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The effect of verbal and illustrative representation on solving statistical problems by students in elementary school

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Abstract

The purpose of this study was to examine the effect of verbal and illustrative representation of the same statistical problems experienced by 56 elementary school students. Students were evaluated on problem-solving skills through six statistical problems as presented in school textbooks. One week later, the students were re-evaluated on problem-solving skills using the same problems that had been illustratively and verbally represented by the researchers. At the same time, we examined the correlation of students' performance to the six given problems in relation to their reading comprehension, verbal and mathematical competence (based on teachers' grade assignment). From the results of the quantitative research method that was used, there was a statistically significant correlation between the verbal representations in a) two out of six problems related to the mean value calculation using α representational table format and b) a bar graph construction based on a representational table format. However, an important finding of the research was the high correlation between students' performance and their reading comprehension, verbal and mathematical competence.

Keywords: Verbal representation of a problem, illustrative representation of a problem, statistics.

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1. Introduction

Statistics is an important part of the elementary school curriculum with data collection and data processing at its core. In particular, in sixth grade, the Greek students are taught to collect and record data, construct tables and graphs, handle the transformation between oral/written data and graphic illustrations, as well as calculate mean values.

According to Goldin (1998), a representation is the formation of signs, characters, images or objects that can somehow constitute or 'represent' something else. Furthermore, Goldin & Kaput (1996) point out that the representations are strictly structured systems, personally or culturally, the 'representational systems'.

The representations are distinguished in internal and external, which are in a continuous two-way interaction (Duval, 1999; Goldin & Kaput, 1996; Lesh, Post & Behr, 1987). The internal representations are mental images created by individuals and include verbal and virtual systems, mathematical formulas, a design, command and control system, as well as an emotional representational system. External representations are natural, observable configurations, such as the words, the diagrams, the images and the microcosms of the computer. The classification of Carney and Levin (2002) is interesting, in which according to their function illustrative representations are divided into decorative, auxiliary-representational, auxiliary-organisational and informative images, aimed at the conceptual understanding and the solution of the mathematical problem. Indeed, for all statistical problems the transformation from one system to another is significant, and thus students are required to demonstrate what is referred to as 'representational flexibility' (Christodoulou & Gagatsis, 2014; Gagatsis, Deliyianni, Elia & Panaoura, 2011) and relates to the operation, the recognition and conversion of representations (Duval, 2006; Goldin, 2003; Lesh et al., 1987).

According to research data, however, many students not only have difficulty in transforming one type of representation into another but also lack understanding of the language required each time to represent and deal with mathematical ideas (Ainsworth, 1999; Even, 1998; Lesh et al., 1987). In particular, the survey researcher Anastasiadou (2007) highlights the difficulties by age, where the majority of students in fourth and fifth grades show weakness in reading, understanding and transforming data between systems and more specifically between graphic and verbal forms. However, students in the sixth grade seem to have the skills to transform and handle the various systems for the representation of statistical systems (Anastasiadou, 2007). In a survey by Gagatsis & Dawn (2000), it is obviously the highest success rates in the handling of tables and graphs rather than in the oral statistical problems that require the ability to transform the spoken language into mathematical terms.

In addition, the research of Tsemeli, Liapi & Chionidou-Moskofoglou (2013) confirms the improvement in performance in arithmetic problems after using verbal representation of mathematical texts with temporal phrases used in the children's daily lives. However, researchers point out that a virtual representation does not always guarantee this improvement because of each student's different cognitive level and learning style. Indeed, reading comprehension and vocabulary seem to play an important role in transformation and problem-solving as it is essential to understand the spoken language in which the data and the relationships are expressed (Maridaki-Kassotaki & Chionidou-Moskofoglou, 2004; Tsemeli et al., 2013). Goldin (1998) also stresses the existence of the external variables that affect the students' performance. These are classified into syntactic variables that relate to the verbal description of the problem, to content and context variables, to structural variables and finally to heuristic variables (Goldin, 1998; Goldin & Kaput, 1996). Verschaffel et al. (1999) point out, yet, the effect of negative beliefs about the mathematical problem, while Gagatsis et al. (2011) identify as performance factors the flexibility of strategies, the teaching method and the teaching environment.

Based on the findings of previous research, as described above, the present study attempts to investigate at an early stage the impact of three of the above-mentioned factors, that is, reading comprehension, mathematical ability and vocabulary, on students' performance on statistical problems in a different verbal and illustrative representation system.

2. Our research

The research was carried out in February and March 2014. The participants were 26 students and 30 students in the sixth grade of two schools in Volos municipality with normal development and intelligence. The research hypotheses were inter alia:

1. Students' performance in the verbal representation of the same statistical problem (from the school textbook) is correlated with the students' performance (teacher rating).
2. The performance of students in the illustrative representation of the same statistical problem (from the school textbook) is correlated with the performance of the students.
3. The performance in solving statistical problems is correlated with the *lexical achievments1* and the *reading comprehension* skills of the students.

3. Method

This study uses the research methodology by Tsemeli et al. (2013). A quantitative research with questionnaires distributed and collected was selected for data collection, as well as a qualitative case study for special cases. The research tools used are presented below.

3.1. Preliminary assessment

The preliminary assessment was conducted in groups by the classroom teacher in two sessions to assess the development of students' vocabulary skills and reading comprehension.

3.2. Assessment of students' vocabulary skills development and reading comprehension

To determine vocabulary development, Tafa's (1995) weighted test was used. The assessment of the students' reading comprehension was based on the 'Text Understanding of the Reading Performance Examination' subscale (Panteliadou & Sideridis, 2002) and incorporated four texts of increasing difficulty, where the student needs to read silently and to respond appropriately to multiple-choice questions.

3.3. Students' mathematics skills assessment

The evaluation of mathematical skills, due to the limited time available, was based on the students' quarterly grades in the Mathematics course and not on the basis of the *WISC – III* weighted scale.

After the preliminary assessment, the survey included the initial and the final evaluation, with a time interval of a school week. The two assessments were conducted by the classroom teacher, who was given clear instructions to avoid problem-solving guidance.

3.4. Initial evaluation

At this stage, students in each grade were given six statistical problems, which were selected from the mathematics textbooks in elementary school.

3.5. Final evaluation

During the final assessment, the same six statistical problems were given to students, which were differentiated as follows:

(i) *Problem Presentation with Verbal Representation.* The problems of the Initial Assessment were distributed to the students with the verbal representation of certain phrases or words in the original problem on the basis of empirical facilitation in the understanding of the problem's data (see Figure 2).
(ii) *Problem Presentation with Illustrative Representation*. The problems of the Initial Assessment were distributed to the students, once enriched with auxiliary organising images – charts with the mathematical data of the problem.

4. Results

4.1. Students' performance in the statistical problems of the study in relation to their verbal representation

The following table depicts, according to the mean values equality assessment, that the total performance in the Initial and Final Evaluation with verbal representation do not differ ($p = 0.22 > 0.05$).

Table 1. Equalisation of mean performance values (and standard deviation) in Initial evaluation and verbal representation

	Mean Value	sig
Overall Score in Initial evaluation - Overall Score in verbal representation	−0.021 (0.125)	0.22

The comparison of the mean performance values among the different statistical problems distributed in both the Initial Assessment and the verbal representation resulted in higher success rates in (a) the construction of a bar graph in verbal representation, (b) the construction of a table in verbal representation and (c) the description of the histogram, even though these performances are compared with those of the other statistical problems.

The results were similar for the Final Assessment with illustrative representation, where no statistically significant difference was observed in the performance compared to the Initial Assessment ($p = 0.34 > 0.05$).

Table 2. Equalisation of mean values (and standard deviation) in initial evaluation and illustrative representation

	Mean Value	sig
Overall Performance in Initial Evaluation - Overall Performance in Illustrative Virtualisation	−0.021 (0.15)	0.34

The performance of comparison for the same problem between the Initial and Final Evaluations concludes that the verbal representation seems to have a statistically significant effect on finding the mean value by using the representational format table ($p = 0.03 < 0.05$), and the same is observed for the construction of a bar graph by using the representational format table ($p = 0.09 < 0.1$).

Table 3. Comparison of average performance for the same problem in initial and final evaluation with verbal representation (*N* = 56)

	Initial evaluation	Verbal representation
Find mean value by using the table	0.79*	0.92*
Construction of bar graph by using the table	0.86**	0.96**

* $p < 0.05$ **, $p < 0.1$

4.2. Preliminary assessment correlations with the mathematical projects

In the correlation analysis, a statistically significant association is observed between the mathematical capability and the overall performance in the Initial Evaluation (p = 0.00 < 0.05, r = 0.60). Similar results are also obtained for the Final Evaluation with verbal representation (p = 0, r = 0.50). On the contrary, there was no statistically significant correlation in the Final evaluation with illustrative representation (p = 0.65 > 0.1).

The level of the vocabulary skills of the students is quite fairly correlated to their overall performance in the Initial evaluation (p = 0.00 < 0.05, r = 0.57) and verbal representation (p = 0.046 < 0.05, r = 0.28). On the contrary, vocabulary skills do not seem to correlate with overall performance in the illustrative representation (p > 0.1).

Table 4. Preliminary evaluation correlations with psychometric parameters (*N* = 56)

	Initial evaluation	Verbal representation	Illustrative representation
Mathematical ability	0.60**	0.50**	0.65
Reading ability	0.24	0.24	−0.35*
Vocabulary	0.57**	0.28*	−0.06

* p < 0.05 **, p < 0.01

The above results are evident in the case of the student below (Figure 1) with moderate mathematical performance (Grade 8) and reading ability, who after the verbal representation of the problem showed more satisfactory results in finding a mean value by using a representational format table.

Initial Evaluation **Verbal Representation**

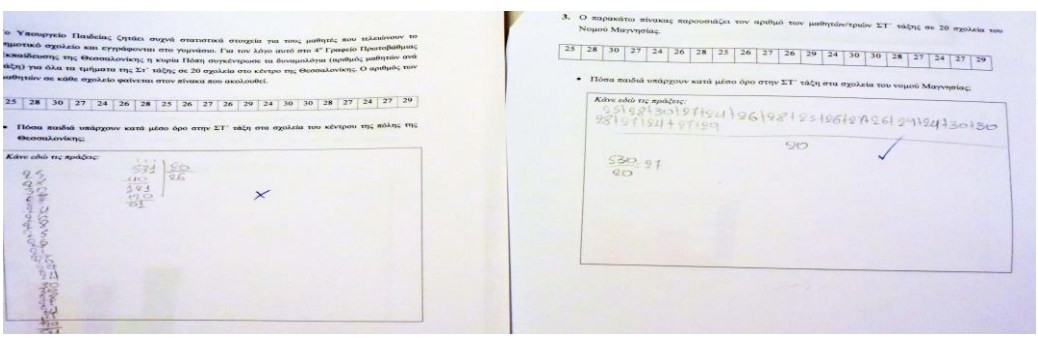

Figure 1. Finding a mean value by using a representational format table

4.3. Qualitative analysis of errors

The incorrect answers of students were classified in the following categories. Construction of a bar graph: *(i) Wrong bar height, (ii) Wrong/absent graph attributes* (title, labels, and spaces). Finding the mean value: *(i) Correct Plan Development – Wrong Algorithms, (ii) Incomplete Solution*. Interpretation of statistical concepts: *(i) Incomplete justification for histogram use, (ii) Incomplete interpretation of mean value*.

In the initial evaluation, the most common type of error was the incorrect placement of numbers on the bar graphs, the correct design development but the wrong algorithms for finding the mean value, as well as the wrong interpretation of the mean value.

In the verbal representation, error rates generally appear to be reduced, while the most common errors are still the wrong interpretation of the mean value and the wrong justification for using the histogram.

In the illustrative representation, the most frequent error is the incorrect placement of the numbers on the bar graph.

5. Discussion

The results of our study showed that the overall performance in the Initial and Final Evaluation did not differ, so verbal representation and illustrative representation did not appear to cause a statistically significant alteration in overall student performance in Statistical Problems. However, the case study is in line with Tsemeli's et al. (2013) research, since it has been shown that the verbal representation improves students' performance particularly in (a) finding the mean value by using the representational format table and b) the construction of a bar graph by using the representational format table in our research.

In addition, the comparison of the average performance values among the different statistical problems that the students were asked to solve reveals both in the Initial Evaluation and in the verbal representation that our elementary school students could more efficiently handle a) the construction of a bar graph by using the verbal representation form, (b) the construction of a table by using the verbal representation form and (c) the description of what a histogram depicts in relation to the other statistical problems given to them.

In addition, Tsemeli et al. (2013) research confirms the significant correlation between reading comprehension and vocabulary with the students' overall performance in the Initial and Final Evaluation with verbal representation. This means that our students with a high level of reading comprehension and vocabulary skills achieve higher success rates in semantically and lexically more demanding problems. Reading comprehension and vocabulary are not correlated with overall performance in illustrative representation, where problems lack in verbal descriptions and excel in the graphical representation of data.

As regards mathematical ability, its positive correlation with the overall performance confirms that the higher it is, the higher the success rates are. This is not, however, the case for illustrative representation, possibly due to the greater flexibility exhibited by students in various mathematical abilities in relation to verbal representation problems. Individual case analysis also reveals the fact that verbal representation can improve the performance of both low and high math skills in statistical problems (Tsemeli et al., 2013), who may have exploited the verbal representation and the presentation of data in the form of tables/charts/images.

Furthermore, the comparative analysis of the performances in each problem separately showed significantly high results in the construction of graphs (bar graph, pictogram) and tables, regarding the problems that required their interpretation, confirming Christodoulou & Gagatsi's (2014) study on the ability to switch between different representations. In particular, our students exhibit higher skills in identifying and describing what a line graph depicts than interpreting and justifying its use for data visualisation. Similar results are observed with respect to the mean value, where its identification by performing the necessary algorithmic calculations yields very satisfactory results. However, difficulties are observed in the association of the mean value with data for drawing conclusions, as well as in understanding its effect on extreme values.

In summary, this study highlights the ways in which students can benefit from the educational process. Future studies of additional factors that affect their performance are important such as the level of representational flexibility, the personal learning style, the use of digital technologies, the emotional state of problem-solving and the teaching processes that influence them.

The results of psychometric and written tests on the specific mathematical problems were correlated with the use of the SPSS PASW Statistics 18 package.

Notes

[1]Vocabulary acquisition refers to students' understanding of semantic concepts on individual words during their development while reading comprehension to students' comprehension of texts through the process of sound or silent reading.

Annex

Sixth grade school notebook/Volume C/Chapter 45/Page 23/1st Exercise

In an exercise on the territory of seven European countries, a group of students constructed the graph you see in the picture. Consult the table on the left and right to draw the bar graph with the territory of the seven countries. Compare and discuss the similarities or differences between your bar graph and the children's bar graph.

Verbal representation

In the European Geography class, you have collected the data for the territory of seven European countries, presented in the table below. (Due to lack of space as above)

Construct the corresponding bar graph and compare it with the bar graph below. (Due to lack of space as above)

COUNTRY	TERRITORY
Belgium	30,153
Greece	131,957
Italy	301,300
Finland	337,009
Sweden	448,750
Spain	505,000
France	547,000

Figure 2. The territory is in thousands of square kilometers

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