebra file of P1, who answered the question correctly. It can be seen from the Geogebra file of P1 that a square is formed again in the second frame with the instructions. P1 made the drawing correctly with the instructions by using Geogebra . After he proved that the innermost shape is a square, he transferred what he did in the process to his paper.

3.1.2. Incorrect answers

In this section, incorrect answers are categorised and analysed.

3.1.2.1. Considering that the centroid of a non-equilateral triangle is the cut-off point of the bisectors

In this section, the responses that are considered to be the cut-off point of the inner bisectors of the centroid of the non-equilateral triangle are included.

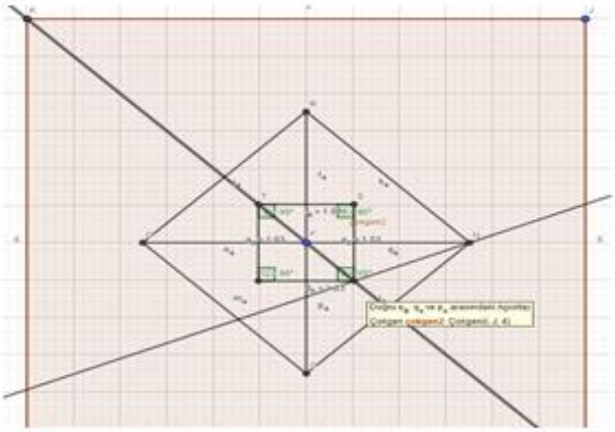

Figure 4. Geogebra file of P5 responding to the first question incorrectly

In Figure 4, we see a screenshot of the Geogebra file of P5, who answered the first part incorrectly. P5 initially made the corner points of the quadrilateral, assuming that the centroids of the triangles were the cut-off points of the inner bisectors. As a result, forming the square does not change the fact that the pre-service teacher made a logical mistake. P5 stated in the answer sheet that the centroid of a non-equilateral triangle is the cut-off point of its bisectors.

3.1.2.2. Terminological errors

In this section, answers are given where mathematical terminology is used incorrectly.

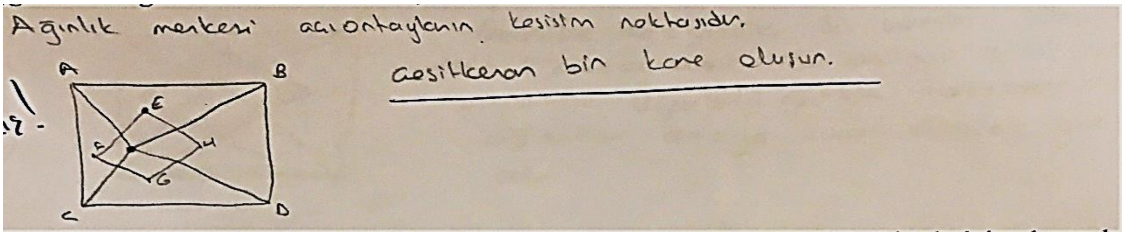

Figure 5. The answer sheet of P17, who responded incorrectly to the first question

P17 stated in the answer sheet that the shape formed is a 'scalene square'. This shows that the pre-service teacher does not have a good grasp of mathematical terminology. He also noted the centroid of the triangle as the cut-off point of the bisectors.

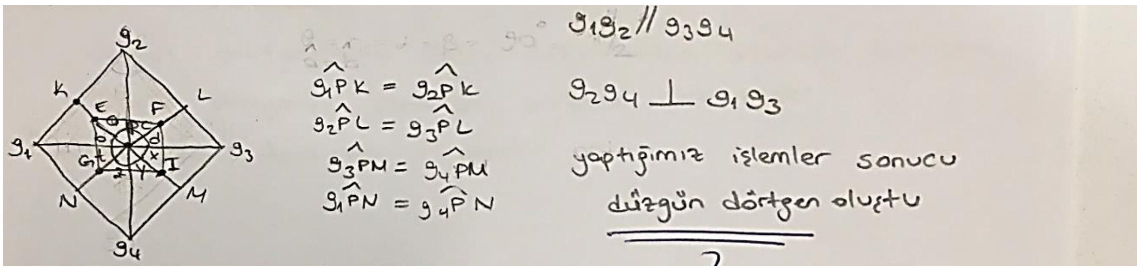

Figure 6. The answer sheet of P24, who responded incorrectly to the first question

Similar to the previous example, P24 stated in the answer sheet that the shape formed is a regular quadrilateral (square or rhombus?). This shows that P24 has problems in using mathematical terminology.

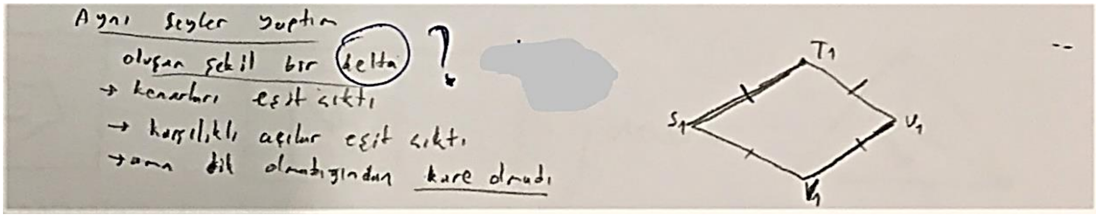

Figure 7. The answer sheet of P35, who responded incorrectly to the first question

Here, we see that P35 misreads 'rhombus' with deltoid and expresses it as 'delta'. This shows that P35 does not have enough knowledge of mathematical terminology. Figures 5–7 show some sort of problem with concept formation and identification.

3.1.2.3. Mistakes arising from misunderstanding the question

In this section, errors arising from misunderstanding the question incorrectly are included and factors causing this situation are investigated.

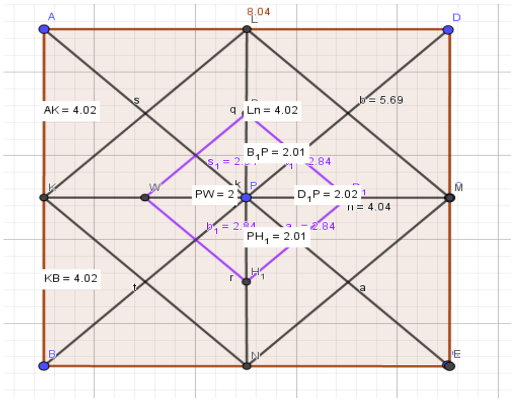

Figure 8. The Geogebra file of P16 who responded incorrectly to the first question

Here, P16 connected the mid-points of the edges of the square, and again obtained a square inside (Figure 8). P6 explained the process by connecting the mid-points of the edges of the square to obtain a square inside.

3.1.2.4. Generalisation cases

In this section, what pre-service teachers did during their generalisation process is examined. It was observed that the vast majority of pre-service teachers tried to reach a general conclusion through only a case. They did not question whether the result they achieved was also valid for other situations.

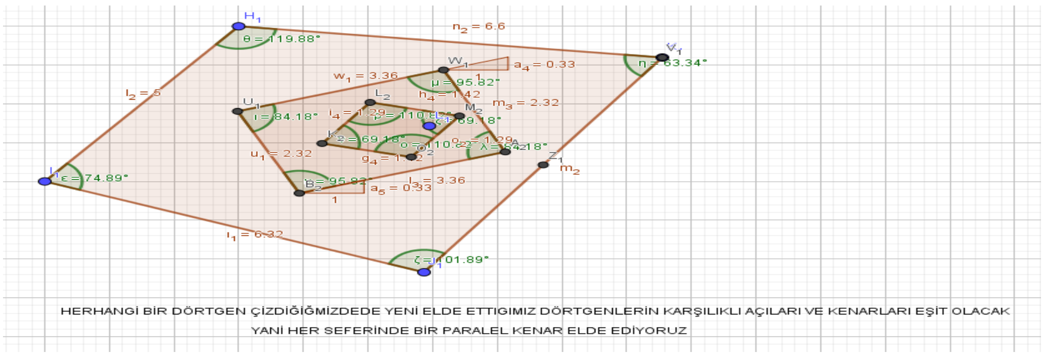

Figure 9. Geogebra file of P57

As seen from Figure 9, P57 tried to reach a general conclusion based on only a quadrilateral.

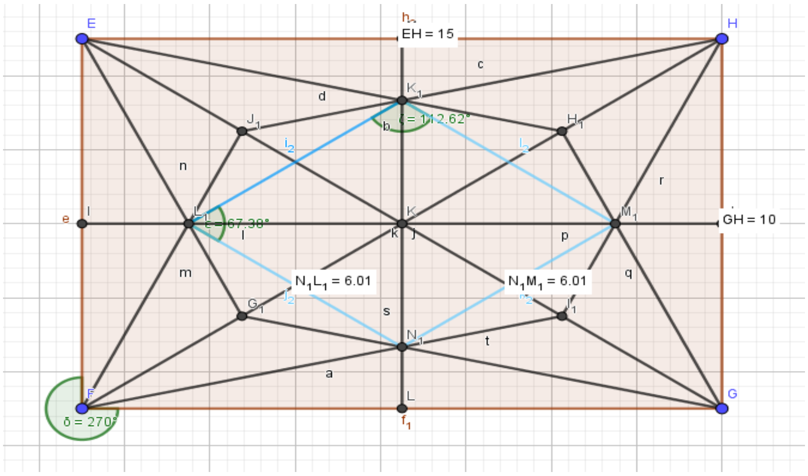

Figure 10. Geogebra file of P48

We can see from Figure 10 that P48 drew only a rectangle and tried to determine a general conclusion based on this drawing. The pre-service teacher identified this as parallelogram. P48 tried to reach a general conclusion based on the rectangle.

3.1.2.5. Reaching a conclusion without looking at the angles of the shape

In this section, the answers of the pre-service teachers' which indicate that 'this shape is a square or rhombus' without checking the internal angles after looking at the side lengths of the shapes created in the squares and seeing that they are equal are included.

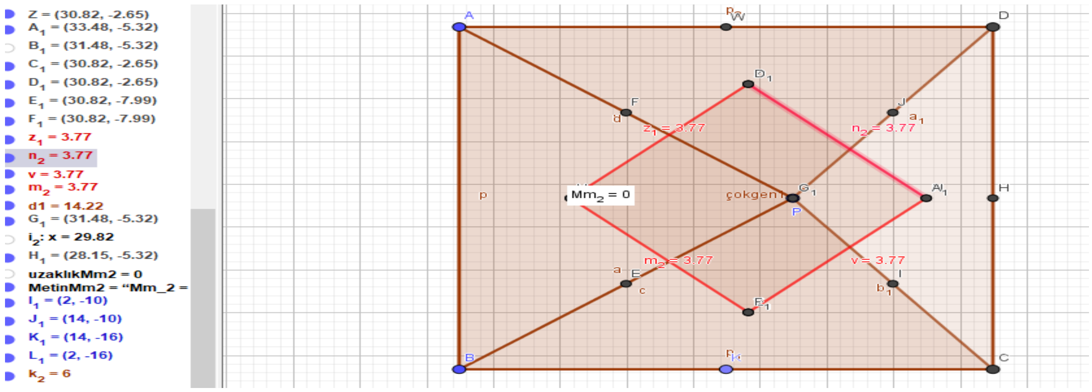

Figure 11. Geogebra file of P29

From Figure 11, it was found that P29 saw that four lengths of the sides of the shape were equal and directly concluded that the shape was a square without checking the angles. Thirteen of the pre-service teachers found that the shape was a square, regardless of the internal angles. P29 saw that the lengths of the shape was equal on all four sides and concluded that the shape was a square without checking the angles. There is no explanation of the angles in the answer sheet of P29.

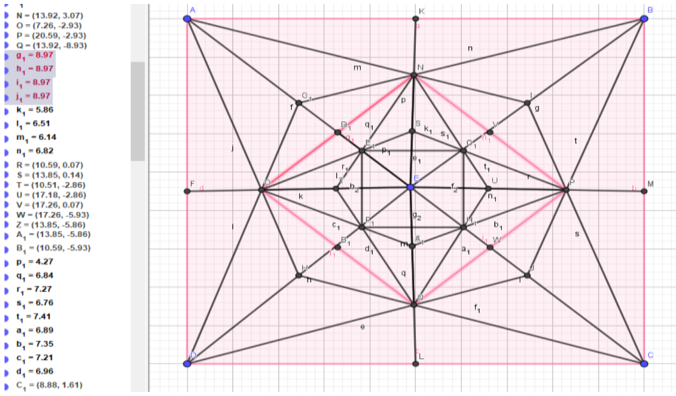

Figure 12. The Geogebra file of P10

From Figure 12, it was found that P10 saw that all the four sides of the shape were equal and concluded that the shape was a rhombus without checking the angles. Although this answer is not wrong, it can be stated that P10 cannot see the shape as a square since he does not control the angles. P10 did not examine the angles after all the edge lengths had an equal quadrilateral; therefore, the inner quadrilateral is also a rhombus. Although this is not a wrong answer, it does not change the fact that P10 did not examine all the things in the process.

3.1.2.6. Other answers

This section contains incorrect answers that were not found in previous categories.

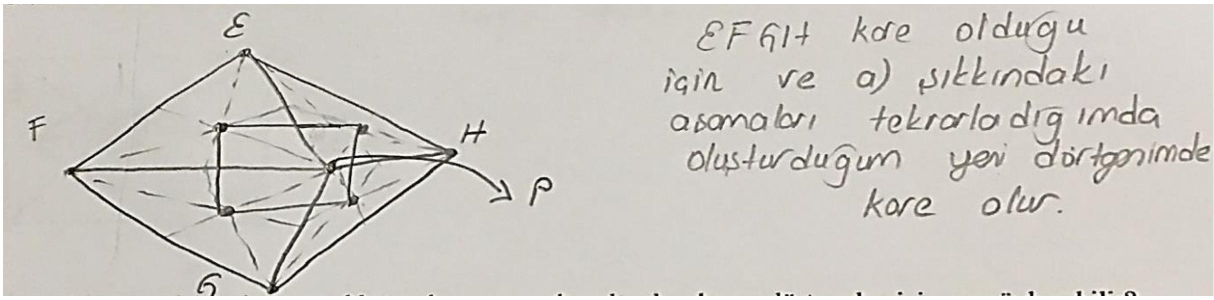

Figure 13. Answer sheet of P11

Figure 13 shows the answer of P11. Based on the conclusion that P11 did not make any drawing using the Geogebra software and that the shape formed in the square is again a square, proves that he thinks of a square instead of any quadrilateral and that a square will be formed in within it.

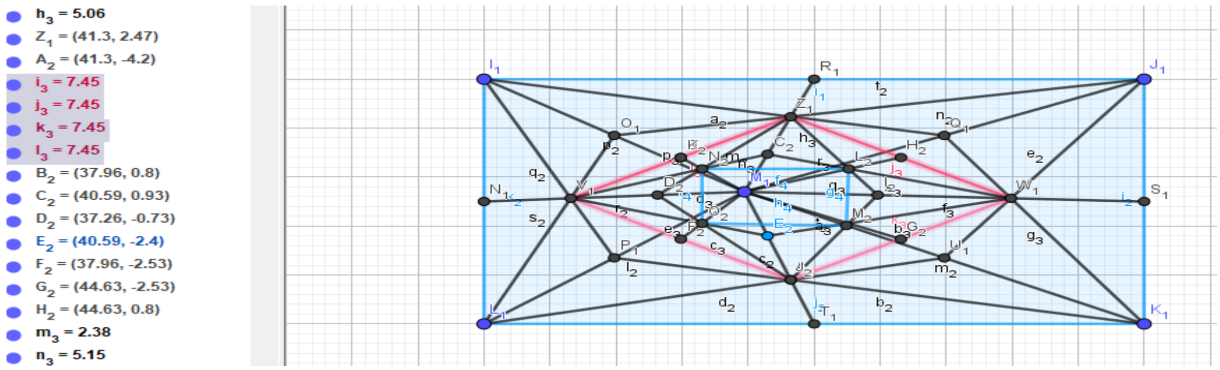

Figure 14. The Geogebra file of P42

When examining the drawing of P42 (Figure 14), we see that the shape formed in the middle is a rhombus. However, despite this result, he stated that the shape formed is parallelogram. When examining the previous drawings of P42 and the answer sheet, we see that the shape formed in the square says that it is a rhombus resembling a square. Based on this, he believed to have reached the conclusion that it is a parallelogram resembling a rectangle in the shape of the rectangle. In the answer sheet, we see that based on the shape drawn in the parallelogram, he also assumes that the innermost shape will be rectangular and considers that not all the edges but opposite sides will be equal.

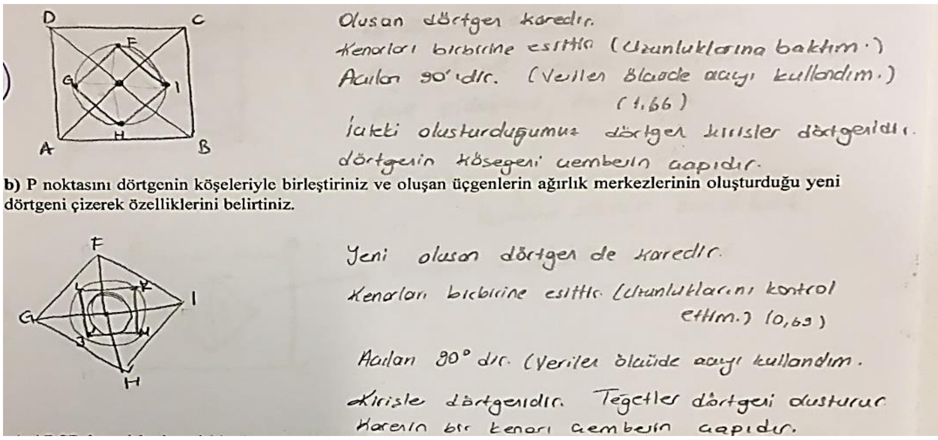

Figure 15. Answer sheet of P13

In Figure 15, although P13 says that the result is a square, it can be seen that he tried to connect the event to the square of tangents, whereas the results he finds are not related to the situation.

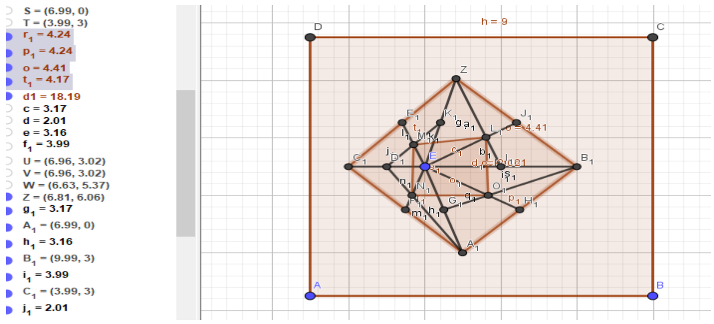

Figure 16. The Geogebra file of P14

In Figure 16, P14 stated that even though the shape formed is neither in the middle nor in the innermost is a square, these quadrilaterals are squares due to the fact that lengths of the sides of the quadrilateral are close to each other. Instead of rechecking the drawings he made, he assumes that it is square because it seems to be square. Although P14 did not obtain a square, he assumes that these shapes are similar to the outermost square.

3.2. To examine the evaluations they made based on their experience in this process

We aimed to examine the evaluations of pre-service teachers with regard to their contributions of such tasks to the students and the teacher in the teaching process. When examining the responses, it was determined that in addition to those evaluating the context of the task in the study, there were also those who expressed their opinions in terms of integrating Geogebra or similar dynamic geometry software into mathematics lessons.

3.2.1. Evaluations of the context of the task

It has been determined that pre-service teachers who evaluated the context of the task (square, rectangle, parallelogram, rhombus, triangle, etc.) emphasised on 'mathematical sense-making', which basically expresses the relationship between different concepts.

P5: 'If the shapes have the same features, the concept of similarity can be explained.'

P57: 'It can be explained that the centroid is the cut-off point of the triangle's edge.'

P38: 'With this task, it is easy to understand parallelism and steepness.'

P25: 'It is provided to see how the square is formed. It is possible to see that a square is formed from a square and a parallelogram from any quadrilateral. '

P4: 'He realises that the square is also a rhombus.'

P38: 'He sees that the ratio between the squares formed is 1/2'

P20: 'Since the nested shapes are drawn, it gives information about the fractal and makes it easier to understand. It is useful in learning the concept of parallelism. '

In the evaluations of pre-service teachers, it was emphasised that the relationships between similar tasks and different mathematical concepts can be easily understood.

3.2.2. Evaluations of the general context

The answers of the pre-service teachers, who addressed the use of Geogebra and technology in mathematics courses and not in the context of the tasks, were collected under two headings: 'mathematical contributions' and 'contributions to the general teaching process'.

3.2.2.1. Mathematical contributions

From the answers of the pre-service teachers, it was observed that the mathematical contributions come to the fore. The first of these states that 'thanks to Geogebra and similar applications, the visuality in the lesson increases', i.e., the use of formal representation increases, and thus it becomes easier to learn the concept. Sample answers are as follows:

P45: 'It helps to see and comprehend how a shape is formed.'

P63: 'Visualisation ensures permanent learning.'

Although P45 and P63's answers are not very detailed, they emphasise on the importance of using formal representations from different representations. This is assumed to be the emphasise of the importance of 'using different representations' in mathematics teaching.

Another pre-service teacher gave the following answer, focusing on the stages and process of the task:

P11: 'In this way, complete understanding is provided with induction.'

Combining this task, which is designed to go from a piece to a whole and from 'pattern to general', P11 stated that full understanding will be realised with the tasks is applied in this way.

Another pre-service teacher who assumed that Geogebra can be used to make 'mathematical proof' gave the following answer:

P27: 'Geogebra can be used for mathematical proof.'

Here, it can be seen that P27 also thinks that the mathematical sense-making process can be realised with Geogebra and similar dynamic geometry software.

3.2.2.2. Contributions to the general teaching process

It is also seen that there are pre-service teachers who emphasise on the contribution of the tasks to be carried out by using Geogebra and similar dynamic geometry software in the general teaching process. In this regard, the answers of the pre-service teachers who stated that 'permanent/full learning' will be realised are as follows:

P34: 'It helps to see and understand how a shape is formed and its features. Permanent learning is provided. '

P23: 'Permanent learning is provided with visuals.'

P41: 'In this way, full understanding is provided with induction. '

P53: 'Understanding what you read and putting it into practice is provided. Permanent learning is provided because it is learned by doing. '

P35: 'It serves to embody abstract concepts.'

It can be seen that pre-service teachers emphasise that permanent learning will be gained with such tasks.

In addition, it has been determined that there are pre-service teachers who think that their learning will be realised by making students wonder about this method. The answers given in this context are as follows:

P14: 'Learning is done by doing-living-testing-trying.'

P13: 'It is an interesting learning without boring, like a game.'

P17: 'The student learns by himself, not through the teacher.'

In the answers of P13, P14 and P17, there is an emphasis on the learning by discovering, as the student perceives the process as a game and has an element of curiosity.

Another important point is that the information learned theoretically can be used practically by using Geogebra .

P67: 'What is learned in theory is seen in practice.'

From the response of P67, it is understood that mathematical concepts can actually be applied in practice. This is important for changing the understanding that mathematics is actually 'a pile of abstract objects independent from the real world'.

Finally, there is an answer of a pre-service teacher who emphasises on the necessity of capturing the information and technology era in mathematics teaching:

P2: 'It allows to train students according to the needs of the age.'

From the answer of P2, we understand that it is possible to teach students in mathematics teaching as required by the age of information and technology with Geogebra and similar software.

4. Discussion, conclusion and recommendations

In this study, we aimed to create a geometric structure and to examine the reasoning of pre-service mathematics teachers by using Geogebra . In this construction process, as well as those who followed all the steps and ended the process without any errors, it was determined that many pre-service teachers had errors and difficulties on different issues. Pre-service teachers who made correct geometric constructions stated that the process was more interesting and permanent learning was realised because the result was achieved by interacting, experimenting and testing with Geogebra in such construction tasks. This overlaps with the conclusion of Budinski et al. (2018) that Geogebra creates opportunities for the evaluation of the mathematical assumptions and results. These results support the situation mentioned by Sengun and Kabaca (2016). In this context, they propose to use software to support skills, such as reasoning, in-depth thinking, problem-solving, creativity, analysis and evaluation, rather than just being used as a presentation and visualisation tool. In this context, the key point is to strengthen the interaction between the software and the user. The efficiency of this interaction also ensures that learning is so efficient (Sengun & Kabaca, 2016).

The issue of identifying patterns and making a general conclusion from the first levels of primary education is included in the mathematics curriculum. In this task, we expected pre-service teachers to reach a generalisation based on special situations. However, we observed that a vast majority of the pre-service teachers had drawn only a figure and reached a general opinion with it. It was determined that they did not question the situation 'Is the situation found for this figure valid for other shapes?' This reveals the deficiencies in the inductive reasoning process. Similarly, in the drawings containing any quadrilateral, they wrote the results they found without trying the special quadrilateral. It was observed that they generalise without examining whether the results they found were also valid for other quadrilaterals. This shows that there are deficiencies in the deductive reasoning process, as well as in the inductive reasoning process. These results are similar to the results of Stylianides, Stylianides and Philippou (2007) as 'pre-service teachers have difficulties in inductive thinking processes'.

Another remarkable result found in the study was that certain pre-service teachers had misconceptions even in the basic concepts included in the mathematics curriculum from the first levels of primary education, such as triangle and quadrilateral. When the steps of the operation in the algebra window were examined, it was seen that there were pre-service teachers who assumed that the centroid of the triangle was the cut-off point of the inner bisectors of the triangle. This task was designed by considering the levels of primary/secondary school students. However, if it is revealed that there are such misconceptions/difficulties, even when they are applied to pre-service mathematics teachers who are believed to have a high level of mathematical understanding, then this matter should be considered seriously. It is quite thought-provoking to see that such deficiencies exist even in the most basic subjects in this group, who will actively teach mathematics in about 3 years. After 12 years of primary school, middle school and high school education, they studied mathematics at the university for 2 years. But they still have difficulties on one of the most basic subjects, like regular polygons, triangles, etc. Therefore, we should think about our education system. On the other hand, we determined that some pre-service teachers did not have a good grasp of mathematical terminology, did not use the mathematical language correctly, did not fully trust themselves in this regard, avoided describing them clearly although the features of the geometric shapes obtained at the end of the process were clear and used more general expressions about the shape. This shows that the current situation of pre-service teachers is very thought-provoking.

On the other hand, it is believed that the use of Geogebra in a task, including different mathematical concepts, is considered to be important for the students. Since the individual interacts directly with the content, the current status of the individual (readiness, knowledge, misconceptions, difficulties, etc.) can be easily determined. This shows us that a qualified measurement and assessment processes can be realised with Geogebra . These findings are similar to the conclusion of Albano and Iacono (2019) that formative assessment can contribute to evaluating open-ended problems, in exams conducted using Geogebra .

This study shows that in the tasks carried out using Geogebra or similar dynamic geometry software, an effective and permanent learning environment can be provided by revealing how geometric shapes are constructed, what are their features and similarities and differences with other shapes. All of these affect the students' mathematical reasoning and inquiry process positively. There are different opinions in the related literature. While Uygan and Bozkurt (2019) stated that dynamic geometry software supports inductive and deductive reasoning processes, Albaladejo, Garcia and Codina (2015), due to the limited social interaction, stated that it reduces the process of reasoning, questioning and proof. It is seen that social interaction between individuals has a share in the formation of these differences. In this context, it is important to design a teaching environment in which students can communicate with both the teacher and each other, in order to use dynamic geometry software effectively.

In this study, pre-service teachers' evaluations regarding the use of technology and Geogebra in mathematics courses were also examined. The vast majority of pre-service teachers stated that it may be easier to establish a relationship between different mathematical concepts and representations by Geogebra . This indicates the contribution of the use of Geogebra to mathematical sense-making and the transition process between different representations. Likewise, using Geogebra and technology, students can be trained in accordance with the requirements of the era, connecting between theory and practice, abstract concepts can be concreted, curiosity and non-boring learning can be realised, discovery processes can be experienced and permanent learning can be realised. These results show similarities with the results obtained in their studies (Baltaci & Baki, 2017; Budinski et al., 2018; Catlioglu, 2010; Hangul & Uzel, 2010; Hohenwarter & Hohenwarter, 2011; Hohenwarter & Lavicza, 2007; Rohaeti & Bernard, 2018; Sengun & Kabaca, 2016; Tatar et al., 2011; Uygan & Bozkurt, 2019).

Finally, in future studies it would be beneficial to investigate the effects of using technology in mathematical lessons with different study groups, different mathematical concepts and different methods that will enable obtaining more in-depth data, such as clinical interviews.

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