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Application of conditional value at risk for credit risk optimization

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Abstract

The article is dedicated to the optimization of credit risk through the application of Conditional Value at Risk (CVaR). CVaR is a risk measure, the expected loss exceeding Value-at-Risk and is also known as Mean Excess, Mean Shortfall, or Tail VaR. The link between credit risk and the current financial crisis accentuates the importance of measuring and predicting extreme credit risk. Conditional Value at Risk has become an increasingly popular method for measurement and optimization of extreme market risk. The use of model can regulate all positions in a portfolio of financial instruments in order to minimize CVaR subject to trading and return constraints at the same time. The credit risk distribution is created by Monte Carlo simulations and the optimization problem is solved effectively by linear programming. We apply these CVaR techniques to the optimization of credit risk on portfolio of selected bonds.

Keywords: value at risk; conditional value at risk; credit risk; portfolio;

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1. Introduction

Risk management is a broad concept involving various perspectives. From mathematical perspective it is a procedure for shaping a loss distribution. On the other side it is a core activity of banks, investment companies, financial institutions and similar institutions in process of asset allocation. Credit risk is a risk of loss of principal or loss of a financial reward stemming from a borrower's failure to repay a loan or otherwise meet a contractual obligation. Traditionally used tools for assessing and optimizing market risk assume that the portfolio return-loss is normally distributed described by mean and standard deviation. This approach has shown up to be quite useful, but it is inadequate for evaluation of credit risk (Kollar, Valaskova & Kramarova, 2015; Valaskova, Gavlakova & Dengov, 2014).

Value at Risk (VaR) has become an increasingly popular method for measuring market risk. VaR measures potential losses over a specific time period within a given confidence level. The concept of VaR is widely used and well understood. Its popularity increased when it was integrated into the Basel Treaty as a required measurement for the determination of capital adequacy for market risk. VaR has also been applied to credit risk through models such as CreditMetrics (Gupton, Finger, & Bhatia, 1997), CreditPortfolioView (Wilson, 1997), and iTransition (Allen & Powell, 2008) (Kollar & Bartosova, 2014).

However, in spite of its popularity, VaR has also some undesirable mathematical properties, such as lack of sub-additivity and convexity. In the case of the standard normal distribution VaR is proportional to the standard deviation. The VaR resulting from the combination of two portfolios can be greater than the sum of the risks of the individual portfolios. (Kliestik, Musa Frajtova-Michalikova, 2015) A further complication is associated with the fact that VaR is difficult to optimize when calculated from scenarios. It can be difficult to resolve as a function of a portfolio position and can exhibit multiple local extreme, which makes it problematic to determine the optimal mix of positions and the VaR of a particular mix (Anderson, Uryasev, Mausser & Rosen, 2000; Kral, Kliestik, 2015).

Conditional Value at Risk was introduced by Rockafellar and Uryasev (2002) represents an alternative method to VaR for measuring market and credit risk. This measure is also called Expected Tail Loss, Mean Excess Loss, Mean Shortfall or Tail Var. CVaR approximately equals the average of some percentage of the worst – case loss scenarios (Grublova, 2010). VaR is heavily used in various engineering applications, including financial ones. VaR risk constraints are equivalent to the so-called chance constraints on probabilities of losses. There is a close correspondence between CVaR and VaR: with the same confidence level, VaR is a lower bound for CVaR. Rockafellar and Uryasev (2002) also showed that CVaR is superior to VaR in optimization applications. The problem of the choice between VaR and CVaR, especially in financial risk management, has been quite popular in academic literature (Frajtova-Michalikova, Kliestik & Musa, 2015). Reasons affecting the choice between VaR and CVaR are based on the differences in mathematical properties, stability of statistical estimation, simplicity of optimization procedures, acceptance by regulators, etc. Conclusions made from this properties may often be quite contradictive (Gavlakova, Kliestik, 2014; Buc, Kliestik, 2013).

2. Methodology

2.1. Basic model

Conditional Value at Risk is an alternative percentile measure of risk and its interpretation is the expected loss given that the loss exceeds the VaR. It is more informative risk metrics than VaR, because VaR does not measure the extent of exceptional losses. VaR merely states a level of loss that we are reasonably sure will not be exceeded: it tells us nothing about how much could be lost if VaR is exceeded. Based on these CVaR shows how much we expect to lose, given that the VaR is exceeded (Alexander, 2014; Dengov, Gregova, 2010).

For random variables with continuous distribution functions, CVARα(X) equals the conditional expectation of X subject to X ≥ VaRα(X). This is the basis for definition of CVaR. The general definition of CVaR for random variables with a possibly discontinuous distribution function is:

CVaR of X with confidence level α ϵ [0,1] is the mean of the generalized α-tail distribution:

$$CVaR_{\alpha}(X) = \int_{-\infty}^{\infty} z dF_X^{\alpha}(z), \tag{1}$$

where

$$F_X^{\alpha}(z) = \begin{cases} 0, & when\ z < VaR_{\alpha}(X), \\ \dfrac{F_X(z) - \alpha}{1-\alpha}, & when\ z \geq VaR_{\alpha}(X). \end{cases} \tag{2}$$

Generally CVARα(X) is not equal to an average of outcomes greater than VaRα(X). For general distributions, one may need to split a probability. So when the distribution is modeled by scenarios the CVaR may be obtained by averaging a fractional number of scenarios. (Yamai, Yoshiba, 2002)

CVARα+(X) (upper CVaR): expected value of X strictly exceeding VaR (also called Mean Excess Loss and Expected Shortfall):

$$CVaR_{\alpha}^{+}(X) = E\left[X \middle| X > VaR_{\alpha}(X)\right] \tag{3}$$

CVARα-(X) (lower CVaR): expected value for X weakly exceeding VaR (also called Tail Var):

$$CVaR_{\alpha}^{-}(X) = E\left[X \middle| X \geq VaR_{\alpha}(X)\right] \tag{4}$$

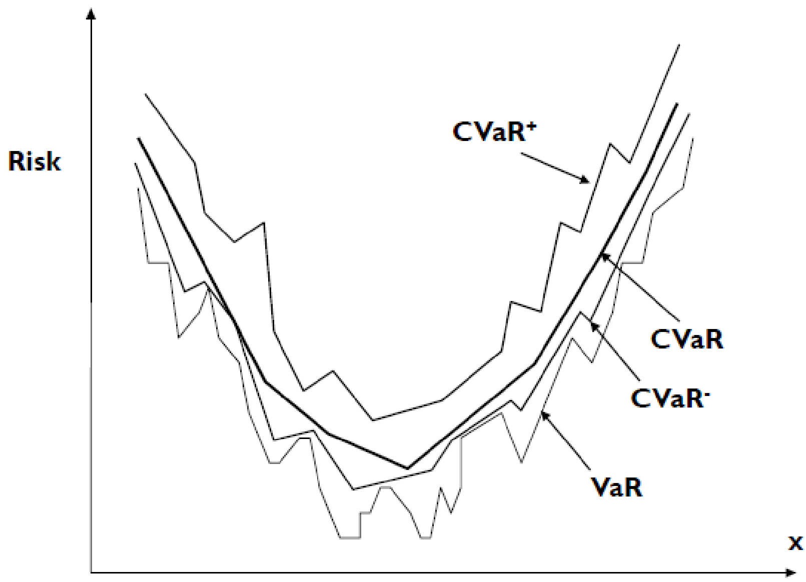

Figure 1. Main relations between risk metrics (Uryasev, 2000)

Figure 1 represents connection between these measures while we can see that CVARα+(X) and VaRα(X) are discontinuous functions. CVaR is convex in X and continuous with respect to α. VaR, CVAR+, CVAR- may be non-convex. The main relations between them are:

$$VaR \le CVaR^{-} \le CVaR \le CVaR^{+} \quad (5)$$

CVaR is considered a more consistent measure of risk than VaR. It supplements the information provided by VaR and calculates the quantity of the excess loss. While CVaR is greater than or equal to VaR, portfolios with a low CVaR also have a low VaR.

The minimum CVaR approach is based on a new representation of the performance function that allows the simultaneous calculation of VaR and minimization of CVaR.

3. Results and discussion

3.1. Application of Conditional Value at Risk for credit risk optimization

Application of Var for optimization of risk leads to a stretch of the tail of the distribution exceeding VaR. Though the minimization of VaR may lead to an increase in the extreme losses the purpose of it is to reduce extreme losses. Larsen, Mausser & Uryasev (2000) suggested two heuristic algorithms for optimization of VaR, which are based on the minimization of CVaR. They concluded that the minimization of Var leads to about 16% increase of the average loss for the worst 1% scenarios in comparison with the worst 1% scenarios in CVaR minimum solution. These results confirm theoretical results that CVaR is coherent while VaR is not coherent measure of risk. Results obtained by Larsen et al. (2000) are shown in the figure 2 and it is clear that how iteratively Var is decreasing than CVaR is increasing.

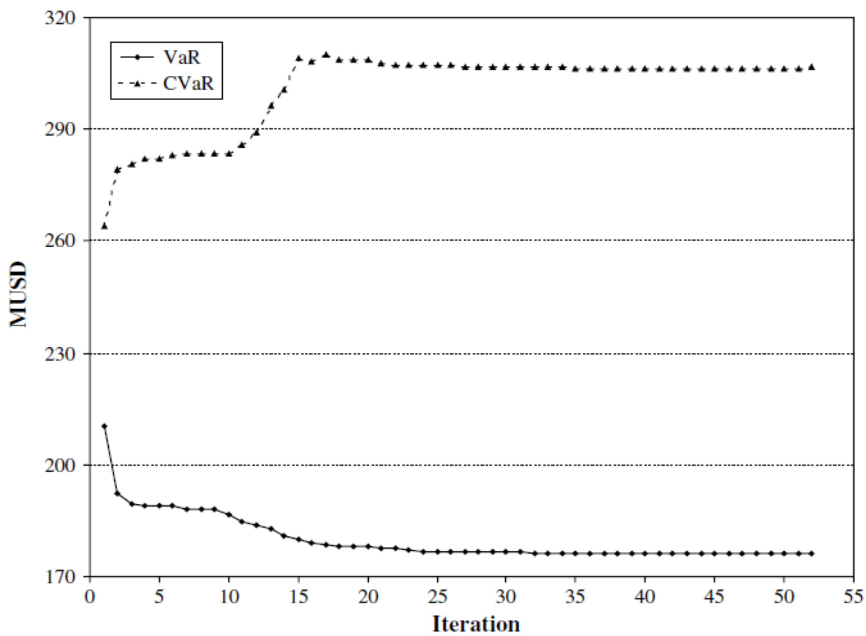

Fig. 2. Main relations between risk metrics. (Larsen, et. al., 2000)

We constructed a portfolio of 10 bonds, which were modeled with 1000 scenarios and subsequently we solved two optimization problems. Firstly is minimized 99%-CVaR deviation of losses and secondly is minimized 99%-VaR deviation of losses.

So the Problem 1 is: min $CVaR_{\alpha}^{\Delta}(x)$ Problem 2: min $VaR_{\alpha}^{\Delta}(x)$

$$\sum_{i=1}^{n} r_i x_i \geq \bar{r} \qquad \sum_{i=1}^{n} r_i x_i \geq \bar{r} \tag{5}$$

$$\sum_{i=1}^{n} x_i = 1 \qquad \sum_{i=1}^{n} x_i = 1$$

where x is the vector of portfolio weights

r_i is the rate of return of asset i

$\bar{r}$ is the lower bound on estimated portfolio return

Table 1. Conditional Value at Risk and Value at Risk functions

	Min $CVaR_{0,99}^{\Delta}$	**min** $VaR_{0,99}^{\Delta}$	**Ratio**
$CVaR_{0,99}$	0,0064	0,0075	1,172
$CVaR_{0,99}^{\Delta}$	0,0342	0,0358	1,047
$VaR_{0,99}$	0,0019	0,0012	0,632
$VaR_{0,99}^{\Delta}$	0,0289	0,0234	0,810
Max loss = $CVaR_1$	0,0124	0,0136	1,097
Max loss deviation = $CVaR_1^{\Delta}$	0,0523	0,0542	1,036

From the table 1 can be concluded that minimization of 99% - VaR deviation leads to 17,2% increase in 99%-CVaR in comparison with 99%- CVaR in the optimal deviation portfolio. So it is clear that minimization of CVaR leads to minimization of credit risk while application of CVaR is more appropriate for the portfolio than VaR.

3.2. Discussion

Based on the provided calculations can be summarized some brief conclusions such as CVaR has superior mathematical properties against VaR. Risk management with CVaR functions can be done quite efficiently. CVaR can be optimized and constrained with convex and linear programming methods, whereas VaR is relatively difficult to optimize VaR CVaR risk may be positive or negative, whereas CVaR deviation is always positive. Therefore, the Sharpe-like ratio (expected reward divided by risk measure) should involve CVaR deviation in the denominator rather than CVaR risk. CVaR of a portfolio is a continuous and convex function with respect to positions in instruments, whereas the VaR may be even a discontinuous function. Standard deviation can be replaced by a CVaR deviation. CVaR accounts for losses exceeding VaR, which can be good or bad:

- CVaR provides an acceptable representation of risks reflected in extreme tails.
- CVaR may have a relatively poor out-of-sample performance compared with VaR if tails are not modeled correctly.

4. Conclusion

The article was dedicated to the optimization of credit risk which was provided by the application of Conditional Value at Risk as a more appropriate risk measure in comparison with Value at Risk. There is a brief theoretical background introducing CVaR as a risk measure supplemented with comparison of CVaR and VaR. The main goal was to apply CVaR for the minimization of credit risk which was fulfilled by calculations provided on the portfolio of 10 selected bonds just to provide example calculations. There is a clear connection between VaR and CVaR and also we have confirmed that application of CVaR can lead to significant minimization of credit risk.

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