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Designing tools for Analytic Geometry: the Quadrics

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Abstract

New versions of GeoGebra, available from www.geogebra.org, allow to create interactive applications in three dimensions that can be used to illustrate the content that requires a representation in space, in addition to the previous possibilities in 2D. The ease with which objects can be changed using sliders, forcing them to take different positions, allows dynamic observation of what teachers want to show.The surfaces represented by a second degree equation in x, y and z are called quadrics. The most important ones are the ellipsoids, the hyperboloids, elliptic cones and paraboloids. For these surfaces, application indicating the traces in the coordinate planes and in planes parallel to them were prepared. Different self-assessments were also designed.The aim of this work is to describe these apps and briefly discuss the experience of using them. The results of a survey in order to give the students' opinion on the tools are also presented.

Keywords: Quadrics, GeoGebra, Interactive Apps

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1. Introduction

In La Geometrie, Descartes offered innovative algebraic techniques for analyzing geometrical problems, a novel way of understanding the connection between a curve's construction and its algebraic equation, and an algebraic classification of curves based on the degree of the equations used to represent these curves (Domski, 2015). In this way, a new subject called Analytic Geometry was created. Nowadays, there exist different tools that helps teaching Analytic Geometry, allowing students to avoid the routine problem of calculation itself to focus on understanding the problems, and to facilitate the visualization of topics that would otherwise be poorly understood. Among other alternatives, commercial software as Maple or Mathematica or free software like Scilab or GeoGebra are excellent options.

New versions of GeoGebra, available from www.geogebra.org, allow to create interactive applications in three dimensions that can be used to illustrate the content that requires a representation in space, in addition to the previous possibilities in 2D. The ease with which objects can be changed using sliders, forcing them to take different positions, allows dynamic observation of what teachers want to show. Embedded in an environment of dynamical geometry, where objects can be manipulated by sliding instead of writing commands, students would be able to understand in a different way some mathematical concepts (Lee & Hollebrands, 2006).

Different authors have analyzed the use of GeoGebra in Linear Algebra (Caglayan, 2015; Caligaris, Schivo & Romiti, 2016; Factor & Pustejovsky, 2016) or Calculus (Hohenwarter, Hohenwarter, Kreis & Lavicza, 2008; Caligaris, Schivo & Romiti, 2015). In particular, for Analytical Geometry issues, circles and lines (Camargo Guedes, 2015), equation of locus for particular geometric constructions (Botana & Abanades, 2014) and conic sections (Gonzalez Concepcion, 2013) were studied.

The aim of this work is to describe the apps prepared for the Algebra and Analytic Geometry course at Facultad Regional San Nicolas, Universidad Tecnologica Nacional, Argentina, to work with quadrics. A surface represented by a second-degree equation in x, y and z is called a quadric surface or a quadric. The most important ones are the ellipsoids, the hyperboloids of one and two sheets, elliptic cones, elliptic paraboloids and hyperbolic paraboloids. It is assumed that the issue is well known, and therefore the included descriptions are brief (Thomas, 2005; Anton, 1995; Lehmann, 1989). For these surfaces, applications indicating the traces in the coordinate planes and in planes parallel to them were prepared. Different self-assessments were also designed. All the tools were developed in Spanish but the Applets presented in this work were translated into English. The results of a survey in order to give the students' opinion on the tools are also presented.

2. The quadrics

The second-degree equation in three variables is:

$$A\,x^2 + B\,y^2 + C\,z^2 + D\,x\,y + E\,x\,z + F\,y\,z + G\,x + H\,y + I\,z + K = 0 \qquad (1)$$

with A, B, C, D, E or F non-zero. A surface represented by an equation like (1) is called a quadric.

By appropriate coordinate transformation, equation (1) can be written taking one of these two forms:

$$M\,x^2 + N\,y^2 + P\,z^2 = R \qquad (2)$$

$$M\,x^2 + N\,y^2 = S\,z \qquad (3)$$

The surfaces like (2) are called centered quadrics and the surface like (3) are called quadrics without center.

2.1. Centered quadrics

When one of the coefficients M, N, P, in (2) is zero, then the surface may be a cylinder or the equation may have no graph. When two of them are zero and the other one is positive, the locus is a pair of parallel planes, but when two of them are zero and the other one is negative, the equation has no graph. When R is zero, then the equation may represent a point, a line (one axis), a plane, a pair of planes or a cone.

When all the coefficients in (2) are non-zero, it can be written as:

$$\pm\frac{x^2}{a^2}\pm\frac{y^2}{b^2}\pm\frac{z^2}{c^2}=1 \qquad (4)$$

called canonical form of a centered quadric.

The first equations that students deal with, are those obtained when the surfaces are located in certain positions relative to the coordinate axes. These quadrics are presented in Table 1, indicating both the Cartesian and the parametric equations.

Table 1. Cartesian and parametric equations of the centered quadrics

Quadric	Cartesian equation	Parametric equation
Sphere	$x^2 + y^2 + z^2 = r^2$	$\begin{cases} x(t,u) = r\,\text{Cos}(t)\,\text{Cos}(u) \\ y(t,u) = r\,\text{Cos}(t)\,\text{Sin}(u) \\ z(t,u) = r\,\text{Sin}(t) \end{cases}$
Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	$\begin{cases} x(t,u) = a\,\text{Cos}(t)\,\text{Cos}(u) \\ y(t,u) = b\,\text{Cos}(t)\,\text{Sin}(u) \\ z(t,u) = c\,\text{Sin}(t) \end{cases}$
Hyperboloid of one sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	$\begin{cases} x(t,u) = a\,\text{Sec}(t)\,\text{Cos}(u) \\ y(t,u) = b\,\text{Sec}(t)\,\text{Sin}(u) \\ z(t,u) = c\,\text{Tg}(t) \end{cases}$
Hyperboloid of two sheets	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	$\begin{cases} x(t,u) = a\,\text{Tg}(t)\,\text{Cos}(u) \\ y(t,u) = b\,\text{Tg}(t)\,\text{Sin}(u) \\ z(t,u) = c\,\text{Sec}(t) \end{cases}$

For these surfaces, interchanging variables has the effect of reflecting the corresponding graph. For example, interchanging the variables x and z in the equation of a surface has the geometric effect of reflecting it symmetrically about the plane x = z.

2.1.1. Ellipsoids

For this surface, the traces in the coordinate planes are ellipses as are the traces in planes parallel to the coordinate planes. Some of these traces are shown in Figure 1a. When a = b = c, the application shows the corresponding sphere as is presented in Figure 1b. In this case, the traces are circles.

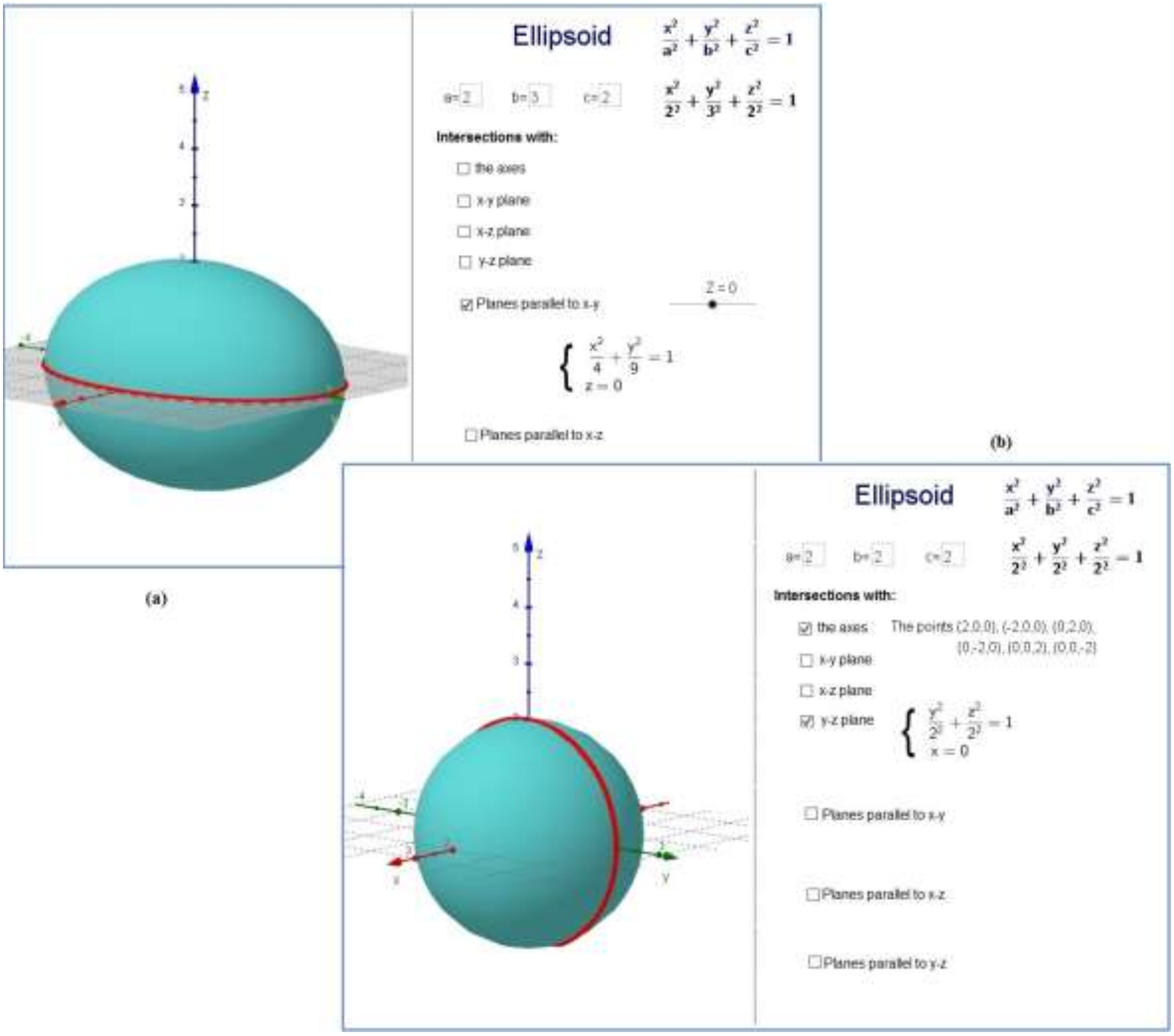


Figure 1. The ellipsoid and the sphere

2.1.2. Hyperboloids

For the hyperboloid of one sheet, the trace in the x-y plane is an ellipse as are the traces in planes parallel to x-y plane (Figure 2a). The traces in the y-z and x-z planes are hyperbolas as are the traces in planes parallel to these (Figure 2b)

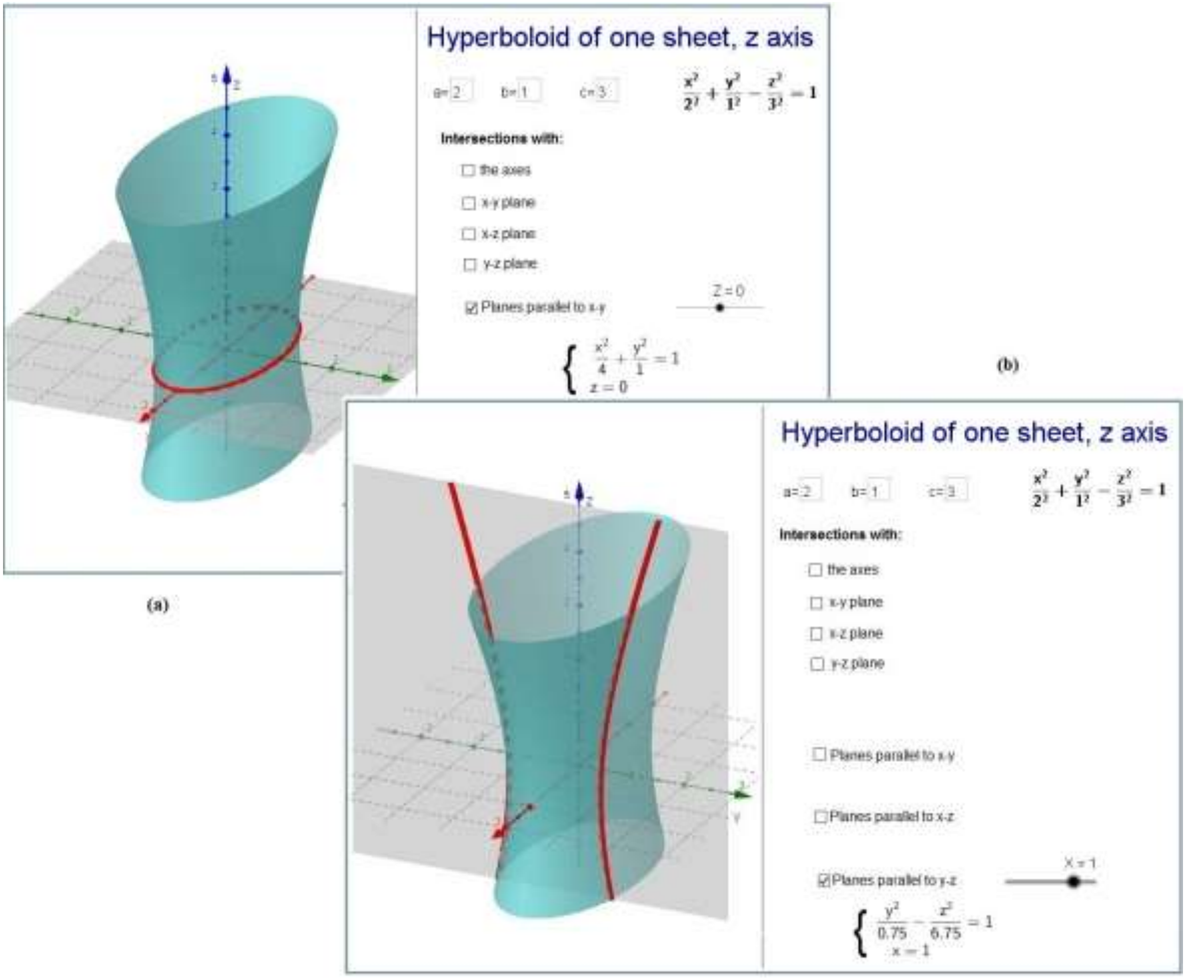
For the hyperboloid of two sheets, there is no trace in the x-y plane. In planes parallel to x-y plane intersecting the surface, the traces are ellipses (Figure 3). The traces in the y-z and x-z planes are hyperbolas as are the traces in planes parallel to these.

2.1.3. Cones

For the cone of equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=z^2 \qquad (5)$$

the trace in the x-y plane is a point, the origin O(0, 0, 0), and the traces in planes parallel to the x-y plane are ellipses. The traces in the y-z and x-z planes are pairs of lines intersecting at the origin.



(a) (b)

Figure 2. The hyperboloid of one sheet

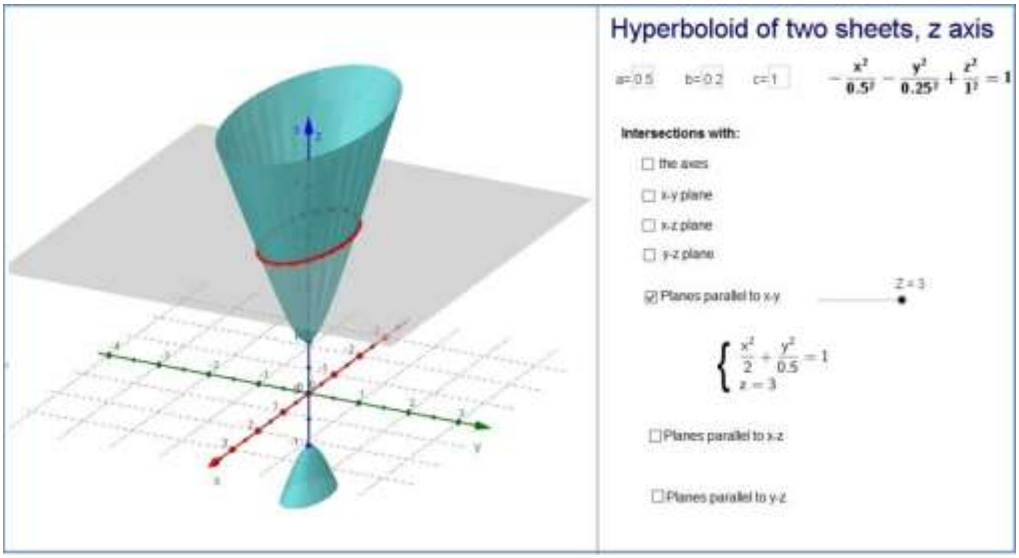


Figure 3. The hyperboloid of two sheets

The traces in planes parallel to these are hyperbolas. Figure 4 shows the application designed to work with cones of equation like (5).

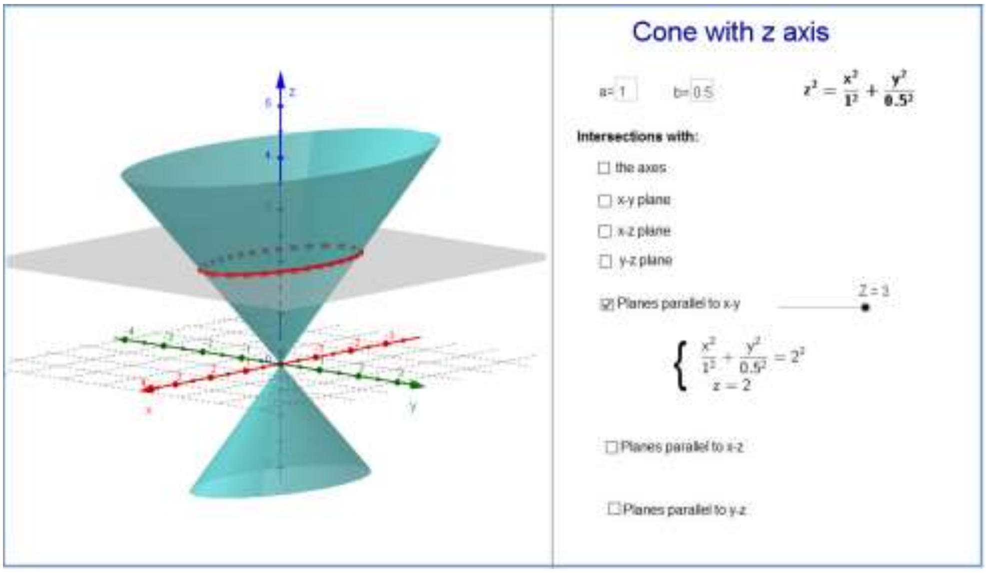


Figure 4. The cone with z-axis

2.2. Quadrics without center

When one of the coefficients M, N, in (3) is zero then the surface is a cylinder. When S is zero and M and N have the same sign the equation represents a line (an axis) but when M and N have different signs the equation represents two intersecting planes. When S and M or S and N are zero, then the equation represents a coordinate plane. When all the coefficients in (3) are non-zero, it can be written as:

$$\pm\frac{x^2}{a^2}\pm\frac{y^2}{b^2} = c\,z \tag{6}$$

called canonical form of a quadric without center. Taking into account the possible combination of signs for the coefficients, there are two different types of surface that can be obtained. These ones are shown in Table 2.

Table 2. Cartesian and parametric equations of the paraboloids

Quadric	Cartesian equation	Parametric equation
Elliptic paraboloid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=z$	$\begin{cases} x\,(t,u) = a\,\mathrm{Cos}\,(t)\,\mathrm{Cos}\,(u) \\ y\,(t,u) = b\,\mathrm{Cos}\,(t)\,\mathrm{Sin}\,(u) \\ z\,(t,u) = (\mathrm{Cos}\,(t))^2 \end{cases}$
Hyperbolic paraboloid	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=z$	$\begin{cases} x\,(t,u) = a\,\mathrm{Cos}\,(t)\,\mathrm{Sec}\,(u) \\ y\,(t,u) = b\,\mathrm{Cos}\,(t)\,\mathrm{Tg}\,(u) \\ z\,(t,u) = (\mathrm{Cos}\,(t))^2 \end{cases}$

For these surfaces, interchanging variables has also the effect discussed earlier. Besides, replacing z by –z has the effect of reflecting the surface symmetrically about the x-y plane.

2.2.1. Paraboloids

For the hyperbolic paraboloid, the trace in the x-y plane is a pair of lines intersecting at the origin. The traces in planes parallel to the x-y plane are hyperbolas. The hyperbolas above the x-y plane open in the y direction and those below the x-y plane, in the x direction. The traces in the y-z and x-z planes are parabolas as are the traces in planes parallel to these. Figure 5 shows the different traces in the x-y plane and in two planes parallel to x-y.

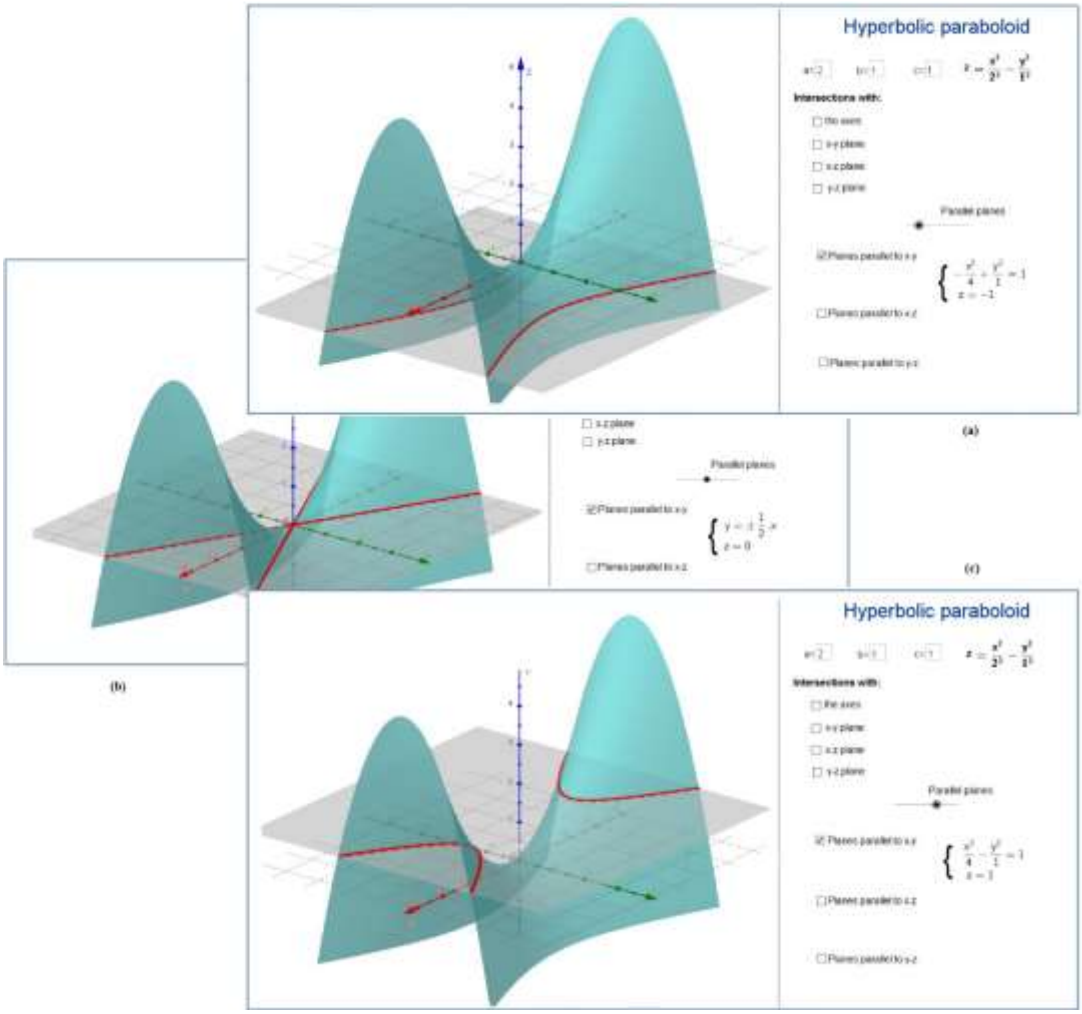


Figure 5. The hyperbolic paraboloid

For the elliptic paraboloid, the trace in the x-y plane is a point, the origin O(0, 0, 0). The traces in planes parallel to and above the x-y plane are ellipses. The traces in the y-z and x-z planes are parabolas. The traces in planes parallel to y-z and x-z are also parabolas. The application prepared to analyze this surface was presented in a previous work (Caligaris, Schivo, Romiti, & Menchise, 2016).

2.3. Self Assessments

Some other tools, which were used in class to conduct a review of the contents, were prepared. Figure 6 shows one of these tools, exhibiting part of the answers.

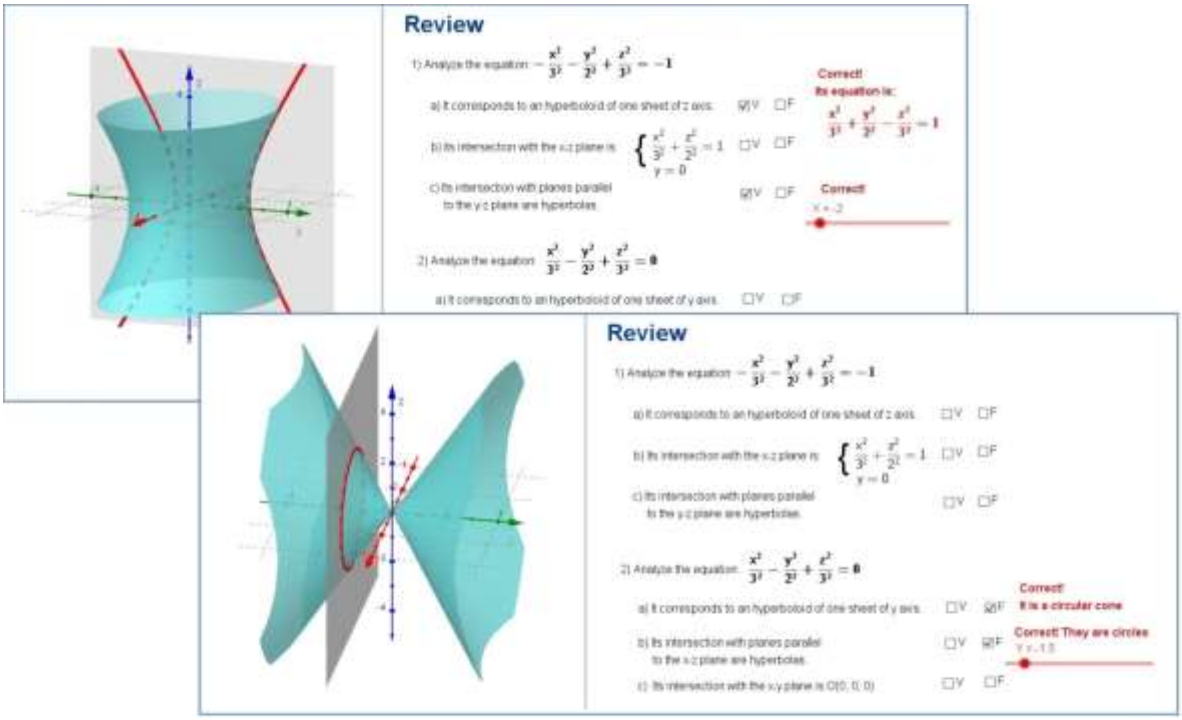


Figure 6. Some issues of the review

3. The experience of using the tools in 2016

The presented applications were offered to students during the course in the academic year 2016 for the first time. In order to evaluate the use of the different tools, students completed a questionnaire at the end of the issue. In this survey, students answered a series of closed questions to be discussed later using a Likert type scale. The options were: strongly disagree, disagree, neither agree nor disagree, agree, strongly agree, with numerical values 1, 2, 3, 4, 5, respectively.

Likert scales, a common measurement method in educational contexts, are based on the idea that some underlying phenomenon can be measured by aggregating an individual’s rating of students’ feelings, attitudes, or perceptions related to a series of individual statements or items (Harpe, 2015). Mathematical attitudes and attitudes towards mathematics are not the same. Attitudes towards mathematics refer to the enjoyment of this discipline, underlining the affective facet more than the cognitive one. Mathematical attitudes, in contrast, refer to the way of using general capacities that are relevant for mathematics, aspects which are all more closely related to cognition (Palacios, Arias & Arias, 2014).

Table 3 shows some of the statements of the questionnaire, with the obtained indexes. As it can be seen, all indexes were over 3.

The applications were designed with the intention to motivate, arouse and maintain interest in the subject, but also an improvement in the academic performance of students is expected. Students' attitudes toward mathematics, may be critically important for success in mathematics (Lipnevich, Preckel & Krumm, 2016). In the midterm exam, better results were observed in exercises with quadrics than in other exercises.

Table 3. Some statements in the questionnaire with the obtained indexes

Statement	Index
The apps helped me to learn the theoretical content about these concepts	4.17
The tools facilitate understanding of each surface intersections with the axes and coordinate planes	4.13
The apps helped me to analyze what kind of surface an equation represented	3.87
The use of the applications in class made them more entertaining	3.17
I paid more attention in class when using the apps	3.26
By using the software I could do exercises outside the class, verifying results	3.39

4. Conclusion

In this work, examples of incorporating technology as a teaching resource to promote dynamic visualization of certain contents of Analytical Geometry have been presented. The applications to work with the quadrics were designed taking into account the difficulties that have been identified by teachers in previous years.

The best tool in the development of a class is to make students interested in what is being taught, and an attractive way to do this is with the support of mathematical software. The incorporation of interactive applications is an effective method to promote the visualization of fundamental concepts, decrease the difficulty the issue presents and improve students' dedication and interest in the subject.

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