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Collaborative learning of differential equations by numerical simulation

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Abstract

This paper presents a didactic proposal designed through the active methodology of collaborative learning to analyse the effect of the use of numerical simulation of a mathematical model on the learning of differential equations in engineering students. A mathematical model of a vibrating string was used, and the Octave Online platform was used for the numerical simulation. The analysis and assessment of this proposal were carried out in a hybrid, quantitative and qualitative manner, through the design of observation and measurement instruments. The results indicate that, in science, technology, engineering and mathematics programmes, the use of numerical simulation in a collaborative learning environment strengthens student learning and fosters the development of disciplinary and soft skills.

Keywords: Collaborative learning, differential equations, Mathematics education, numerical simulation, STEM skills.

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1. Introduction

Higher education institutions periodically update their educational programmes and improve the design of learning environments with the purpose of fostering in their students the development of competencies and skills demanded by the exercise of their profession (Ananiadou & Claro, 2009; UNESCO, 1998). Therefore, these updates include innovative methods that allow students to go beyond mere cognitive mastery in their discipline, adapting information and communication technologies (ICT), as well as virtual systems with new pedagogical and didactic approaches that facilitate learning (UNESCO, 2015).

In engineering education programmes, one way to know if the updating was adequate is through compliance with internationally accepted standards in the Washington Accord (IEA, 2020), through criteria and parameters evaluated by accreditation bodies (ABET, 2020; CACEI, 2020; CEAB, 2020). In the criterion of student outcomes or attributes, it is considered that during their university career, they develop hard skills (methods, approaches and knowledge to identify, formulate and solve complex problems specific to the discipline) and soft skills (teamwork, effective communication, leadership, organisation and critical thinking).

For this purpose, in science, technology, engineering and mathematics (STEM) programmes, depending on the type of course, methodological innovations are implemented to develop these competencies, which, in turn, have a positive effect on the teaching and learning processes. The most used active methodologies in STEM are problem-based learning, project-oriented learning (POL), collaborative learning (CL), gamification, competency-based learning and flipped classroom, among others (Beier et al., 2018; Capraro & Slough, 2013; Hernández et al., 2016; Lara-Quintero et al., 2017; Nunez-Lopez et al., 2017; Rehmat & Kendall, 2020; Tsichouridis et al., 2020; Zastavker et al., 2006).

All of them have their own characteristics in their actions, or some analogies between them, so that the students are the protagonists of their own learning and, in contexts similar to the workplace, are involved in processes of analysis, reasoning, relationship, debate and discussion of a topic. For example, it is evident that using the POL or the CL causes an interdependence among the members of a team, who have different skills and interests but share the responsibility of obtaining a product.

Theoretical framework

The theoretical foundations of this research are linked to the psychological theory of Vygotsky (1978, 2012), which considers that in order to understand the individual it is necessary first of all to understand the social relations in which the individual develops, emphasising that the individual emerges from the collective. For Vygotsky, all mental functions occur first among people in social interactions and then at the psychological level of the individual. Furthermore, in order to know the ontogenesis of cognitive functions, it is necessary to observe the individual in interaction with the most expert in their culture and to study how he appropriates these interactions and internalises them.

The internalisation of concepts is not achieved through only one of their representations; it requires the use of all the forms of expression that the concept involves. Research conducted with students shows that learning designs that emphasise only the conceptual, for example, accompanied by typical exercises, do not achieve internalisation. On the other hand, the role of mediators, according to Vygotsky, is to collaborate in the formation of human consciousness and mental processes. The development of consciousness and mental processes depends on social interaction, and this necessarily involves signs as mediating mechanisms.

Related research

In teaching and learning mathematics through didactic activities such as mathematical modelling, the student develops some hard competencies and meaningful learning when the modelling is of real systems observed in everyday life or their professional practice (Lopez-Reyes et al., 2018). In addition, learning theories with approaches based on social interaction point out that the use of mediators contributes to generating an environment conducive to forming new concepts in the student (John-Steiner, 1995; Wertsch, 1991). In this sense, the potential of ICT through a virtual platform can originate the necessary interactions to foster some soft skills, individual and group, with the discussion of new concepts as proposed by the CL (Panitz, 1999).

Within a mathematical modelling context, the student has very important challenges for learning ordinary differential equations (ODEs), from the understanding of the meaning of each term in ODEs

(Rowland & Jovanoski, 2004), the dimensional analysis of parameters and variables (Rowland, 2006), the ODE resolution itself and its respective error analysis (Barbaran-Sanchez & Fernandez-Bravo, 2014) to the difficulties in the interpretation of ODE solutions (Guerrero-Ortiz et al., 2010). It is not easy for a single teacher to cover all these aspects, hence the importance of experimenting with active methodologies such as CL and didactic proposals that support students to navigate through these complex conceptual and cognitive processes (Camacho-Machin et al., 2012).

Nowadays, numerical simulation of mathematical models is an essential technique in science and engineering, placed at the same level of importance as theoretical and experimental studies (Wouwer et al., 2014). Supported by high-performance computing and scientific software, it is used for sensitivity analysis, estimation of unknown parameters or state variables and the design of control schemes, in general, in the study of the dynamics of a system. The numerical simulation of mathematical models using Octave, a free software with platforms on the web and for mobile devices, can benefit teachers and university students who wish to integrate it into the teaching and learning processes of ODEs (Peramunugamage et al., 2020).

The vibrating string is a real physical system that students in STEM programmes can observe in everyday life in stringed musical instruments or their professional practice in sensors to measure different variables or parameters in industrial processes and even in special installations (Arutunian et al., 2007; Mei et al., 2016). Through its mathematical model and its corresponding numerical simulation, the analysis of this system is fundamental for the study of vibrations of continuous systems and complex wave phenomena. In the framework of this mathematical education research, the vibrating string is the reference topic for the didactic proposal presented herein; however, for the learning of ODEs through numerical simulation, there are several topics and software to induce students in a CL environment (Lin et al., 2020; Marani & Perri, 2016).

Purpose of the study

A didactic proposal was implemented and evaluated in this research on mathematics education. It aimed to analyse the effect of numerical simulation of a mathematical model of a vibrating string in a CL environment, using the Octave Online platform, on the learning of ODE by students in STEM programmes.

Method

The evaluation of the didactic proposal was hybrid, quantitative and qualitative (Hernandez-Sampieri & Mendoza-Torres, 2018), for which observation and measurement instruments were developed, such as tests, a rubric and a questionnaire. A pre-experimental design (pre-test, treatment and post-test) was carried out to observe if there was a significant effect on ODE learning.

The didactic proposal consisted of the following activities: i) Session 1 – Formation of work teams. Knowing the generalities of Octave Online and practicing the necessary commands (algebra, calculus, ODE and graphing) to simulate the model in the platform; ii) Session 2 – Developing a mathematical model of a vibrating string also obtaining its analytical solution; iii) Session 3 – Performing the numerical simulation of the mathematical model obtained in the previous session; iv) Session 4 – Comparing both results and analysing the dynamics of the system; v) Session 5 – Presentation of the results. During all sessions, the students worked in teams, except during the second session, which was carried out with the teacher's support for the development of the mathematical model.

Research population and sample

The study population consisted of undergraduate students enrolled in STEM programmes. Two groups formed the sample with different profiles registered in differential equations: a control group with 44 students and an experimental group with 34 students; for university administrative reasons, the teacher did not intervene in the selection of the sample. Due to the students' background, in this research, it was considered that the sample members met the assumptions of having had previous contact with ODE and could work without problems in a friendly computing environment.

During the experiment, the control group focused on studying the mathematical model and the analytical solution of the ODE involved. While the treatment in the experimental group began by presenting the purpose of the didactic activity, highlighting the importance of mathematical modelling within STEM programmes and the potential of numerical simulation to understand the dynamics of a system comprehensively, it was explained that the activity would be using Octave Online through an institutional email account; none of the students knew the software.

Data collection tools

The quantitative analysis was carried out by applying the pre-test in both groups to measure basic disciplinary knowledge such as the classification of an ODE or examples of mathematical models. The students who made up the groups had previously studied first- and second-order ODEs and the basic techniques for their analytical solution; for this reason, the application of the pre-test to both groups was justified to have a starting point for the evaluation of the subject. A post-test was applied to identify students' understanding of the meaning of terms or behaviour of the solution in the mathematical model. The questions that integrated the pre-test and post-test are convergent type (multiple choice).

The qualitative evaluation was carried out by applying a rubric to evaluate the level of achievement in soft skills, such as preparing a report, teamwork or communication, and also through a questionnaire applied to the experimental group students to evaluate the activities performed.

Data analysis

A statistical analysis of variance and comparison of means with Student's t-test was applied to the data observed and collected during the experiment, using SPSS Statistics 25 software. The qualitative evaluation of the activities performed by the experimental group was carried out using a rubric. The evaluation of the didactic proposal was carried out with feedback provided by the experimental group students through a questionnaire.

Results

As a starting point, the pre-test results were used to verify compliance with the assumptions of normality and equality of variance, applying Kolmogorov–Smirnov test (see Table 1) and Levene's test (see Table 2), respectively. The assumptions are met because p-value > 0.05; i.e., the pre-test data have a normal distribution, and there is equality of variances in both groups. Then, considering independent samples and based on the result of the t-test, it is verified that there are no significant differences between the groups (see Table 2); this means that at the beginning of the experimentation, the level of prior knowledge of the ODEs was homogeneous; therefore, there was no difference in the academic background that influenced the result of the treatment applied later.

Table 1. Normality test on the pre-test results of both groups.

Kolmogorov-Smirnov Test for a sample			
Pre-test		*Control*	*Experimental*
N		44	34
Normal Parameters	Mean	67.95	70.15
	Deviation	15.224	16.582
Maximum extreme differences	Absolute	.128	.115
	Positive	.072	.068
	Negative	-.128	-.115
Test statistic		.128	.115
Sig. asymptotic (bilateral)		.070	.200

Table 2. Test of equality of variances in the pre-test results of both groups.

Independent samples' test									
	Levene's test for equality of variances		t-test for equality of means						
	F	Sig.	t	df	Sig. (bilateral)	Difference of means	Std error	95% confidence interval of the difference	
								Lower	Upper

Pre-test	Equal variances are assumed	.352	.555	-.607	76	.546	-2.193	3.614	-9.391	5.006
	Equal variances are not assumed			-.600	67.885	.551	-2.193	3.654	-9.485	5.100

After the treatment to the experimental group and post-test applied to both groups, the statistics for mean and standard deviation in the control group (74.77 with 12.293) and the experimental group (83.82 with 13.487) were observed; in addition, with a calculation of confidence intervals and a boxplot, more information is obtained from the experiment. However, for greater formality, an analysis of variance was applied, assuming the post-test as the dependent variable, with type of group as a fixed factor, and the pre-test as a covariate (see Table 3). Regarding the p-value of each factor, if it is less than 0.05, it is concluded with a confidence level of 95% that this factor has a significant effect on the result; therefore, it is observed that the group factor has a significant effect on the result of the treatment. Subsequently, an analysis of the post-test provided valuable information on which items were easier for one group than for the other (see Table 4).

Table 3. Analysis of variance of the post-test

Tests of inter-subject effects

Dependent variable: Post-test

Source	Type III sum of squares	df	Quadratic mean	F	Sig.
Corrected model	1605.975[a]	2	802.988	4.831	.011
Intersection	25679.809	1	25679.809	154.501	.000
Pre-test	34.849	1	34.849	.210	.648
Group	1596.132	1	1596.132	9.603	.003
Error	12465.820	75	166.211		
Total	497400.000	78			
Corrected total	14071.795	77			

a. R squared = .114 (R squared adjusted = .091)

Table 4. Percentage of correct answers to the post-test.

	Questions					
	1	*2*	*3*	*4*	*5*	*6*
Control	55%	44%	39%	65%	10%	63%
Experimental	70%	62%	84%	78%	35%	72%

The rubric results showed that the sequence of activities with CL methodology using online platforms as didactic support generates an environment conducive to ODE learning. Adequate development of soft skills was observed in students, such as solving as a team the deficiencies of some with the strengths of others, showing during the sessions close collaboration in the planning of individual and team activities, communication among its members and adapting very quickly to the use of the software that served as a mediator in the development of the experiment. Likewise, it is perceived that the role played by the teacher was modified in the experimental group because the questions asked by the students were focused on conceptual questions, being less the operative ones (those referring to the use of the software).

With regard to evaluation of the didactic proposal, most students found the numerical simulation easy using the online platform; on the other hand, they considered that the mathematical modelling did contribute to making the learning of ODEs easier; in addition, they thought that the model used was adequate for the course. It is worth mentioning that a significant percentage of the students indicated that the time spent analysing the mathematical model and its numerical simulation was not enough; they indicated that they would have liked more time to practice with Octave Online and continue using it for more topics in the ODE course itself or others in their career. This opinion is interpreted as reflecting that many students had not previously used this tool as didactic support in mathematics classes. Only a small percentage of students commented that it was better if all the activities were worked on individually.

Discussion and conclusion

In summary, it can be concluded that the objective of experimenting and evaluating a didactic proposal for ODE learning through numerical simulation of a mathematical model was achieved. In response to the question 'How does the use of numerical simulation influence students' collaborative learning of ODE?' it was observed that there was a statistically significant difference between the experimental group and the control group. This difference is precisely due to the effect produced by different methodologies in the study of ODEs and that the covariate (pre-test) did not significantly influence the final performance; i.e., the students' academic background in the sample was not a determinant in the result.

The following can be highlighted from the results obtained: i) teachers should carry out continuous research based on their reflections on teaching that make possible the structuring of an adequate learning environment to help students raise their potential for understanding mathematical concepts; ii) the active methodology known as collaborative learning, used for the development of the didactic proposal, significantly promoted the interaction between the students themselves, students–media and students–teacher; iii) the use of new technologies improves the learning process in activities such as the numerical simulation of mathematical models; however, the teacher must select the topics that can be worked on with the use of this resource; iv) mathematics teachers should make a reflection related to their performance within the classroom and if it meets the institutional and personal objectives of the student, it should be considered, therefore, to incorporate the use of media and materials in the teaching of mathematics, to make the teaching practice a more fluid and meaningful process for students; v) it is proposed that in the subsequent updates of the mathematics courses, didactic activities with the use of diverse active learning methodologies, be incorporated within the study programmes; vi) finally, it is advisable to take full advantage of the online platforms available for the teaching and learning of mathematics since they can have an impact on a better performance of the students and thus propitiate that their theoretical support is solid enough to link mathematics to the physical sciences and engineering.

References

ABET. (2020). Criteria for accreditation engineering program. Accreditation Board for Engineering and Technology. https://www.abet.org/accreditation

Ananiadou, K., & Claro, M. (2009). 21st century skills and competences for new millennium learners in OECD countries. OECD Education Working Papers, 41, 33. https://doi.org/10.1787/218525261154.

Arutunian, S. G., Mailian, M. R., & Wittenburg, K. (2007). Vibrating wires for beam diagnostics. Nuclear Instruments and Methods in Physics Research A, 572(3), 1022–1032. https://doi.org/10.1016/j.nima.2006.12.043

Barbaran-Sanchez, J. J., & Fernandez-Bravo, J. A. (2014). El analisis de errores en la resolucion de ecuaciones diferenciales ordinarias. Una metodologia para desarrollar la competencia matematica. Ensenanza de Las Ciencias, 32(3), 173–186. https://doi.org/10.5565/rev/ensciencias.1122

Beier, M., Kim, M., Saterbak, A., Leautaud, V., Bishnoi, S., & Gilberto, J. (2018). The effect of authentic project-based learning on attitudes and career aspirations in STEM. Journal of Research in Science Teaching, 56(1), 3–23. https://doi.org/10.1002/tea.21465

CACEI. (2020). Marco de Referencia 2018. Consejo de Acreditacion de La Ensenanza de La Ingenieria.

http://www.cacei.org/docs/marco_ing_2018.pdf

Camacho-Machin, M., Perdomo-Diaz, J., & Santos-Trigo, M. (2012). Procesos conceptuales y cognitivos en la introduccion de las ecuaciones diferenciales ordinarias via la resolucion de problemas. Ensenanza de Las Ciencias, 30(2), 9–32. https://www.raco.cat/index.php/Ensenanza/article/view/254501/391048

Capraro, R. M., & Slough, S. W. (2013). Why PBL? Why STEM? Why now? An introduction to STEM project-based learning: An integrated science, technology, engineering, and mathematics approach. In R. M. Capraro, M. M. Capraro, & J. R. Morgan (Eds.), STEM project-based learning (2nd ed., pp. 1–7). Sense Publishers. https://doi.org/https://doi.org/10.1007/978-94-6209-143-6

CEAB. (2020). Accreditation criteria and procedures. Canadian Engineers Accreditation Board. https://engineerscanada.ca/accreditation/accreditation-resources

Guerrero-Ortiz, C., Camacho-Machín, M., & Mejia-Velasco, H. R. (2010). Dificultades de los estudiantes en la interpretacion de las soluciones de ecuaciones diferenciales ordinarias que modelan un problema. Ensenanza de Las Ciencias, 28(3), 341–352. https://raco.cat/index.php/Ensenanza/article/view/210804

Hernandez-Sampieri, R., & Mendoza-Torres, C. P. (2018). Metodología de la Investigacion: las rutas cuantitativa, cualitativa y mixta. (1st ed.). Mc Graw-Hill Interamericana.

Hernández, A., Luna, S., & Arellano, J. (2016). La formacion integral de estudiantes de ingeniería mecanica y electrica para desempenarse en contextos globalizados. ANFEI Digital, 5. http://www.anfei.org.mx/revista/index.php/revista/article/view/317

IEA. (2020). Washington accord. International Engineering Alliance. https://www.ieagreements.org/accords/washington/

John-Steiner, V. (1995). Spontaneous and scientific concepts in mathematics: A Vygotskian approach. In L. Meira & D. Carraher (Eds.), 19th Annual Conference of the International Group for the Psychology of Mathematics Education. (pp. 30–44). PME. http://www.igpme.org/publications/current-proceedings/

Lara-Quintero, V., Avila-Palet, J. E., & Olivares-Olivares, S. L. (2017). Desarrollo del pensamiento crítico mediante la aplicacion del Aprendizaje Basado en Problemas. Psicologia Escolar e Educacional, 21(1), 65–77. https://doi.org/10.1590/2175-3539201702111072

Lin, Q., Zhao, S., Gao, D., Lou, Y., Yang, S., Musa, S. S., Wang, M. H., Cai, Y., Wang, W., Yang, L., & He, D. (2020). A conceptual model for the coronavirus disease 2019 (COVID-19) outbreak in Wuhan, China with individual reaction and governmental action. International Journal of Infectious Diseases, 93, 211–216. https://doi.org/10.1016/j.ijid.2020.02.058

Lopez-Reyes, L. J., Jimenez-Gutierrez, A. L., Lara-Ramirez, L. E., & Costilla-Lopez, D. (2018). Mathematical modeling in STEM programs: The vibrating string. Journal-Mathematical and Quantitative Methods, 2(2), 25–31. https://rinoe.org/spain/mathematical_and_quantitative_methods_articles.html

Marani, R., & Perri, A. G. (2016). A simulation study of analogue and logic circuits with CNTFETs. Journal of Solid State Science and Technology, 5(6), 38–43. https://doi.org/10.1149/2.0241606jss

Mei, B., Lucas, J., Hole, S., Lamarque, I., & Cheron, N. (2016). Excitation mode influence on vibrating wire sensor response. Experimental Mechanics, 56, 331–338. https://doi.org/10.1007/s11340-015-0101-6

Nunez-Lopez, S., Avila-Palet, J. E., & Olivares-Olivares, S. -L. (2017). El desarrollo del pensamiento crítico en estudiantes universitarios por medio del Aprendizaje Basado en Problemas. Revista Iberoamericana de Educación Superior, 8(23), 84–103. http://www.scielo.org.mx/scielo.php?script=sci_arttext&pid=S2007-28722017000300084

Panitz, T. (1999). Collaborative versus cooperative learning: A comparison of the two concepts. https://eric.ed.gov/?id=ED448443

Peramunugamage A., Ratnayake H. U. W., Karunanayaka S. P., & Halwatura, R. U. (2020). Designing online learning activities for collaborative learning among engineering students. In M. Auer, H. H, Hortsch, & P. Sethakul (Eds.), The impact of the 4th industrial revolution on engineering education. Springer. https://doi.org/http://dx.doi.org/10.13140/RG.2.2.16241.22882

Rehmat, A. P., & Kendall, H. (2020). Building engineering awareness: Problem-based learning approach for STEM integration. The Interdisciplinary Journal of Problem-Based Learning, 14(1), 1–15. https://doi.org/10.14434/ijpbl.v14i1.28636

Rowland, D. R. (2006). Student difficulties with units in differential equations in modelling contexts. International Journal of Mathematical Education in Science and Technology, 37(5), 553–558. https://doi.org/10.1080/00207390600597690

Rowland, D. R., & Jovanoski, Z. (2004). Student interpretations of the terms in first-order ordinary differential equations in modelling contexts. International Journal of Mathematical Education in Science and Technology, 35(4), 503–516. https://doi.org/10.1080/00207390410001686607

Tsichouridis C., Batsila M., Vavougios D., & Tsihouridis, A. (2020). From an inquiry-oriented instruction to the connectivist approach to science teaching for the 21st century learners. In M. Auer, H. Hortsch, & P. Sethakul (Eds.), The impact of the 4th Industrial Revolution on Engineering Education. ICL 2019. Springer. https://doi.org/http://dx.doi.org/10.1007/978-3-030-40274-7_40

UNESCO. (1998). Higher education in the twenty-first century: Vision and actions. World Conference on Higher Education. https://unesdoc.unesco.org/ark:/48223/pf0000113878

UNESCO. (2015). Education 2030: Incheon declaration and framework for action. World Education Forum. https://unesdoc.unesco.org/ark:/48223/pf0000245656

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. In(M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, (Eds.). Harvard University Press.

Vygotsky, L. S. (2012). Thought and language. In A. Kozuli(n (A. Kozulin, Ed.). The MIT Press.

Wertsch, J. V. (1991). Voices of the mind: A sociocultural approach a mediated action. Harvard University Press.

Wouwer, A. Vande, Saucez, P., & Vilas, C. (2014). Simulation of ODE/PDE models with MATLAB®, OCTAVE and SCILAB: Scientific and engineering applications. Springer. https://doi.org/10.1007/978-3-319-06790-2

Zastavker, Y. V., Ong, M., & Page, L. (2006). Women in engineering: Exploring the effects of project-based learning in a first-year undergraduate engineering program. Frontiers in Education. 36th Annual Conference, 1–6. https://doi.org/10.1109/FIE.2006.322485