World Journal of
Environmental Research

Volume 8, Issue 1, (2018) 017-025

www.wjer.eu

Theoretical study to calculate the vibration modes of a wind turbine blade with a new composite material

Rabie ElAlaoui*, Equipe Energetic, Mechanical and Industrial Systems (EMISys) Research Team, Engineering Smart and Sustainable Systems (Engineering 3S) Research Center, Mohammadia School of Engineers, University Mohammed V in Rabat, Morocco

Hamid Mounir, Equipe Energetic, Mechanical and Industrial Systems (EMISys) Research Team, Engineering Smart and Sustainable Systems (Engineering 3S) Research Center, Mohammadia School of Engineers, University Mohammed V in Rabat, Morocco

Boudi El Mostapha, Equipe Energetic, Mechanical and Industrial Systems (EMISys) Research Team, Engineering Smart and Sustainable Systems (Engineering 3S) Research Center, Mohammadia School of Engineers, University Mohammed V in Rabat, Morocco

Abdellatif El Marjani, Equipe Energetic, Mechanical and Industrial Systems (EMISys) Research Team, Engineering Smart and Sustainable Systems (Engineering 3S) Research Center, Mohammadia School of Engineers, University Mohammed V in Rabat, Morocco

Suggested Citation:

ElAlaoui, R., Mounir, H., Mostapha, E. L. & Marjani, E. L. (2018). Theoretical study to calculate the vibration modes of a wind turbine blade with a new composite material. *World Journal of Environmental Research. 8*(1), 017-025.

Received from December 2, 2017; revised from February 21, 2018; accepted from March 10, 2018;
Selection and peer review under responsibility of Prof. Dr. Haluk Soran, Near East University, Cyprus.

---

* ADDRESS FOR CORRESPONDENCE: **Rabie ElAlaoui**, Equipe Energetic, Mechanical and Industrial Systems (EMISys) Research Team, Engineering Smart and Sustainable Systems (Engineering 3S) Research Center, Mohammadia School of Engineers, University Mohammed V in Rabat, Morocco.
E-mail address: elalaoui.rabie@gmail.com / Tel.: +223 71 99 02 82

Abstract

A precise understanding of the dynamics of a blade is essential for its design, especially in the development of new structures and the resolution of noise and vibration problems. This understanding involves the study of experimental and/or theoretical modal analysis. These latter present effective tools for describing, understanding and modelling the dynamic aspect of each structure, in the present work, we are going to establish the Eigen-mode of a wind turbine blade made by a new composite material 'hemp fibre' using theoretical calculation for flap-wise, edge-wise and torsional mode using the finite element method applied to a structure consisting of a beam embedded at one end, based on the Euler-Bernoulli hypothesis and the equation of beam's motion. Furthermore; we compare the obtained results with those of composite material made by fibreglass.

Keywords: Blade, Eigen-mode, hemp fibre, flap-wise, edge-wise, torsional, fibreglass.

1. Introduction

Energy is essential to the achievement of sustainable development in any country. Unlike fossil fuels, which contain high percentages of carbon (Labdine, s. d.), renewable energies are generated from sources that are obviously inexhaustible—water, sun, biomass, geothermal heat and wind (Ottmar, Edenhofer. Ramon, Pichs-Madruga. Youba, 2011). Among these renewable sources is the wind that is considered one of the best energy sources to reduce imports of fossil fuels and greenhouse gases. By using wind power, we can reduce our dependence on oil and protect the planet.

The blades are the important elements of a wind turbine (Larsen, Hansen, Carl & Baumgart, 2002). The length of the blade determines the amount of energy that can be extracted depending on the wind speed (Shokrieh & Rafiee, 2006). To recover the maximum energy from the wind, the blades must be positioned higher under very variable wind speed conditions which generate vibrations that can cause problems for the structure of the blade (Shokrieh & Rafiee, 2006). The purpose of manufacturing blades with variable lengths is to capture the different wind speeds. The modal analysis provides information on the dynamic properties of wind turbine components and thus leads to an understanding of their dynamic behaviour (Tartibu, Kilfoil & Van Der Merwe, 2012). Many materials have been used, both for the industrial production of wind turbines power and the small domestic applications. Indeed, the prime objective in wind turbine design is to have a great blade in terms of length, width, profile and material (Rabie, Mounir, Boudi & Marjani, 2015; Rabie, Mounir, Boudi & Marjani, 2016).

The rupture of the blade is caused in particular by vibrations (Gherbi, 2011). The study of the vibrations of a blade is one of the most important issues in the design process of a wind turbine. Carbon fibre is widely used in the manufacture of wind turbine blades (Herbert, Iniyan, Sreevalsan & Rajapandian, 2007). The blade of a wind turbine machine has a complex geometry and great bending changes (Labdine, s. d.).

The origin of modal analysis is the study of the characteristics of the vibration (Tartibu, 2013), that is to say, the study of the natural frequencies and the Eigenmodes of a structure or a component of the machine (Cai, Pan, Zhu & Gu, 2013). It can also serve as a starting point for another more dynamic and detailed study, such as a transient dynamic study, a harmonic study or a spectral study (Muzard, 1994). Natural frequencies and mode shapes are necessary parameters in the design of a structure for dynamic loading conditions (Larsen et al., 2002).

In mechanics, the blade is assimilated to a homogeneous beam, in order to simplify the system in order to obtain an estimate of its frequencies and its own modes (Wang, Kolios, Delafin, & Nishino, s. d.). The method consists in associating to the real structure an equivalent structure of the mass-spring to perform the vibration computation, Euler-Bernoulli theorem (Nour, Gherbi, & Chevalier, s. d.).

2. Theoretical study

For a flush beam subjected to free vibration and the system is considered as a continuous system in which the mass of the beam is considered to be distributed with the rigidity of the shaft (Muzard, 1994) (Banerjee, Su & Jackson, 2006).

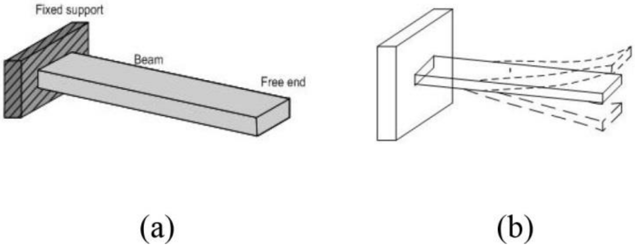

Figure 1. (a), Cantilever beam. (b), The beam under free vibration

Figure 1 shows a rectangular section recessed beam which can be subjected to a bending vibration giving a small initial displacement at the free end; Figure1 (a) and Figure 1 (b) show a beam embedded under the free vibration.

To determine the behaviour of the wind turbine blade theoretically, the researchers have developed a mathematical model based on the equation of motion of a beam to calculate the frequencies and the Eigen-modes (Liu, 2010) (Bramwell, Done & Balmford, 2000).

2.1. Equation of motion for a uniform beam

Consider a free-body diagram of an element of a beam shown in Figure 2. The external force applied to the beam per unit length is denoted $f\left(x,t\right)$. The bending moment $M\left(x,t\right)$ is related to the deflection of the beam $w\left(x,t\right)$ by (Tartibu, 2013):

$$M\left(x,t\right)=E\left(x\right)I\left(x\right)\frac{\partial^2 w\left(x,t\right)}{\partial x^2} \tag{1}$$

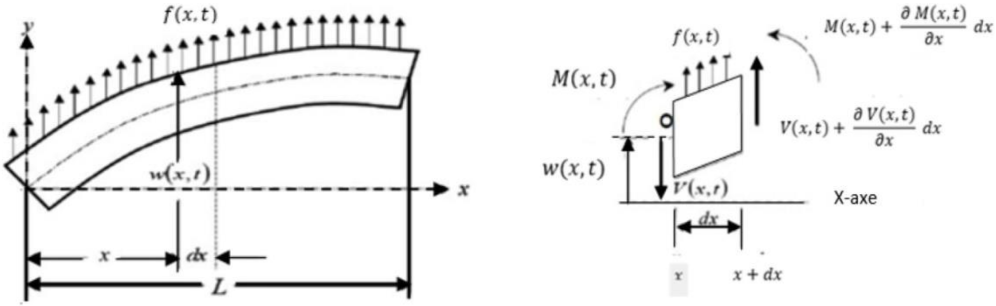

Figure 2. Beam in bending (Talukdar, s. d.)

One note;

$w\left(x,t\right)$ is the transverse vibrations

$A\left(x\right)$ is the cross-section

$E\left(x\right)$ is the modulus of elasticity

$\rho(x)$ is the density

$I\left(x\right)$ is the moment of inertia

$V\left(x,t\right)$ is the shear force at the left end of the dx element

$V\left(x,t\right)+\frac{\partial V\left(x,t\right)}{\partial x}dx$ The shear force at the right end of the element

They considered that the deformation is rather small so the shear deformation will be sufficiently smaller than transverse vibrations. What we in the equation of motion in the y-direction.

$$\left[V\left(x,t\right)+\frac{\partial V\left(x,t\right)}{\partial x}dx\right]-V\left(x,t\right)+f\left(x,t\right)dx=\rho(x)A\left(x\right)\frac{\partial^2 w\left(x,t\right)}{\partial t^2} \tag{2}$$

By simplifying the term V (x, t), we find the following expression

$$\frac{\partial V\left(x,t\right)}{\partial x}dx+f\left(x,t\right)dx=\rho(x)A\left(x\right)\frac{\partial^2 w\left(x,t\right)}{\partial t^2} \tag{3}$$

The equation of motion around the z-axis passing through the point O leads to the following expression.

$$\left[M\left(x,t\right)+\frac{\partial\, M\left(x,t\right)}{\partial x}dx\right]-M\left(x,t\right)+\left[V\left(x,t\right)+\frac{\partial V\left(x,t\right)}{\partial x}dx\right]dx+\left[f\left(x,t\right)dx\right]\frac{dx}{2}=0 \tag{4}$$

The simplification of equation (3.4) gives;

$$\left[\frac{\partial M\left(x,t\right)}{\partial x}+V\left(x,t\right)\right]dx+\left[\frac{\partial V\left(x,t\right)}{\partial x}+\frac{f\left(x,t\right)}{2}\right]\left(dx\right)^2=0 \tag{5}$$

They considered that the moment of inertia of rotation for the element (dx) is negligible, and if dx is small, then $\left(dx\right)^2$ is negligible; therefore equation (5) becomes;

$$\left[\frac{\partial M\left(x,t\right)}{\partial x}+V\left(x,t\right)\right]dx=0$$

$$V\left(x,t\right)=-\frac{\partial M\left(x,t\right)}{\partial x} \tag{6}$$

By replacing the term V (x, t) in equation (3), we find;

$$-\frac{\partial}{\partial x^2}\left[M\left(x,t\right)\right]+f\left(x,t\right)dx=\rho(x)A\left(x\right)\frac{\partial^2 w\left(x,t\right)}{\partial t^2} \tag{7}$$

By assembling equation (1) and (7) we find

$$\rho(x)A\left(x\right)\frac{\partial^2 w\left(x,t\right)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[E\left(x\right)I\left(x\right)\frac{\partial^2 w\left(x,t\right)}{\partial x^2}\right]=f\left(x,t\right) \tag{8}$$

$f(x,t)=0$ For there is no external force, Hence, the equation of motion for the free vibration of the beam is given by the expression of a fourth order differential equation;

$$\rho(x)A(x)\frac{\partial^2 w(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[E(x)I(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right]=0 \tag{9}$$

This equation is the Euler-Bernoulli vibration equation for a non-uniform beam, as long as $E(x)$, $A(x)$, $I(x)$ and $\rho(x)$ are constants, can simplify the equation in the following form

$$\frac{\partial^2 w(x,t)}{\partial t^2}+C^2\frac{\partial^4 w(x,t)}{\partial x^4}=0 \tag{10}$$

Where

$$C=\sqrt{\frac{EI}{\rho A}}$$

The resolution of this differential equation gives the natural frequency equation of the beam as a function of the geometry parameter and the material uses

$$\mathrm{f}=\frac{\omega}{2\pi}=\frac{(\beta \mathrm{L})^2}{2\pi}\sqrt{\frac{EI}{\rho A L^4}} \tag{11}$$

The value of β in Equation 11 can be determined from the following formula:

$$\beta_n=(2n-1)\frac{\pi}{2L} \qquad avec \quad n=1,2,\dots\dots$$

The natural frequency is related to the circular natural frequency as

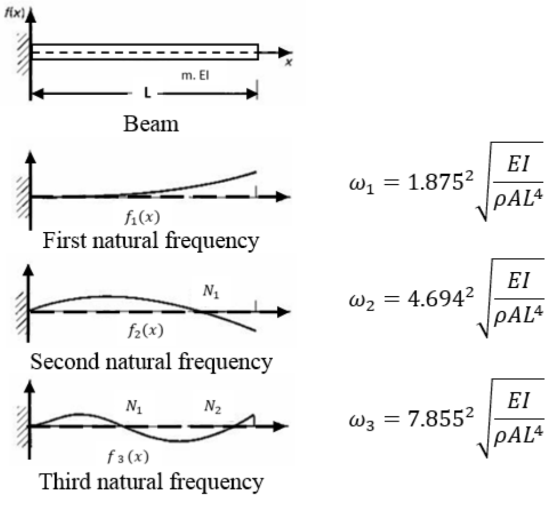

Figure 3 The first three undamped natural frequencies and mode shape of the beam (Talukdar, s. d.)

The blade of the wind turbine can be considered as a beam of Euler-Bernoulli (Bot, 1994). For calculating the first natural frequencies and modes of vibration of the beam from equation (11) for a non-rotating state, to validate the theoretical results, these natural frequencies of our composite material will be compared to the results of the fibreglass.

The theoretical modal analysis was performed for a beam of dimension L = 4,200 mm, W = 690 mm, T = 240 mm

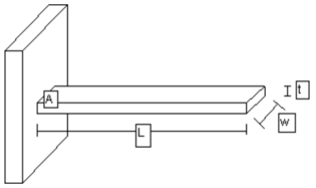

Figure 4 Typical cantilever beam studied

The natural frequency of the beam can be calculated from equation 11 as follows

$$f = \frac{(\beta L)^2}{2\pi}\sqrt{\frac{EI}{\rho A L^4}}$$

The cross-section A and the area moment of inertia I are given as follow $A = W * T$

✓ For flap-wise modes: $I = \frac{1}{12}\left(W * T^3\right)$

✓ For edge-wise modes: $I = \frac{1}{12}\left(W^3 * T\right)$

✓ For torsional modes: $f_n = \frac{(2n-1)}{4L}\frac{2T}{W}\sqrt{\frac{G}{\rho}}$

✓ With shear modulus G is given by $G = \frac{E}{2(1+\nu)}$

3. Proprietes de materiau

The composite materials used in our study are hemp fibre and fibreglass with Epoxy matrix.

Table 1 Hemp fibre and glass fibre properties according to the matrix

	Hemp/Epoxy	E-glass/Epoxy
Density (g/m3)	1.252	1.8
Tensile modulus E (GPa)	30.4	31.6

4. Results and discussion

The results obtained by the two materials are very close which gives validation to our composite material that was used.

Table 2. Flap-wise modes

Mode	Hemp	E-glasse	Ecarte type (%)
F1	6.321	5.374	14.97
F2	39.611	33.681	14.97

F3	110.912	94.309	14.97
F4	217.344	184.807	14.97
F5	359.285	305.500	14.97

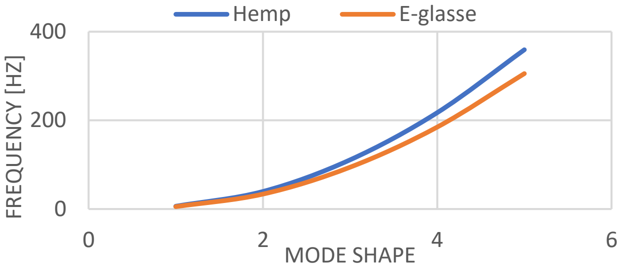

Figure 5 Flap-wise mode in function of frequency

Table 3. Edge-wisemodes

Mode	**Hemp**	**E-glasse**	**Ecarte type (%)**
F1	27.089	23.034	14.97
F2	169.762	144.349	14.97
F3	475.338	404.180	14.97
F4	931.473	792.032	14.97
F5	1539.791	1309.286	14.97

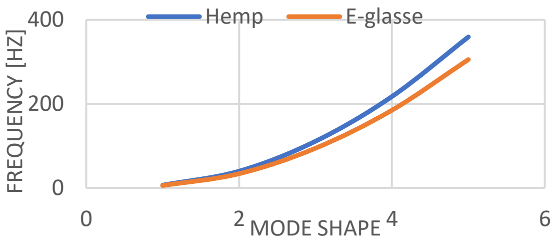

Figure 6 Flap-Edge mode in function of frequency

Table 4. Torsional modes

	Hemp	**E-glasse**	**Ecarte type (%)**
F1	88.354	75.127	14.97
F2	265.062	225.383	14.97
F3	441.770	375.638	14.97
F4	618.478	525.892	14.97
F5	795.186	676.149	14.97

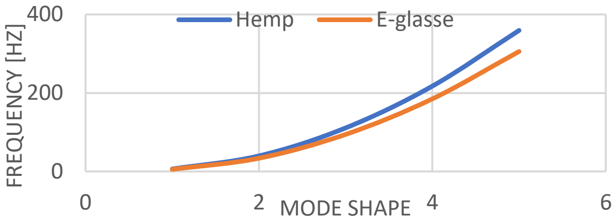

Figure 7 Torsional mode in function of frequency

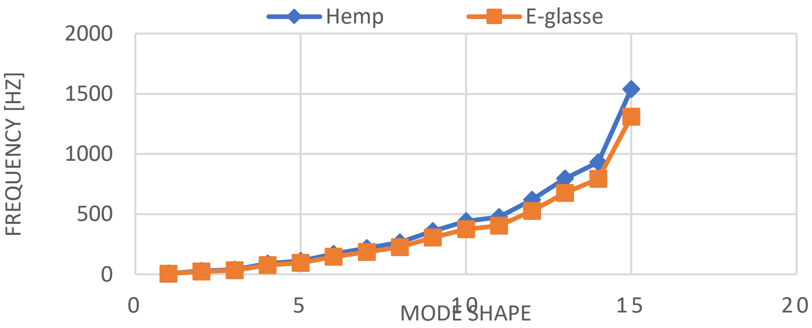

Figure 8 Comparison all mode shape for the both materials

We can see that the natural frequencies obtained for the hemp composite material are greater than those of glass composite material which gives more advantage to our material.

5. Discussion

In this paper, we have presented a numerical modal analysis of a wind turbine blade considered as a Euler-Bernoulli beam. The modal calculations of the fifth order of the hemp composite material in flap-wise, edge-wise and the torsional mode are very closed to these of the fibreglass, especially for the first and the second modes, physically, this result can be explained by the low density of hemp fibre compared to glass fibre and the natural frequency of a structure is calculated without external exciter force or dissipative forces. It proves that the hemp material will be of great value in practical engineering applications of the wind turbine. Indeed, to approve this finding, we are invited to combine the modal test and the numerical calculations, also analysed and investigated frequency variations of the blades with the impact of structural damping. This purpose will be our main interest in future work.

6. Conclusion

The results give us an approach on the vibration modes of a hemp fibre beam, but the dynamic study of slender and flexible structures like the blades of a wind turbine usually requires resolutions based on the use of finite element models. Because these allow an accurate description of the blade displacement by taking a high number of modes into attention, and this will be our next work carried out in a more detail article on modal analysis of a blade using the element method finished the results of hemp fibre are compared with the results of glass fibre composite, it is shown that the natural frequencies of the hemp fibre composite are higher than those of the fibreglass composite. This result indicates that the hemp fibre composite exhibits advantageous characteristics for future use in the manufacture of the wind turbine blade. The results of this modelling can also be used to calculate the dynamic stresses, in order to then estimate the fatigue of the blade.

Reference

Banerjee, J. R. A., Su, H. & Jackson, D. R. (2006). ARTICLE IN PRESS Free vibration of rotating tapered beams using the dynamic stiffness method. *Journal of Sound and Vibration*, *298*, 1034–1054. https://doi.org/10.1016/j.jsv.2006.06.040

Bot, L. (1994). *Equations energetiques en mécanique vibratoire : Application au domaine des moyennes et hautes frequences*. Ecole centrale de Lyon.

Bramwell, A. R. S., Done, G. & Balmford, D. (2000). *Bramwell's Helicopter Dynamics*. (2nd ed.). *British Library Cataloguing* (second). https://doi.org/10.1016/B978-075065075-5/50005-1

Cai, X., Pan, P., Zhu, J. & Gu, R. (2013). The analysis of the aerodynamic character and structural response of large-scale wind turbine blades. *Energies*, *6*(7), 3134–3148. https://doi.org/10.3390/en6073134

Gherbi, M. (2011). *Modelisation par elements finis d'une pâle d'helicoptere sous chargement aerodynamique*.

Herbert, G. M. J., Iniyan, S., Sreevalsan, E. & Rajapandian, S. (2007). A review of wind energy technologies. *Renewable and Sustainable Energy Reviews*, *11*, 1117–1145. https://doi.org/10.1016/j.rser.2005.08.004

Labdine, M. Z. (s. d.). *Etude Dynamique Et Optimisation Des Pales d'un Aerogenerateur*.

Larsen, G. C., Hansen, M. H., Carl, I. & Baumgart, A. (2002). *Wind turbine blades*.

Liu, X. (2010). Dynamic Response Analysis of the Blade of Horizontal Axis Wind Turbines. *Journal of Mechanical Engineering*, *46*(12), 128. https://doi.org/10.3901/JME.2010.12.128

Mccosker, J. (2012). *Design and optimization of a small wind turbine by*. Faculty of Rensselaer Plytechnic Institue.

Mohan, R. S., Sarkar, A. & Sekhar, A. S. (s. d.). Vibration analysis of a steam turbine blade. Dans *inter.noise 2014 mobourne australia* (p. 1–10).

Muzard, P. (1994). *ETUDE DU COMPORTEMENT DYNAMIQUE LINEAIRE ET NON-LINEAIRE D'UN ROTOR D'HELICOPTERE APPLICATION AU COUPLAGE ROTOR - FUSELAGE*. Ecole centrale de Lyon.

Nour, A., Gherbi, M. T. & Chevalier, Y. (s. d.). Modes shape and harmonic analysis of different structures for helicopter blade I . Introduction. Dans *30th European Conference on Acoustic Emission Testing & 7th International Conference on Acoustic Emission University of Granada,* (p. 1–16).

Ottmar, Edenhofer. Ramon, Pichs-Madruga. Youba, S. (2011). *Rapport spécial sur les sources d'énergie renouvelable et l'atténuation du changement climatique*. https://doi.org/978-92-9169-231-6

Rabie, E. A., Mounir, H., Boudi, E. M. & Marjani, A. El. (2015). Mechanical Performances Investigation of New Materials for Wind Turbine Blade. Dans *2015 3rd International Renewable and Sustainable Energy Conference (IRSEC)* (p. 1–4). Marrakech: IEEE. https://doi.org/10.1109/IRSEC.2015.7455142

Rabie, Ela., Mounir, H., El Mostapha, B., Marjani, A., Echab, H. & Abdellah, M. (2016). Performances Compraison of Wind Turbine. Dans *2016 4rd International Renewable and Sustainable Energy Conference (IRSEC)*. Marrakech: IEEE.

Shokrieh, M. M. & Rafiee, R. (2006). Simulation of fatigue failure in a full composite wind turbine blade. *Composite Structures*, *74*(3), 332–342. https://doi.org/10.1016/j.compstruct.2005.04.027

Talukdar, S. (s. d.). *Vibration of continuous systems*.

Tartibu, K. (2013). *A SIMPLIFIED ANALYSIS OF THE VIBRATION OF VARIABLE LENGTH BLADE AS MIGHT BE USED IN WIND TURBINE SYSTEMS*. cape peninsula university of technology. https://doi.org/10.1017/CBO9781107415324.004

Tartibu, L. K., Kilfoil, M. & Van Der Merwe, A. J. (2012). Vibration analysis of avariable length blade wind turbine. *International Journal of Advances in Engineering & Technology*, *4*(1), 630–639.

Wang, L., Kolios, A., Delafin, P., & Nishino, T. (s. d.). Development and Validation of Structural Models for Wind Turbine Composite Blades, *44*(0).