World Journal on Educational Technology: Current Issues

Volume 13, Issue 1, (2021) 82-95

www.wj-et.eu

The cultivation of self-directed learning in teaching mathematics

Saied Bishara*, Department of special education and psychology, Beit Berl college and Open university, Israel https://orcid.org/0000-0002-6440-0586

Suggested Citation:

Bishara, S. (2021). The cultivation of self-directed learning in teaching mathematics. *World Journal on Educational Technology: Current Issues*. *13*(1), 82-95. https://doi.org/10.18844/wjet.v13i1.5401

Received from July 31, 2020; revised from October 25, 2020; accepted from January 15, 2021.
Selection and peer review under responsibility of Prof. Dr. Servet Bayram, Yeditepe University, Turkey.

Abstract

This study examined students' ability to learn mathematics in a self-directed teaching environment. One of the main goals of the educational system is to nurture independent learners who can grow up to be inquisitive, critical, creative, and capable of piloting their own learning. This implies making a change in the way the role of the mathematics teacher is perceived in that the teacher must now become a mentor who supports and mediates learning, enabling students to construct a knowledge base of rules and methods in mathematics and acquire and experience ways of thinking that enable them to construct this knowledge.

This qualitative study is based on interviews with four ninth-grade mathematics teachers and on in-class observations of teaching styles and teacher-student interactions. Our findings show that applying self-directed learning methods in class based on a constructivist approach to teach mathematics is an important factor in developing students' creativity and thinking. These findings suggest that developing this model of teaching should be recommended to teachers. Accordingly, this study also proposes a model for staff development programs that foster self-directed learning in mathematics. The model proposes that increasing teachers' awareness of their teaching process and training them to prepare learners to cope effectively with unfamiliar mathematical problems are goals to include in teacher training. This model of teaching may have far-reaching effects in pedagogy, e.g.: reducing drop-out numbers, improving achievements, and improving social interactions.

Key words: constructivist approach, fostering thinking, self-directed learning, teaching mathematics.

* ADDRESS FOR CORRESPONDENCE: Saied Bishara, Department of special education and psychology, Beit Berl college and Open university, Israel
E-mail address: saiedbs@gmail.com / Tel.: 972505914363

Introduction

Over the last few decades, a significant change has taken place in the pedagogical goals of teaching and in how we perceive and define knowledge. The goal of instruction today is not merely to transmit knowledge by way of traditional instruction, but to enable students to independently acquire knowledge and create content knowledge. The idea of self-directed learning was presented as a central goal in education in the early 1990's (Emanuel, 2003). Its implementation required charting a new course in teacher training: that of developing the skills needed to create learning environments that foster self-directed learning. In 2002, the NCATE (National Council for Accreditation of Teacher Education in the United States) published a list of goals for educating teachers-in-training to develop self-directed learning skills in their students, and for cultivating a self-directed approach to teaching and to learning (Michlasky & Kramarsky, 2008).

Changes and innovations in how we perceive knowledge in teaching require changes in teaching and learning processes as well as in our understanding of teachers' function and the nature of their interactions with their students. Schools should therefore be directing their efforts toward professional development of their teachers and supporting them in adjusting to the changing needs dictated by the evolving reality (Harpaz, 2008). This study offers a perspective on how professional development of ninth grade mathematics teachers in the areas of self-directed learning and creation of the appropriate teaching tools can be applied in school.

Ninth grade mathematics curriculum in Israel includes both algebra (and probability) and geometry which are taught concurrently. Ministry of Education directives (2013) allocate 80 hours a year for algebra. The official curriculum emphasizes how algebra can be used to explain patterns and form mathematical generalizations. Geometry in ninth grade is a 70-hour course in which students are introduced to proving theorems by inference (based on axioms and definitions), and learn to use the formal language of geometry.

Self-Directed learning and The Constructivist Approach

Constructivism is a general name for philosophical, psychological, pedagogical, sociological, linguistic, and methodological approaches which assert that knowledge is constructed rather than transmitted. Learning, according to this approach, is not based on copying existing information, but is a process by which a learner creates meaning (Yahieli, 2008).

Lev Vygotsky (1962), a well-known developmental psychologist, claimed that all high order mental functions develop in social contexts, and therefore social interaction has a central role in cognitive development. Exploiting the potential for development depends on full social interaction within the learning group. Accordingly, the learning unit is not a single individual, but a group that conducts discussions that require thinking and reasoning, providing feedback on the decision-making and learning processes.

Piaget (1972) was convinced that knowledge is not a product that can be transmitted from “head to head”, but a long process of acquisition in which learners construct personal knowledge for themselves, and spurring development in the brain. The constructs or knowledge structures are the mental or cognitive structures derived from a learner's personal experiences and engagement with bodies of knowledge.

According to the constructivist approach, one of the most important objectives of education is to cultivate an independent, thinking, self-directed learner. This is achieved by allowing leaners to learn at their own pace and in their own way, and also by establishing learning, social, and personal objectives that are in line with the learners' abilities and learning style. In a self-directed teaching process, the teacher becomes a mentor who guides the learner through the learning process by creating opportunities and, activities that are consistent with the learner's level of readiness.

Educating in this way requires a material change in the way we view a teacher's role in the learning process, a change that will allow us to see a teacher assisting students in their learning process and creating learning opportunities in which students understand on their own (Birenboim, 1997).

Among other things, self-directed learners will develop the ability to evaluate their own learning processes and products by self-reflecting. Socially, students develop their ability to debate, persuade, decide, and collaborate. On a personal level, learners acquire dedication of purpose, flexibility, curiosity, independence, and the ability to take the initiative and accept responsibility (Brooks & Brooks, 1999).

Clearly, students who are taught this way do not simply store and recycle information. They become engaged, search for meaning, evaluate information based on their own experience and goals, and are ready to modify and revise their interpretations and constructed knowledge. Learners like this are curious, seek links between ideas and concepts, and assume responsibility for their own learning. They become problem solvers who seek to identify problems, explore new ideas, and reveal new ways of thinking (Yehieli, 2008).

Teachers who adopt this approach use the learning cycle model (exploration, concept development, concept application) to foster natural curiosity and emphasize the important role of a self-directed learning process (Marshall, 1998). Such teachers encourage autonomy, ultimately aiming to nurture a thinking inquisitive learner who makes use of source materials, raw data, and concrete interactive, engaging sources. The constructivist teacher sets tasks that promote high-order thinking (by requiring analysis, interpretation, prediction, making or seeking connections), and allows the responses of the learners to drive their lessons.

Teachers who adopt this approach are aware that learners must understand relevant concepts before they present their own interpretation of the concepts, and so, they encourage dialogue among peers and with teachers with the goal of arriving at a deeper understanding of the subject under discussion. They also encourage active investigative tasks by asking open-ended stimulating questions, and wait for answers allowing sufficient time for the learner to construct meaning and create metaphors. In this way, teacher and students form novel constructions through the reflective abstractions taking place inside them (Apatow, 1999; Keiny, 1994).

Self-Directed learning and promoting Thinking

One of the challenges in pedagogy is to systematically foster a culture of thinking and create an environment that encourages learners to be inquisitive and imaginative (Tishman, Perkins & Jay, 2000).

The need to teach for thinking stems from the phenomenon called the "information explosion". In the last few decades, information has multiplied within a short space of time making it imperative to develop people's ability to think methodically so that they can acquire new knowledge efficiently, understand, critique, produce information, decide wisely, and find solutions to problems (Tishman, Perkins & Jay, 2000).

Teaching for thinking requires addressing qualities of thinking, such as critical, creative, and effective thinking. Creative thinking brings forth new ideas or original products, and critical thinking is the ability to evaluate an intellectual product. Creative thinking requires a measure of critical thinking and vice versa, and effective thinking triggers thinking tools quickly and accurately (Harpaz, 2008).

Self–Directed learning and The Environment That Promotes it

Cultivating a self-directed learner requires a physical and social learning environment that fosters such learners (Yehieli, 2008). This type of learning environment encourages thinking and provides learners with opportunities for evaluating ideas, rational presentation of arguments, asking questions, and imagining and explaining phenomena. As such, it stimulates intellectual inquisitiveness, and cultivates metacognitive skills (Eilam & Reiter, 2014).

Developing a language of thinking, in which students learn to distinguish between facts, opinions, theories, and claims (Emanuel, 2003) is also part of the learning environment.

Teaching Mathematics and Developing Thinking

Mathematics is a discipline based on posing questions, diligently searching for answers, and immediately posing a new question when the latest one has been answered (McClain & Cobb, 2001). Mathematics education is designed to enable students to construct knowledge of mathematical products that is consistent with the knowledge codified by the modern mathematical community, and at the same time help them acquire and experience ways of thinking that will enable them to construct this knowledge (Ministry of Education, 2013).

A learner's motivation to learn mathematics is defined as the inclination to discover and think independently, use social interactions with peers and teachers to improve problem solving skills and develop higher mathematical abilities. Aided by social cues, students critique themselves and their actions (ones that derive from needs, goals, and desires) as members of a family, an organization or a group (in this case a class). The class as a group is aware of its goals, formulates strategies to achieve them, and promotes feedback processes to improve how it learns mathematics. In a constructivist class general social norms develop, such as legitimacy of not understanding, attention to thinking strategies, and respect for people who make mistakes and for the errors they make (Clements, 2000; Emanuel, 2003).

In a mathematics teaching-learning process, socio-mathematical norms develop which although they are norms accepted by the group that is engaged in researching or solving a problem together, they are based on the language of mathematics (Cobb, Yackel & McClain, 2000). Because they are specific to mathematics, socio-mathematical norms are different from the general social norms that dictate student participation and normative classroom interactions and activities. The teacher plays a key role in creating the socio-mathematical climate in class that is necessary to teaching mathematics. This climate develops, guided by the teacher, as the class searches for solutions to problems (Fogarty, 1991; Hirst & Hughes, 2015).

Teacher Training and Professional Development in Self-Directed Learning and in Mathematics

Teachers tend to stick to the same pedagogical approaches and behaviors they have always used and, in fact, tend to teach in the way they themselves were taught, without fostering self-direction in learning)Ball, 2000). So, for teacher training to stress elements of self-directed learning over transmitting knowledge, teacher trainers must become guides to learning rather than transmitters of knowledge. This will empower their students, in turn, to guide the children in school to take an active role in their own education rather than becoming passive consumers of knowledge (Ben-Eliyahu & Linnenbrink-Garcia, 2015).

It is important to establish teacher training and professional development programs in self-directed learning that are based on reflective and critical models and which emphasize the link between professional development ideas and real-life experience in school (Clarke & Clarke, 2003; Michlasky & Kramarsky, 2008).

Direct practical experience and experiencing peer instruction in school and at staff development programs are the major reflective ways for teachers to develop professionally, enabling them to create learning environments that will nurture the self-directed learner (Emanuel, 2003).

The mathematics educator, Debora Ball, (2000) claimed that building the content necessary for teaching should begin with practice not curriculum. We must better understand the work that teachers do, carefully analyze the role of content knowledge at work, and thus prepare teachers who not only know the content, but also use it to help their students learn. She further claims that teachers must also be able to break down their own compressed understanding, which is the hallmark of an expert, into it constituents, because an explanation will only lead to understanding if it presents the uncompressed steps.

When developing mathematics curricula and learning materials, it is imperative to consider the socio-mathematical norms and social processes that are essential in a mathematics-nurturing environment. Students' willingness and motivation to engage in solving problems and mathematical tasks are pronounced in classes where there is no competition and where there is openness to feedback and mutual evaluation (Sabar-Ben Yehoshua, 2001). Achieving this broadmindedness requires professional training in self-directed learning.

There are indeed many challenges in the pedagogical aspects of teaching mathematics. In addition to individual differences between students, multiple environmental factors affect mathematics learning. It is therefore important to try and establish which methods will produce the best results in the shortest time.

This study examined specific features of self-directed learning and their association with teaching mathematics and is based on the idea that by applying self-directed learning to solving unconventional mathematics problems teachers may achieve increased learner motivation and improved achievements.

Research Questions

The research questions are:

1. How do constructivist characteristics, if present, facilitate the cultivation of thinking in the classroom, and how do they help foster self-directed mathematics learning in students?

2. From the standpoint of the mathematics teachers who participated in the research, what are the conditions that enabled them to foster functions of thinking and of knowledge construction? What conditions, in their estimation, interfere with advancement of these functions in the classroom?

Methodology

Participants

The participants in the study were four mathematics teachers in a junior high school in the center of Israel who teach two grade nine classes as well as the students of their two classes. Of the first two teachers, one is 48 years old, has an MA in mathematics, has been teaching for 27 years, and is also the mathematics coordinator in the school. The second is 36 years old, has a BA in mathematics, and has been teaching for 12 years. The class they teach is heterogeneous in terms of academic achievement including in mathematics. There were 18 males and 22 female students, aged 14 to 15 years from similar socio-economic backgrounds in this class.

The other two teachers who participated in the study teach a second ninth grade class in the same school. One of the teachers is 26 years old, and has a BA in mathematics and computers and five years of teaching experience. The other teacher is 24 years old, has a BA in mathematics and science and two years of teaching experience. There were thirty-eight ninth grade students in these teachers' class and also this class was heterogeneous in terms of academic achievement (see distribution in Table 1). There were 20 male and 18 female students, aged 14 to 15 years old from similar socio-economic backgrounds in this class.

Table 1 shows the distribution of participants' background information: gender, and age group. The average and standard deviation of age and years of teaching experience are given for teachers. Data was reported by the teachers.

Table 1. Distribution of participant background information

Student Sample (N= 78)		
Gender	**n**	**%**
boys	38	(48.7%)
girls	40	(51.3%)
Ninth grade age range n- (%)	78	(100.0)
Teacher Sample (N=4)		
Age	**m**	**sd**
	33.5	(11.0)
Teaching experience (years)	**n**	**%**
1-9	2	(50.0)
+10	2	(50.0)

Research tools

For the purpose of the research, a semi-structured questionnaire titled "Self-Direction in Learning and Teaching Mathematics to Advance Thinking and Constructing Knowledge" was prepared and administered to the four teachers during the interview. This instrument examined the ways in which teachers cultivate thinking, how they believe they should promote thinking in their students, and the conditions that facilitate or interfere with the development of thinking in their students during mathematics lessons. A 10-item observation sheet was also developed for marking characteristics of constructivism and development of thinking in-class. The author also received documentation from the teachers which assisted in understanding the learning processes that took place in their classes.

A two-part questionnaire was administered to the teachers. The first part collected teacher information such as: gender, age, teaching experience, discipline taught, and age of students and the second part contained open-ended questions designed to assess their position regarding the study questions. The questions were formulated to reflect the study goals as reflected in its title and questions were analyzed for reliability and validity. Internal reliability was provided by another researcher who was given the questions to review. Questions that proved difficult to understand were revised and submitted to other researchers for review, to ensure accuracy. Data was collected by three routes: interviews, observations, and document analysis which ensured that the desired information was collected.

The questionnaire included the following questions:

1. How would you define self-directed learning in the context of teaching mathematics?

2. How would you define fostering thinking and how would you define a constructivist approach to teaching mathematics?

3. Is there a link between either self-directed learning or developing thinking and teaching mathematics?

4. Does using a constructivist approach help foster thinking and does it help students get ahead in mathematics?

5. Do you have any suggestions regarding self-directed learning and promoting thinking in the teaching of mathematics?

Research Model

After obtaining consent of the schools' authorities to participate in the study, the investigators visited the school and met with the mathematics teachers and students of both 9th grades.

We held a 30 minute semi-structured interview with each of the teachers and observed each one during five ninth grade mathematics lessons providing a total of 20 observations. Data from the interviews and observations were collected and used for research purposes only.

Teachers also provided information about background variables: age, years of teaching experience, number of students in class, and student gender ratio.

Date analysis approach

The approach in this study was qualitative (Sabar-Ben Yehoshua, 2001; Shlasky & Alpert, 2007). All data was reviewed carefully and underwent analysis for meaning, interpretation, and generalization of the studied problem (promoting self-directed learning in mathematics at schools).

FINDINGS

1. Constructivist characteristics in teaching for thinking in the mathematic classroom

Based on the data from the interviews and the in-class observations, the four teachers were classified by their constructivist approach: one group consisted of two teachers who used a more constructivist approach to teaching math, and the second group consisted of two teachers who took a less constructivist approach.

Table 2 shows the frequency distribution of observed behavior in the classroom by constructivist approach to cultivating thinking in mathematics and by teaching approach.

The data in Table 2 indicate that teachers in the first group (who used the constructivist approach) made wide use of questions to develop thinking, used more methods that apply constructivism to foster thinking, employed teaching that made maximum use of time. These teachers led discussions, formulated questions that drove the lesson ahead, and afterwards grounded the discussions held in class based on their students' perspective. In this way, for example, Teacher A asked students to compose a question whose answer makes use of an equation with one unknown. Teacher B held a discussion during which she asked students to arrive at a mathematical generalization following a discussion held in class.

Table 2: Frequency distribution of observed behavior in the class constructivist characteristics of cultivating thinking in mathematics and by teaching approach

	Teachers who adopt a more constructivist approach				**Teachers who adopt a less constructive approach**			
	Not at all	**Very little**	**To a great extent**	**To a very great extent**	**Not at all**	**Very little**	**To a great extent**	**To a very great extent**
Develop thinking								
Use of directed questions to stimulate thinking	---	---	50%	50%	---	100%	---	---
Use of questions which stimulate search for new points in the material being studied	---	---	100%	---	---	100%	---	---
Use of questions regarding comparing quantities that are represented in different units of measure	---	---	100%	---	50%	50%	---	---
Teaching Methods								
Perceiving the topic being studied as a whole and breaking it down to its elements to simplify and clarify.	---	---	---	100%	50%	50%	---	---
Use of words of praise and appreciation for correct answers	---	---	---	100%	---	100%	---	---
Dividing students into small groups and encouraging dialogue in the group before it presents an answer.	---	---	50%	50%	50%	50%	---	---
Use of graphic representation to help students understand internal difficulties in the logic of components of the question	---	---	---	100%	50%	50%	---	---
Time management								
Students are allowed sufficient time to complete tasks so there is time for comprehensive thinking	---	---	50%	50%	---	100%	---	---
Lesson is concluded with a review of the topics learned during the lesson.	---	---	---	100%	50%	50%	---	---
Homework is assigned at the end of each lesson to extend students understanding of the topic.	---	---	50%	50%	---	100%	---	---

In their questions teachers sought new points in the material. They began their instruction with a complete, major idea and entered into the details by breaking it down into its components. This intentionally thought-provoking approach gave the students multiple entry points. Teacher A, for example, responded to her students' questions in the following way, "That is an excellent answer. The way you solved the problem shows that you understand unknowns and how to create equations." Teacher B told students: "Good start. Keep it up." Both teachers asked students to collaborate with others to find the solution and to take their time to think things over.

Teacher A frequently used expressions such as: "Take your time, don't rush to find an answer, think things through. We are not in a hurry." Teacher B addressed her student by name and said, "I know that you can solve this problem. Yesterday you solved a similar problem." In other words, both of these teachers encouraged the students and reinforced their answers, which created a dynamic process of interaction between peers and with the teacher and guided students to a deeper understanding.

The teachers in the first group often used thought organizers and cognitive terms, raised questions around mathematical problems, adapted the topics to the ability of the students, encouraged deep thinking, and gave the students the time they needed to think. In this way, for example teacher A encouraged the students to make use of graphic representations of the problem. Teacher B encouraged the use of a table which included titles such as "Type of problem investigated," "Number of unknowns," "How the problem was solved," and "Examining the obtained answer." The teachers guided the students to find a variety of solutions to the same problem.

Both teachers maintained continuity in content in order to enable their students to acquire skills, in depth understanding, and insight into the material. For example, teacher A asked the students questions about a comparison between quantities represented by fractions, percentages and decimals. To do this, the students had to unify quantities and only then make the comparison. Teacher B asked the students to calculate the angles of polygons with different areas, and the students arrived at the correct conclusions by trying it themselves.

In contrast, the two teachers in the second group, who made less use of the constructivist approach, conducted their lessons in a structured way based on the textbook and made far less use of discussion. The emphasis in their lessons was on the need to complete the material in the curriculum. They rarely praised their students or used teaching aids beyond the textbook.

2. Conditions that promote or inhibit thinking functions in mathematics teaching

As emerged from the interviews, the teachers believe that to promote the development of self-direction functions and of knowledge they must provide problems, assignments and challenging questions that encourage high-order thinking and help the learner develop and construct knowledge. The teachers also consider it valuable to use examples based on everyday life and on topics that students can relate to, encourage debate and active discussion in class, and develop intellectual skills. Their answers show that they believe in the need to give their students questions and problems requiring in-depth thinking and in relating to students based on their individual ability. In practice, they tried to show their students more than one method of arriving at a solution.

Self-directed learning in mathematics is a personal process of knowledge building in which a learner raises questions, identifies sources of knowledge, processes information, and generates new knowledge that is appropriate to the mutual relationship. Self-direction touches the individual, both the learner and the instructor. By drawing on different aspects of a person's intellectual, emotional, social, and creative experiences self-directed learning allows the individual to express their abilities and excel, as well as to develop a deeper understanding of subjects they are interested in and which satisfy their needs. The pedagogical and psychological experiences in self-directed processes complement and reinforce each other. In mathematics, self-directed learning obliges all participants to consider their learning goals, the learners' characteristics, the curricula, and learning, teaching, and evaluation processes from a fresh perspective.

The teachers pointed out certain conditions that interfered with their personal teaching style. In general, they said that mathematics is a difficult and complex discipline, and this makes it harder for them to find the ideal way to teach. They also pointed out the difficulty of developing a sense of self-efficacy in students and the need to construct new knowledge based on what they were previously supposed to have learned, but might not have understood. This is an obstacle encountered by teachers who want to push ahead with the material and adhere closely to the curriculum schedule.

Discussion and Conclusions

Self-directed learning and a constructivist approach to teaching mathematics

The findings of this study indicate that teachers who use a constructivist approach in teaching mathematics, formulated preview and follow-up questions based on discussions held in class and on their students' perspectives. In doing so, they took a constructivist approach which views learning as an individual, internal process of constructing knowledge. In this process, individuals are active in constructing their world of knowledge in their own unique way that is also meaningful to themselves (Salomon, 2003).

Facilitation of discussions during a math lesson, as was observed in this study, is consistent with the constructivist approach, which emphasizes the central role of social interaction in cognitive development. The learning unit is not the individual, but is rather a group which conducts discussions accompanied by thinking and deliberation, and provides feedback on the decision-making process and learning processes (Vygotsky, 1962).

Establishing socio-mathematical norms and directed social processes in a mathematics environment may increase motivation of students to learn mathematics. Self-directed learning assisted by peers and teacher is likely to lead to improved problem solving skills, better achievements and can boost mathematical abilities to a higher level (Emanuel, 2003). Creativity is a skill that students can and should be taught and as they acquire creativity they also learn to evaluate their learning process and products , which produces better learning (Watts, 2017).

Through their questions, the teachers in group A (the constructivists) sought to facilitate new ways of looking at the material. They began with a large and complete idea and then went into greater detail by breaking down that whole into its component parts. In doing so, they deliberately provided their students with multiple entry points, which encouraged thinking.

This approach is characteristic of constructivism, which is based on constructing knowledge around "major ideas" or fundamental concepts. Students must see the whole before they are capable of finding meaning in its components. Instead of including as many details as possible, teachers should delve deeper into the fundamental concepts of the discipline. Constructing the instruction program around "major ideas" enables learners to approach these ideas in a variety of ways, consistent with their interests and individual way of thinking. This is the reason that asking a broad question or setting curriculum tasks around broad ideas gives students multiple entry points, and promotes their thinking and understanding of what they are learning in class (Brooks and Brooks, 1997). In this way, teachers develop in their students' extensive knowledge of cognitive functions that affect learning, and foster high level thinking. Moreover, the distinction between general meta-cognitive strategies and specific meta-cognitive strategies may advance teachers' ability to address the content they are required to teach to students with different backgrounds, abilities, and thinking styles (Marshall, 1998).

Teachers in the first group asked each student to contribute to a solution in their own way and tried to allow each student the time required to think and express themselves. By reinforcing students' answers they encouraged them, which created a dynamic process in which the interaction between the teachers and students propelled the lesson. According to the Brooks and Brooks (1997) metaphor, a student's point of view is a window into her thought process. Awareness of a student's perspective helps teachers challenge their students, and the viewpoint of each student is the beginning of the path towards individualized instruction.

The interactions between teachers in the first group and their students reflect these teachers' constructivist approach to their role as teachers, which is to guide their students' learning processes and create learning opportunities for them that steer them toward understanding on their own (Birenboim, 1997; Harpaz, 2008). Such mediator teachers are better teachers because they not only know the material and how to transmit it well, they understand the cognitive platform on which the knowledge is based and are able to design lesson plans that consider students' needs (Keiny, 1994). Teachers must serve as role models when they guide students through the material.. By expanding on the basic curriculum and by assigning questions that go beyond it teachers advance students' thinking and develop their knowledge.

The Movshovitz-Hadar (2018) study notes that to teach mathematics successfully requires a combination of raising and maintaining student motivation, seeking excellence, handling difficulties, having suitable teachers, addressing language problems, and more. Indeed mathematics teachers must both practice as well as keep up to date with innovative research and policies in mathematics education in the international community.

The findings regarding teachers' declared beliefs indicate that the teachers in the first group were committed to self-directed learning (Michlasky & Kramarsky, 2008). These teachers emphasized that their goal was to develop intellectual skills in their mathematics students and explained that in order to do this they use topics that are relevant to their students' lives and set them problems which direct them to in depth thinking. The teachers pointed out that it is important to relate to each student according to his or her ability. Findings also show that they attempted to stimulate in their students the understanding that there is more than one way to solve a problem (Bulotsky-Shearer & Fernandez, 2011).

Getting the thinking trajectories of teacher and student to intersect is the heart of self-directed learning. This meeting is dependent on the teacher's awareness that each student may think differently, based on different intuitive knowledge and unique ways of thinking (Brooks and Brooks, 1997). The teacher is a guide in the maturation of cognitive, social and personal processes and in this way helps develop learners who are able to identify their own learning goals, weigh a choice of strategies, and produce self and group feedback in order to advance their learning over time (Birenboim, 1997; Tishman, Perkins & Jay, 2000).

This study's findings about teachers' stated convictions regarding teaching-for-thinking show that the teachers tried to develop in their students learning patterns that are conducive to thinking, investigating, and using their imagination while creating a supportive learning environment (Tishman & Perkins, 2000).

Conscious self-direction requires conscious thinking and indeed the teachers encouraged their students to think about their thinking thereby raising their students to a higher level of metacognition. The teachers also expanded on the official curriculum and asked questions to foster students' thinking while still allowing them sufficient time to independently arrive at an organized answer.

The findings from this study can be used to outline a constructivist model for staff development programs in self-directed learning in mathematics. Such a program would establish two central goals. The first is the creation of a shared knowledge-base about self-directed learning which can be achieved by exposing teachers to the fundamental concepts and overall philosophy of self-directed learning, and how it affects teaching and learning. As part of the program, participants will learn about the physical and human factors that make a learning environment that fosters self-directed learning (as described in the review of the literature above). According to Marshall (1998), it is important to help teachers become more aware of the content and processes of instruction and of how these are relevant to their student's cultural context. Analyzing events in class will allow a discourse on the physical and human teaching-learning environments that promote self-direction in learning (Zimmerman, 2001).

McGrath (2015) proposes teaching mathematics using story telling. He notes that stories can inspire both mathematics teachers and students. This could make it possible to use stories to foster children's mathematical thinking. He notes the potential association between story and mathematics and gives practical illustrations of how stories can be used to help children connect to, express, and understand mathematical concepts through the language of stories. Also Ashkenazi (2015) emphasizes the self-directed route in teaching mathematics, using fun activities and games to improve mathematics perception in school children. Jaquith (2015) focuses on art as a means of developing thinking, promoting motivation, and developing mathematics teaching skills. Art is a more flexible means of helping students understand and internalize the material**.**

Teaching mathematics and fostering thinking

The second goal of staff development programs should be to develop the ability to critically analyze and evaluate texts for aspects of self-directed learning, with an emphasis on the principles of the constructivist approach in mathematics classes. Lesson plans that apply constructivist theory ensure the advancement of students as individuals and as members of a group thus promoting the independent learner. As part of the training participants should be given opportunities to analyze peer experiences, and reflect with their peers on their own experiences. In this way, participants build meaning and insight into their experience in the practice classroom (Emanuel, 2003).

Schools whose teachers foster self-direction in learning can become places where students are encouraged to hypothesize, test their own and others' ideas, create links between disciplines, investigate problems, collaborate, and become lifelong learners. The teachers' role in this setting will change accordingly: teachers will no longer be bound by their historical role as transmitters of information, but rather initiate and provide opportunities for learning that enable students to understand on their own (Birenboim, 1997). Teachers should motivate learners, guide them, and shape a learning environment that inspires them to construct and cultivate their knowledge. In order to implement these assumptions, teachers must adopt a renewed perception of their role and become the initiators and navigators of learning opportunities, enabling their students to think and self-direct their learning (Brooks & Brooks, 1997; Yee, 2005).

However, mathematics is considered one of the most difficult subjects to teach, meaning it is harder for teachers to find the correct route to their students' minds, one that sparks thinking and promotes understanding. Teachers are also greatly challenged by the diversity in abilities among students, which further complicates decisions about time allocation and the use of demonstration tools.

To boost thinking and self-directed learning in mathematics we must integrate pedagogy and content knowledge in our teaching. This is mandatory for building students' confidence and giving them both the skills and knowledge they need. In this way we can successfully teach students of all ages (Cotton, 2016). A study conducted among students with learning disabilities showed how important the mathematical discourse is between students and teacher during mathematics classes. When correct mathematical terms are used, they can aid understanding, promote thinking and boost achievements (Rozinfeld, 2016).

Study limitations and future research

The limited scope of the present study dictated a qualitative approach. Further quantitative research is suggested to examine intervention programs based on the proposed staff development program model. In these follow-up studies, the ways in which socio-mathematical norms emerge in the classroom, and the ways in which the teaching-learning environment affects the cognitive, behavioral, and social-emotional contexts should be examined.

References

Apatow, R. (1999). Socratic dialogue*. Executive Excellence*, *16*(5), 10-11.

Ashkenazi, Y. (2015). *Learning math through play*. Ariel: Ariel University (In Hebrew).

Ball, D. L. (2000). Bridging practices: Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education***,** *51*(3), 241-247.

Ben-Eliyahu, A., & Linnenbrink-Garcia, L. (2015). Integrating the regulation of affect, behavior and cognition into self-regulated learning paradigms among secondary and post-secondary students. *Metacognition Learning, 10*(1)*,* 15-42.

Birenboim, M. (1997). *Alternatives in Assessing Achievements*. Tel Aviv: Ramot (In Hebrew).

Brooks, M. and Brooks, G. (1997). *Towards Constructivist Teaching*. Jerusalem: The Branco Institute Weiss (In Hebrew).

Brooks, M. G., & Brooks, J. (1999). The Courage to be constructivist. *Educational Leadership, 57*(3), 18-24.

Bulotsky-Shearer, R. & Fernandez, V. (2011). Behavior problems in learning activities and social interactions in head start classrooms and early reading mathematics, and approaches to learning. *School Psychology Review*. v. 40, no. 1. 39-56.

Clarke, D.M., & Clarke, B.A. (2003). Encouraging perseverance in elementary mathematics: Tale of two problems. *Teaching Children Mathematics, 10(4),* 204-209.

Clements, D. (2000). From exercises and tasks to problems and projects: unique contributions of computers to innovative mathematics education. *The Journal of Mathematical Behavior*. v. 19. 9-47.

Cobb, P., Yackel, E., & McClain, K. (2000). *Symbolizing and Communicating in Mathematics Classrooms*. New York and London: Routledge.

Cotton, T. (2016). *Understanding and teaching primary mathematics*. Abingdon, Oxfordshire: Routledge.

Eilam, B., & Reiter, S. (2014). Long-term self-regulation of biology using standard junior high school science curriculum. *Science Education, 98*(4), 705-737.

Emanuel, D. (2003). *Self-Direction in Learning Challenge for School Growth: A Description of Action Research.* Tel Aviv: Ramot (In Hebrew).

Factors that support and inhibit .*Thinking Classroom, 6(4),* 14-46.

Fogarty, R. (1991). Ten models of integrative curriculum. *Educational Leadership,* 49*.*

Harpaz, Y. (2008). *The Third Model: Teaching and Learning in A Thinking Community*. Tel Aviv: Poalim Library (In Hebrew).

Hirst, J. & Hughes, N. (2015). Reverse mathematics and marriage problems with unique solutions. *Mathematical Logic, 54*. 49-57.

Jaquith, D. (2015). *The Learner directed classroom: developing creative thinking skills through art.* New-York: Teachers College Press.

Keiny, S. (1994). Constructivism and teachers professional development. *Teaching & Teacher Education*, *10*(2), 157-167.

Marshall, H. (1998). Teaching educational psychology: Learner-centered constructivist perspectives. In N. M. Lambert, & B. L. McCombs (Eds.), *How Students Learn: Reforming Schools Through Learner-Centered Education* (449- 474). Washington, DC: Library of Congress.

McClain, K., & Cobb, P. (2001). An analysis of development of socio-mathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education, 32*(3), 236-266.

McGrath, C. (2015). *Teaching mathematics through story: a creative approach for the early years.* London: Routledge.

Michlasky, T. and Kramarsky, B. (2008). Cultivation of self-direction in learning among teaching students in the environment in relation to perceptions of learning and teaching. *Megamot, 4 (4),* 798-765 (In Hebrew).

Ministry of Education (2013). *Planning of Mathematics Studies in the High School*. Jerusalem: Ministry of Education (In Hebrew).

Movshovitz-Hadar, N. (2018). *K-12 mathematics education in Israel: issues and innovations.* Singapore: World Scientific Publishing.

Piaget, J. (1972). *The Child's Worldview*. Tel Aviv: Poalim Library (In Hebrew).

Rozinfeld, M. (2016). *Fostering mathematical thinking through discourse with students with learning disabilities.* Beit Berl College (In Hebrew).

Sabar-Ben Yehoshua, N. (2001). *Traditions and Trends in Qualitative Research*. Lod: Dvir (In Hebrew).

Salomon, G. (2003). A constructivist teacher in a constructivist school. *Opinions, 7*, 29-26 (In Hebrew).

Shlasky, S. & Alpert, B. (2007). *Ways of writing qualitative research*. Tel-Aviv: Mofet institute (In Hebrew).

Tishman, S., Perkins, D. N. & Jay, E. (2000). *The Thinking Classroom: Learning and Teaching in A Culture of Thinking*. Boston: Allyn and Bacon.

Vygotsky, L. S., (1962). *Thought and language*. Cambridge, MA: The MIT Press.

Watts, L. (2017). *Creative Learning in higher education: international perspectives and approaches*. New-York: Routledge.

Yee, F.P. (2005). Developing creativity in the Singapore primary mathematics classes:

Yehieli, T. (2008). How to construct constructivist learning. *Hed Hahinuch, 82 (4),* 44-40 (In Hebrew).

Zimmerman, B. J. (2001). Theories of self-directed learning and academic achievement: An overview and analysis. In B. J. Zimmerman, & D. H. Schunk (Eds.), *Self-Regulated Learning and Academic Achievement* (1-37). New York: Lawrence Erlbaum Associates.